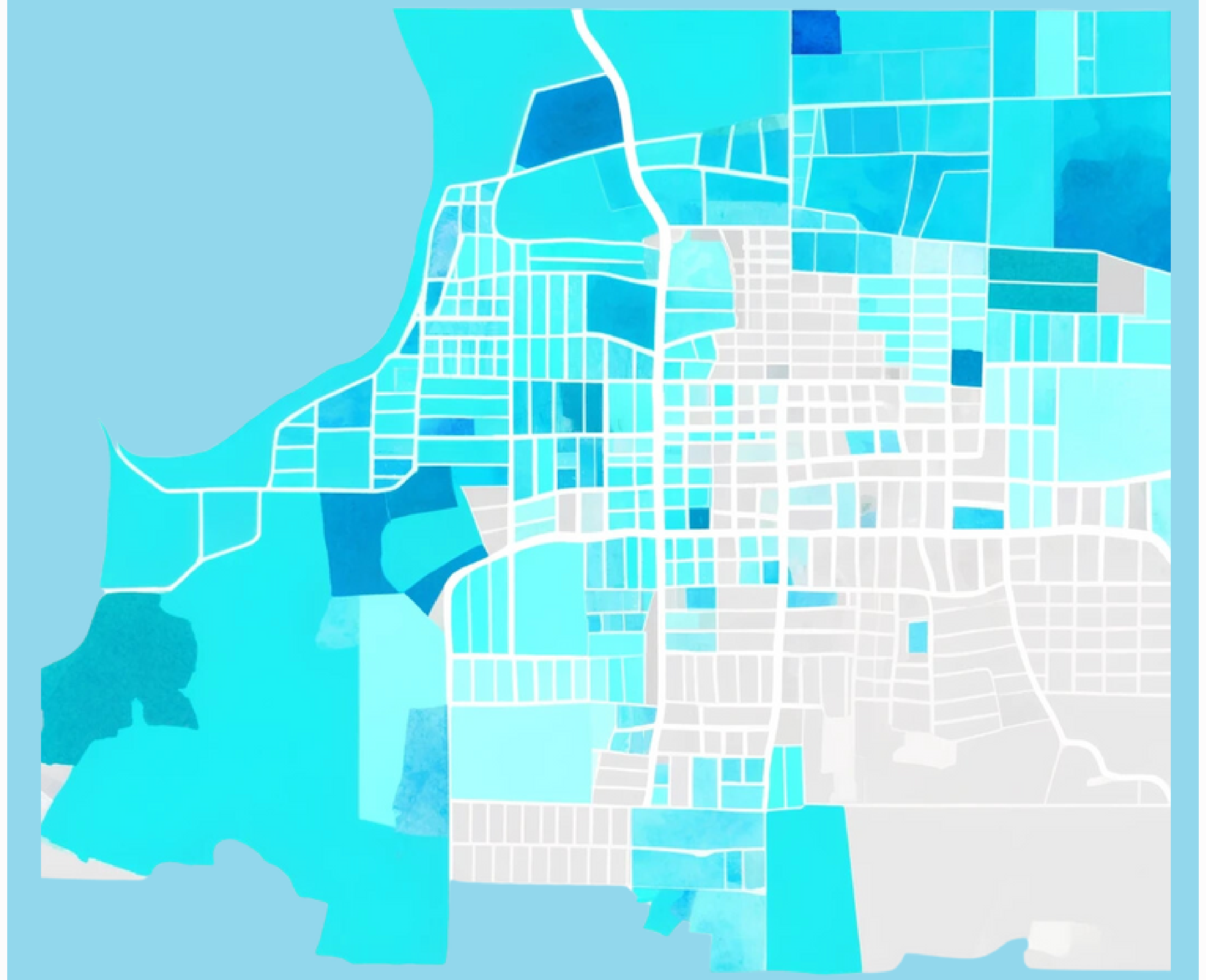


UNIVERSITÄT KONSTANZ

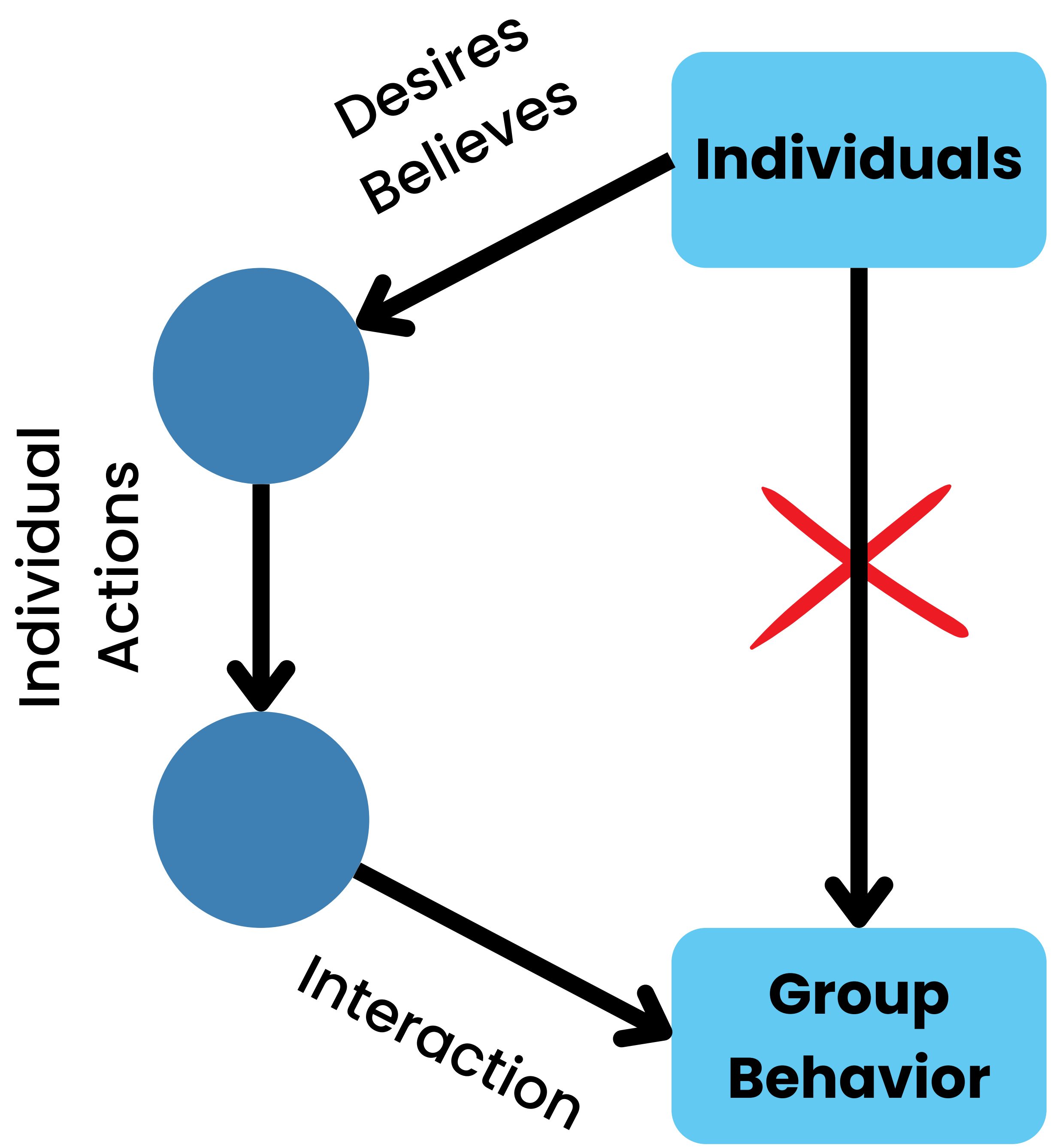
Modelling Segregation: Schelling's Model

Computational Modelling of Social Systems

Giordano De Marzo
Max Pellert

Universität Konstanz

Recap



Emergence of Complex Social Behavior

- Humans behave differently in groups as in isolation: collective behavior emerges spontaneously

Agent-Based Modelling (ABM)

- A computational approach to formalize and analyze social systems

ABM Example: Date Choice Model

- A simple model shows that seeking attractiveness in a finite dating pool generates the observed correlations in couples

Outline

1. Segregation
2. Schelling's segregation model
3. Analyzing Schelling's model
4. Cellular Automata: The game of life

Segregation

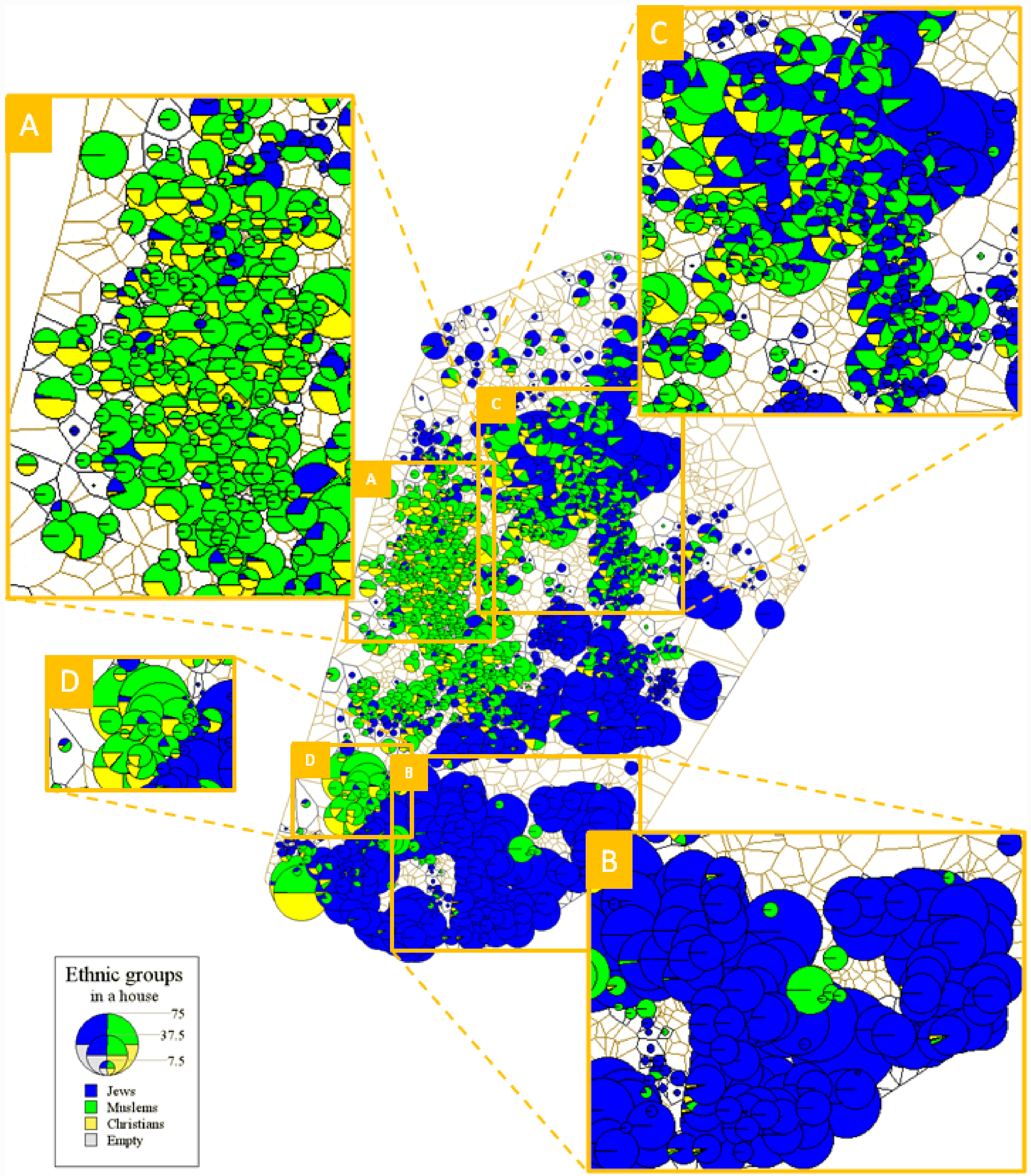
Urban
Segregation

Definition of Urban Segregation

Urban segregation is the unequal distribution of different social groups in the urban space, based mainly on occupation, income and education, as well as on gender and ethnicity.

The Future of Cities. European Research Comission

Rome at Night



Segregation in Jaffa, Israel

- Neighborhood units with shares of **ethnic groups** as pie charts
- Share of Jewish, Muslim, and Christian inhabitants
- **Segregation**: Jewish vs non-Jewish areas
- Observation: some areas are less segregated (area C)

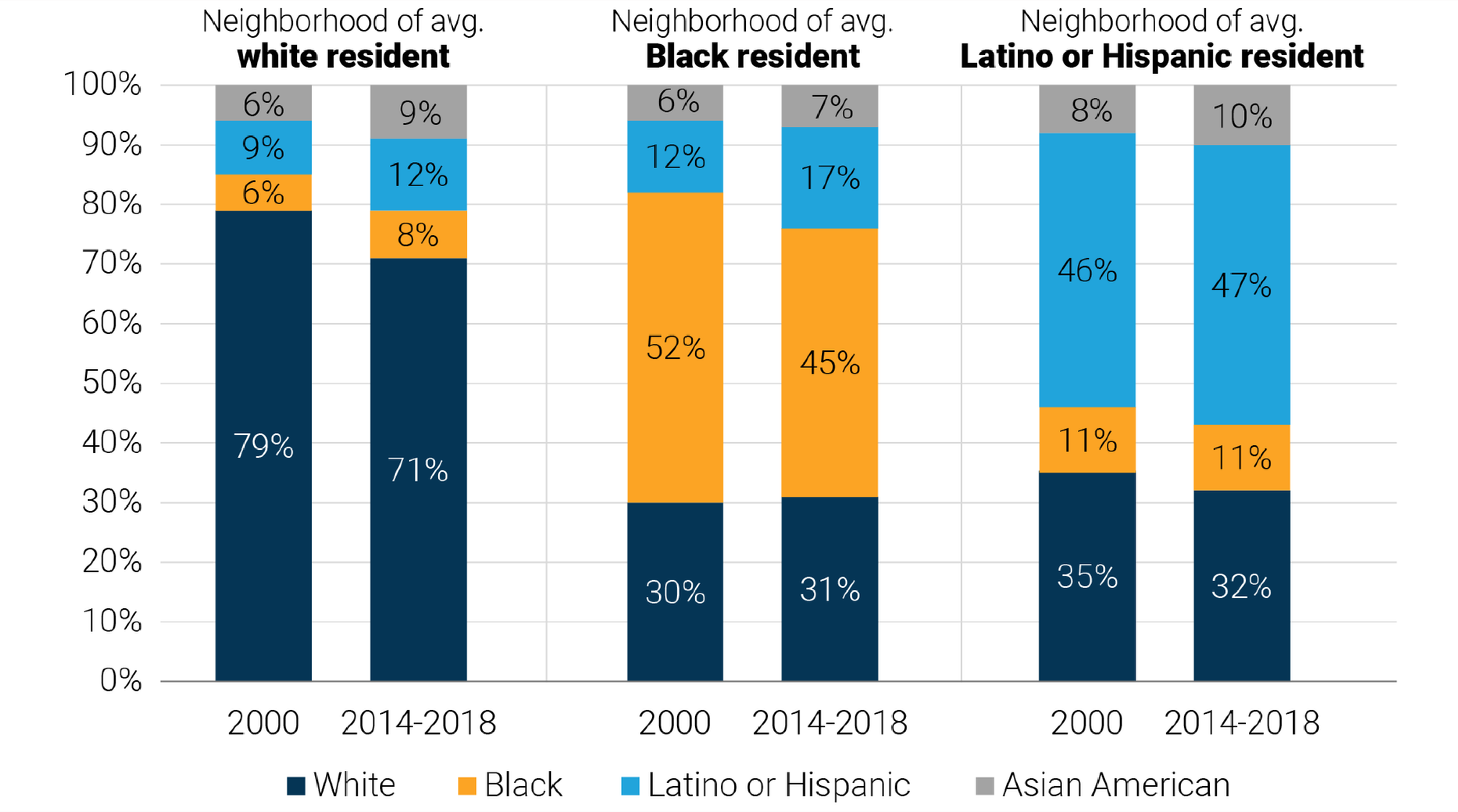
[Hatna & Benenson. JASSS, 2012](#)

Segregation in the District of **Columbia**

Since 1990, more than 90 percent of U.S. metro areas have seen a decline in racial stratification … yet cities like Detroit and Chicago still have large areas dominated by a single racial group.

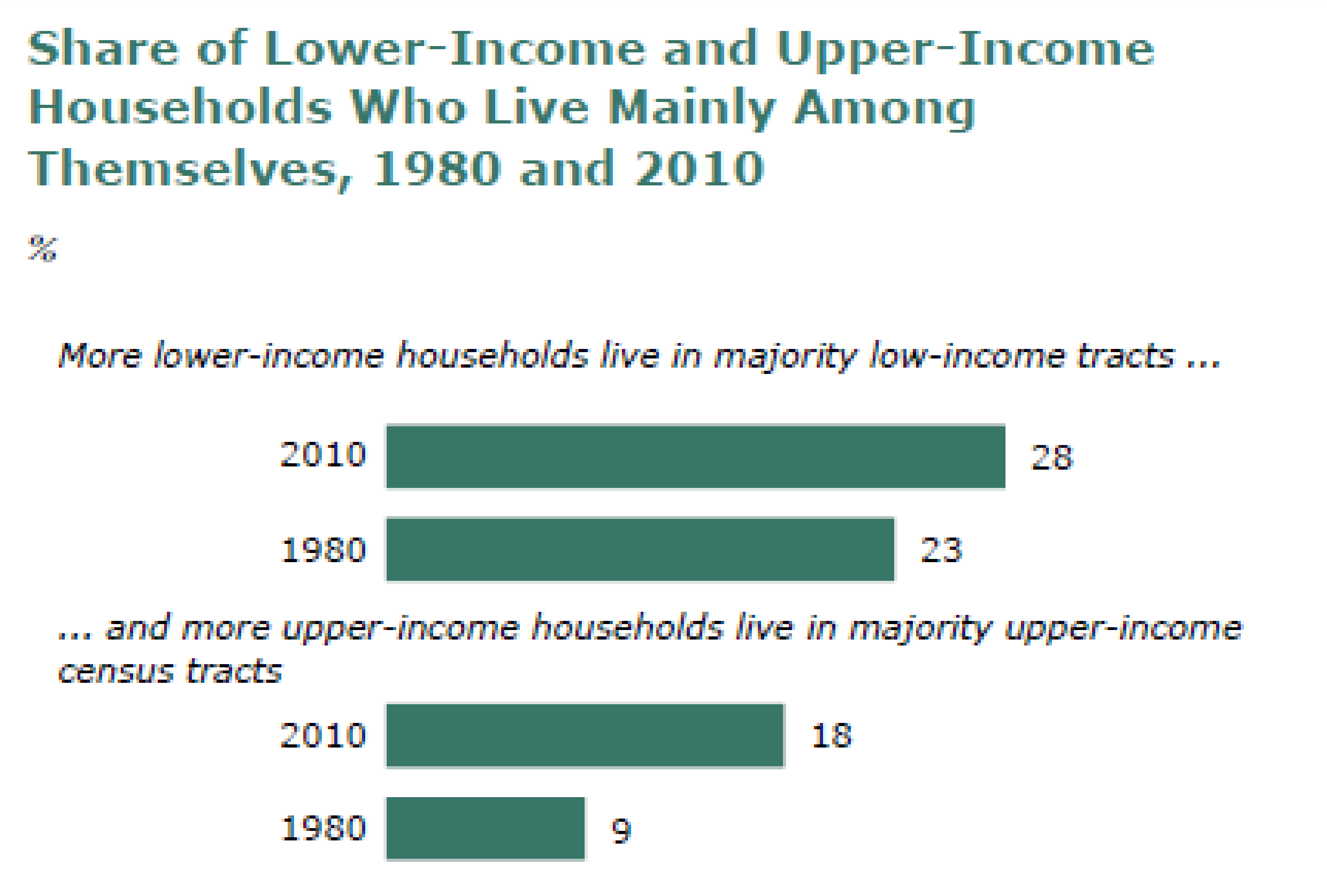
[America is more diverse than ever — but still segregated. The Washington Post]

Persistence of Urban Segregation



The Great Real Estate Reset. Loh, Coes, & Buthe, 2020

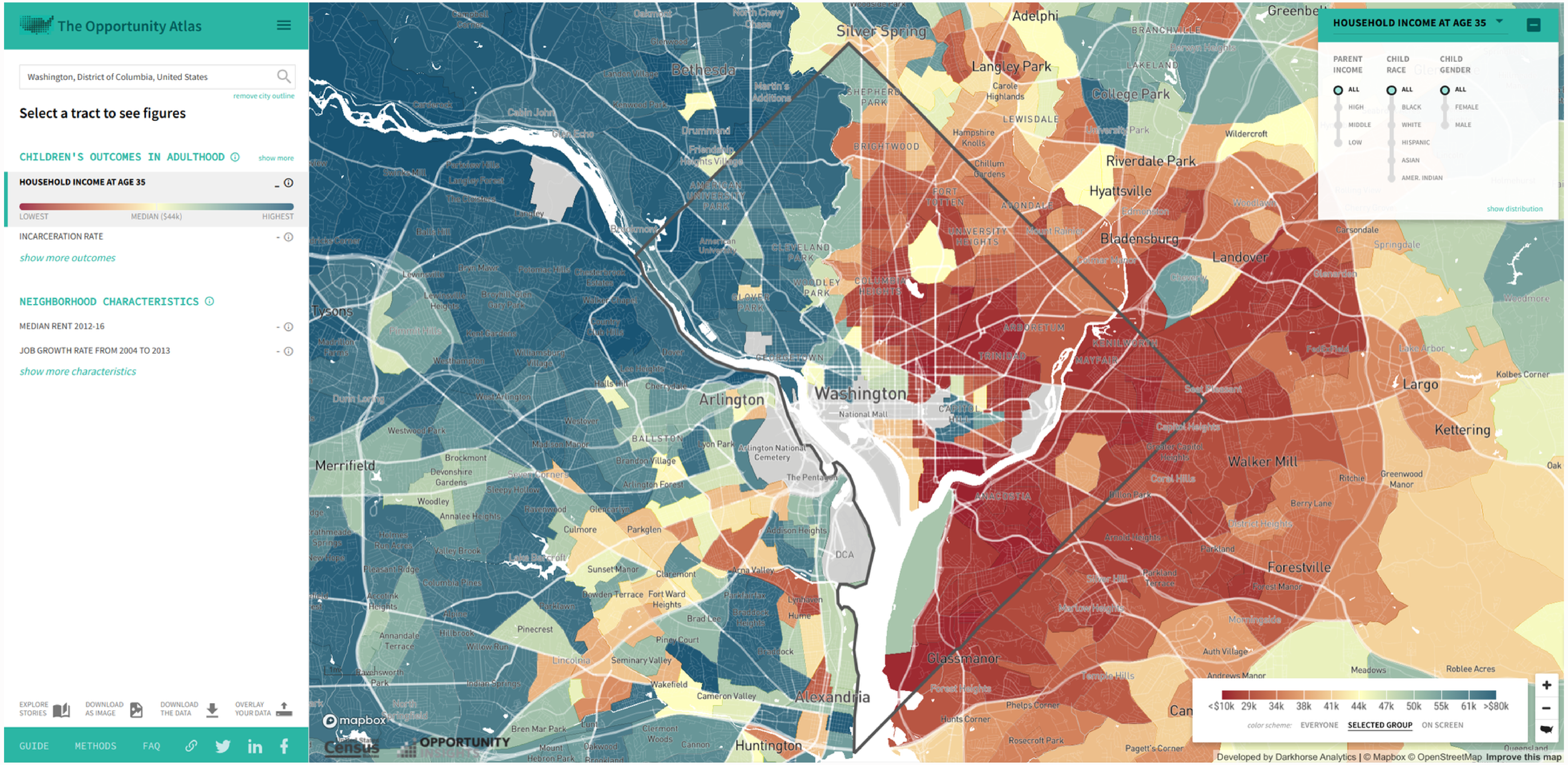
Increase of Income Segregation



The Rise of Residential Segregation by Income. Pew Research Center.

- In 2010, **28% of lower-income** households were located in areas where the majority of households were also lower-income.
- In 2010, **18% of upper-income** households were situated in areas where the majority of households were upper-income.
- There is a **rise in urban income segregation**

ZIP code at birth predicts life outcomes



https://www.opportunityatlas.org/

Questions about Segregation

Is segregation Top-Down or Bottom-Up?

- Top-down segregation: caused by discriminatory policies. Not spontaneous
- Bottom-up segregation: Individual location choices influenced by:
  - Price of housing and services
  - Access to religious centers and education in a language
  - Tenant exclusion and bias

What guides the process?

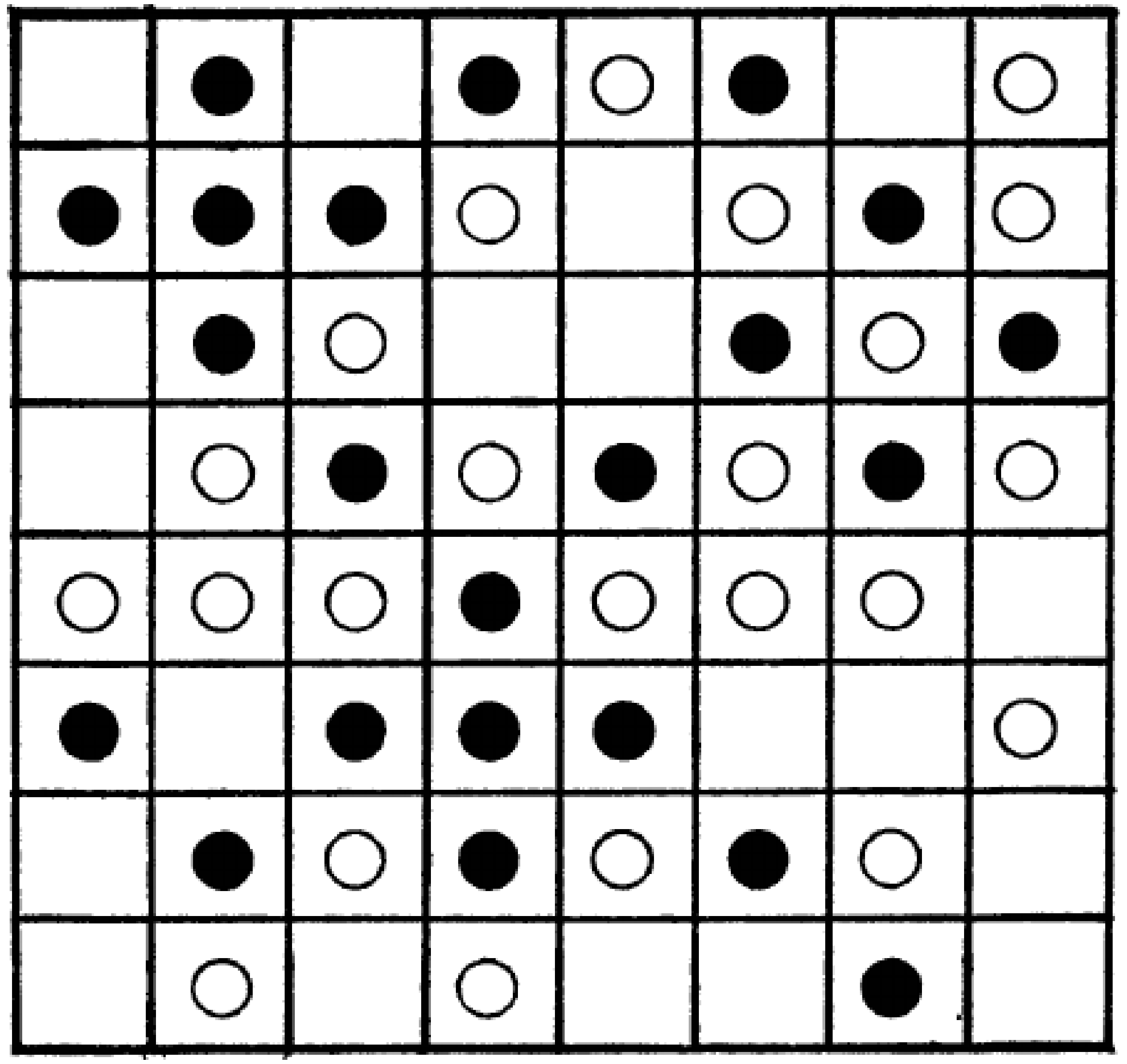
What is the role of inhabitant intolerance to different neighbors?

Is segregation spontaneous?

Without central discrimination, can segregation emerge even when individuals tolerate living in a neighborhood in which they are in the minority?

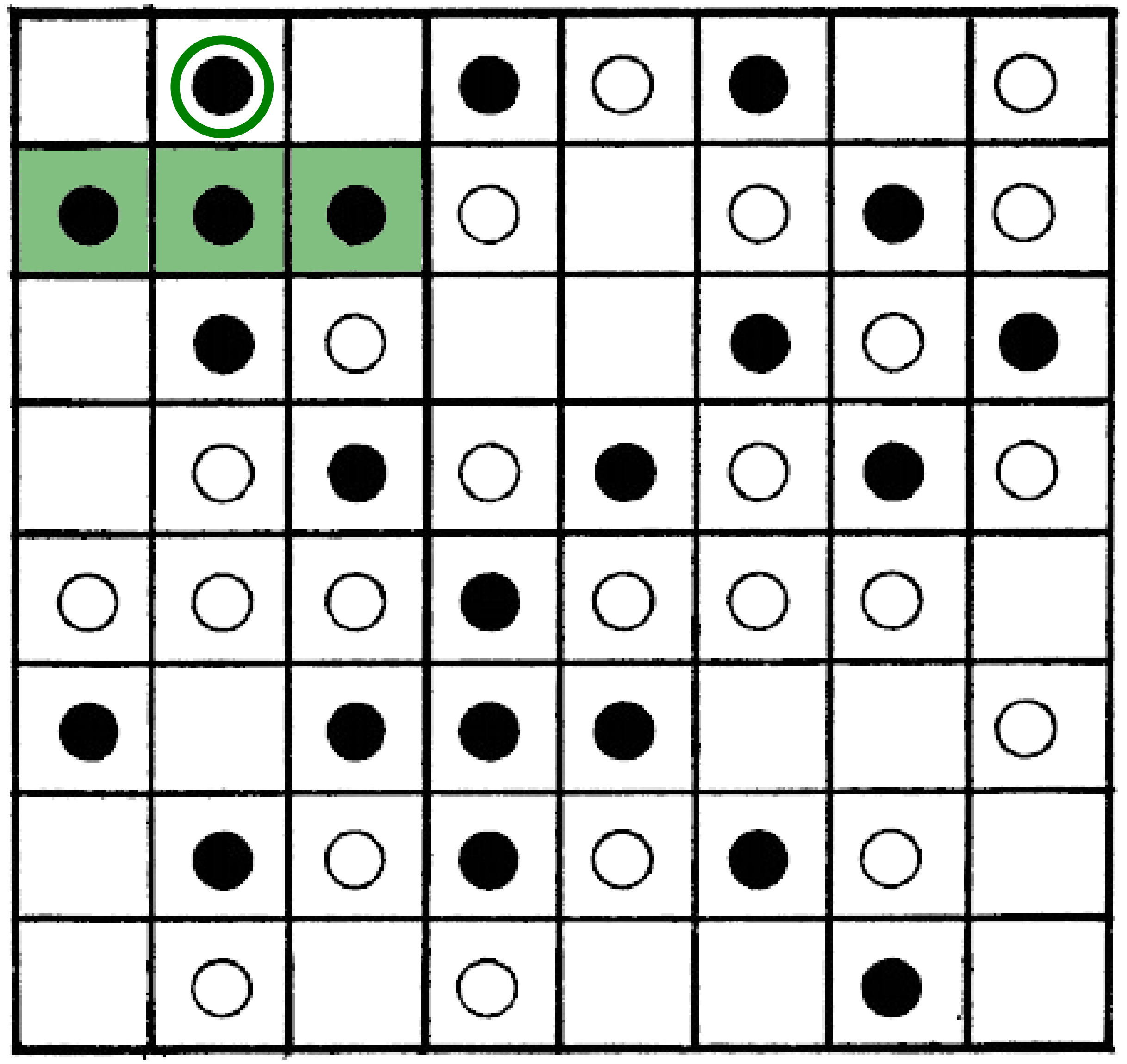
Schelling's segregation model

Shelling's Segregation Model



- Agents of **two kinds**, similar number (here 22 and 23)
- Low but nonzero fraction of **empty cells** (here 19)
- Cells have a **neighborhood** of 8 cells (3x3 - 1)
- Border and corner cells have smaller neighborhood
- Agents are aware of the fraction of **similar agents** in their neighborhood f
- Agents are **satisfied** with f≥F, F measures **intolerance**
- Unsatisfied agents **relocate**

Relocation Examples



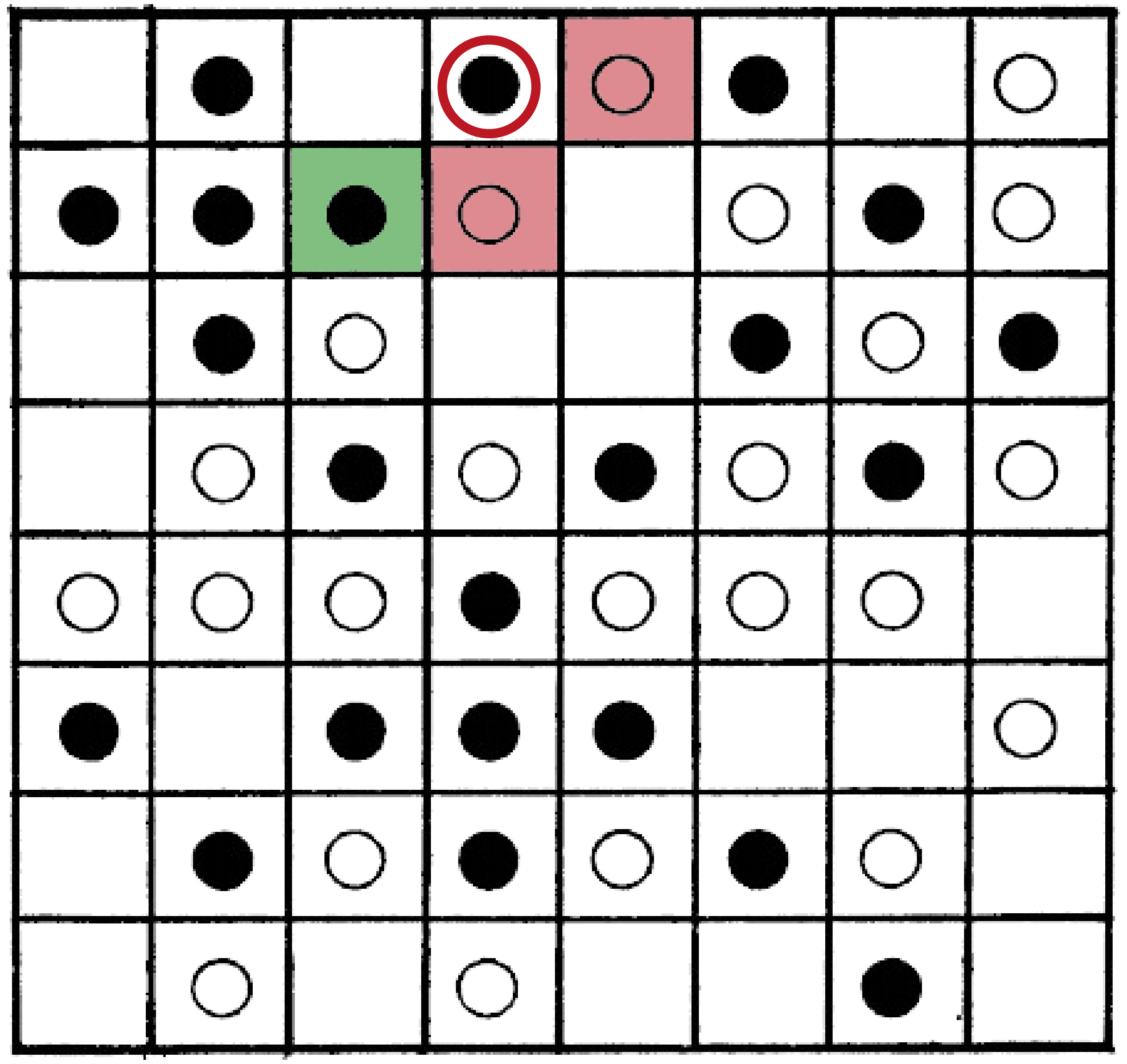
We set the intolerance F=0.35

- 3/3 (100%) neighbors are equal
- **Agent stays**

Note that:

- only occupied neighbor cells count
- cells close to the border have smaller neighborhoods

Relocation
Examples



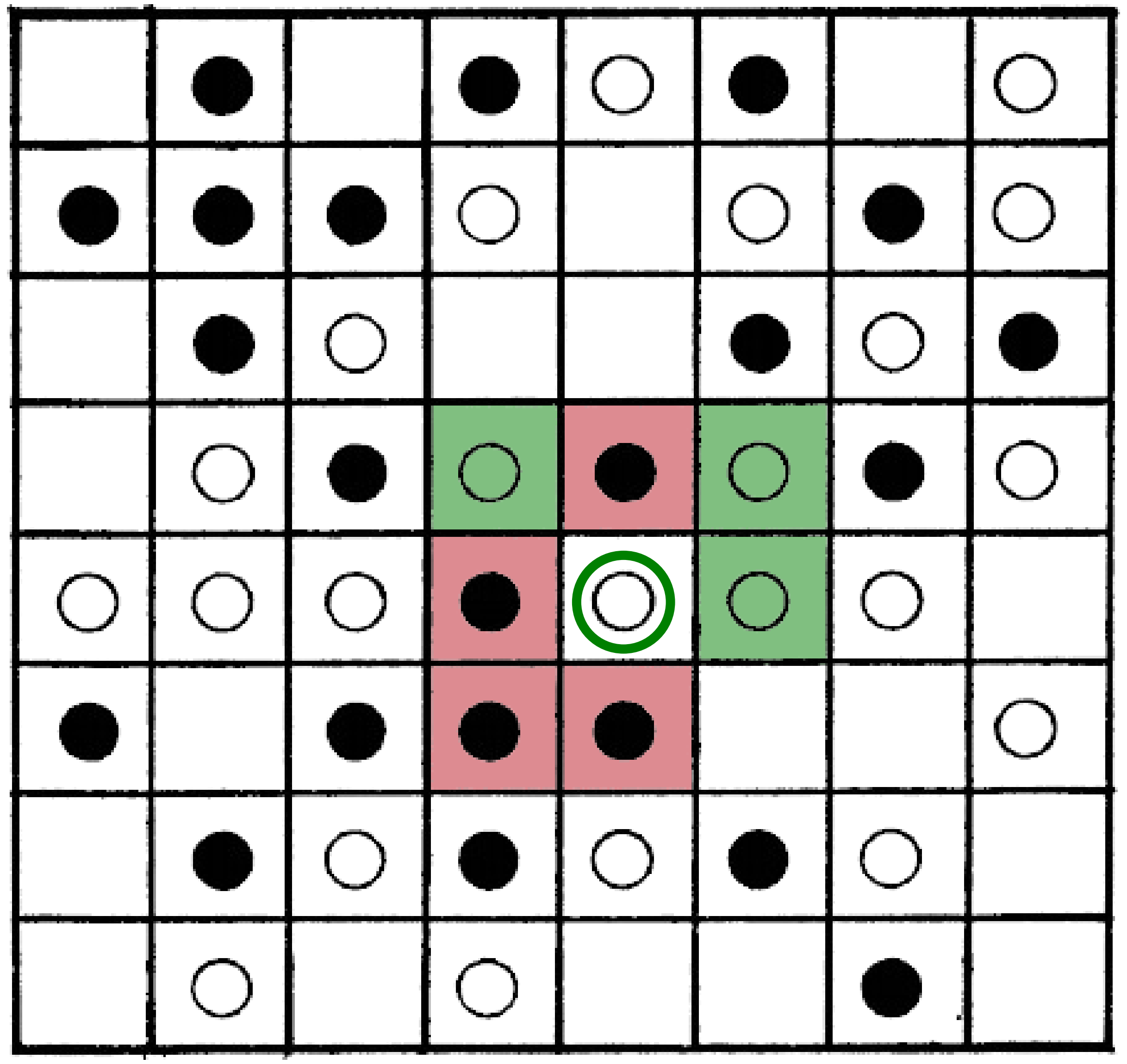
We set the intolerance F=0.35

- 1/3 (33%) neighbors are equal
- **Agent moves**

Note that:

- only occupied neighbor cells count
- cells close to the border have smaller neighborhoods

Relocation
Examples



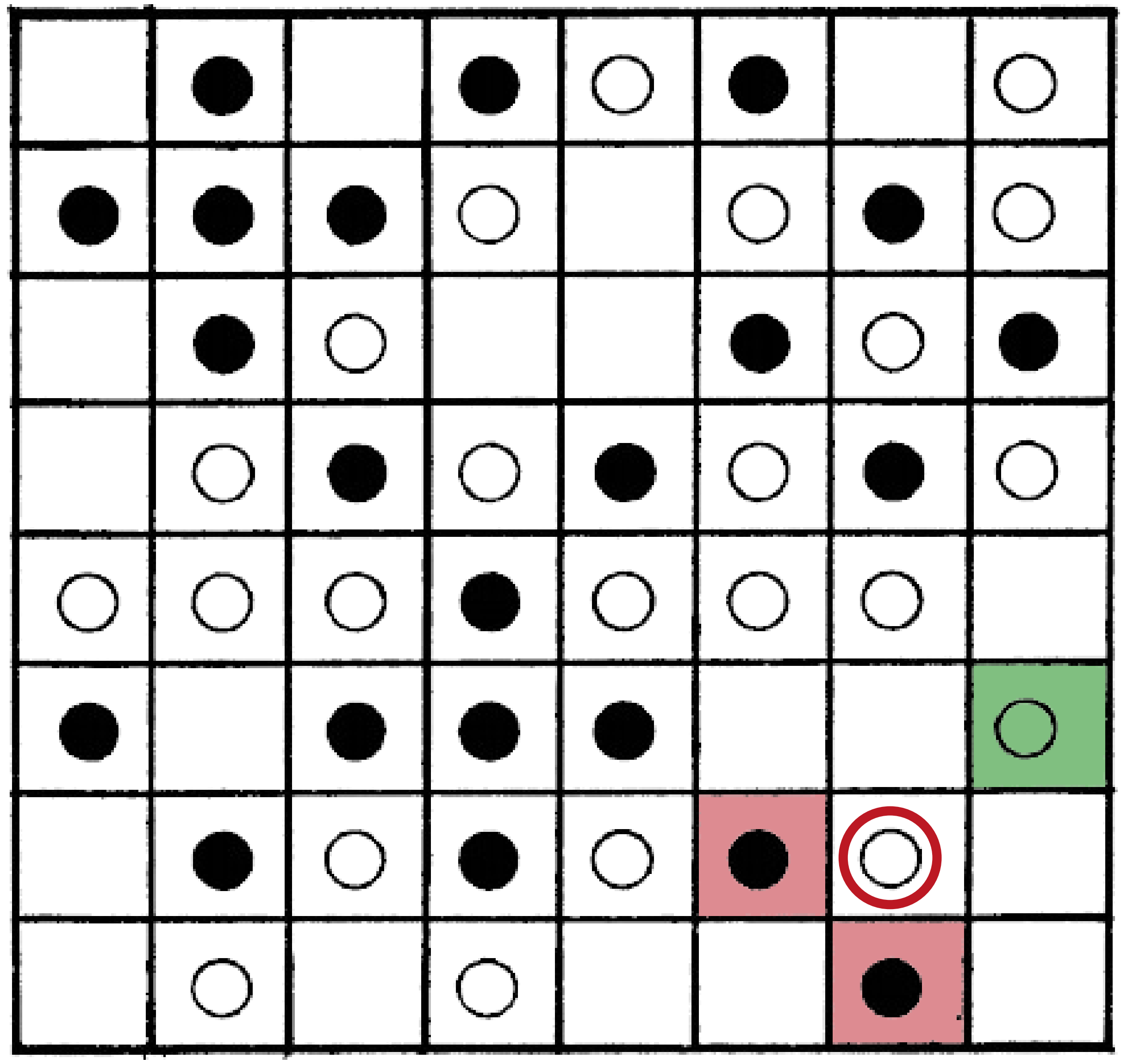
We set the intolerance F=0.35

- 3/7 (43%) neighbors are equal
- **Agent stays**

Note that:

- only occupied neighbor cells count
- cells close to the border have smaller neighborhoods

Relocation
Examples



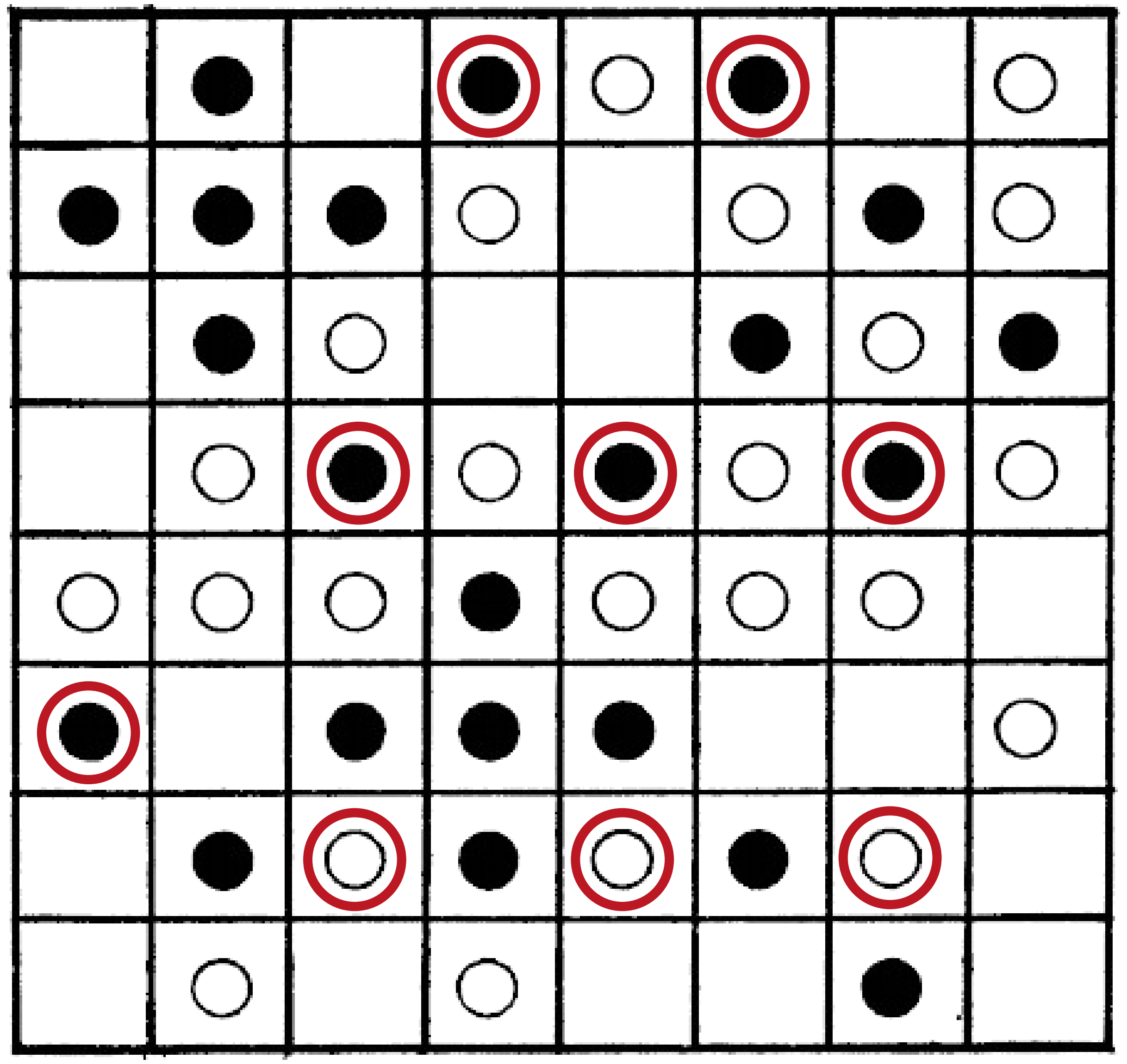
We set the intolerance F=0.35

- 1/3 (33%) neighbors are equal
- **Agent moves**

Note that:

- only occupied neighbor cells count
- cells close to the border have smaller neighborhoods

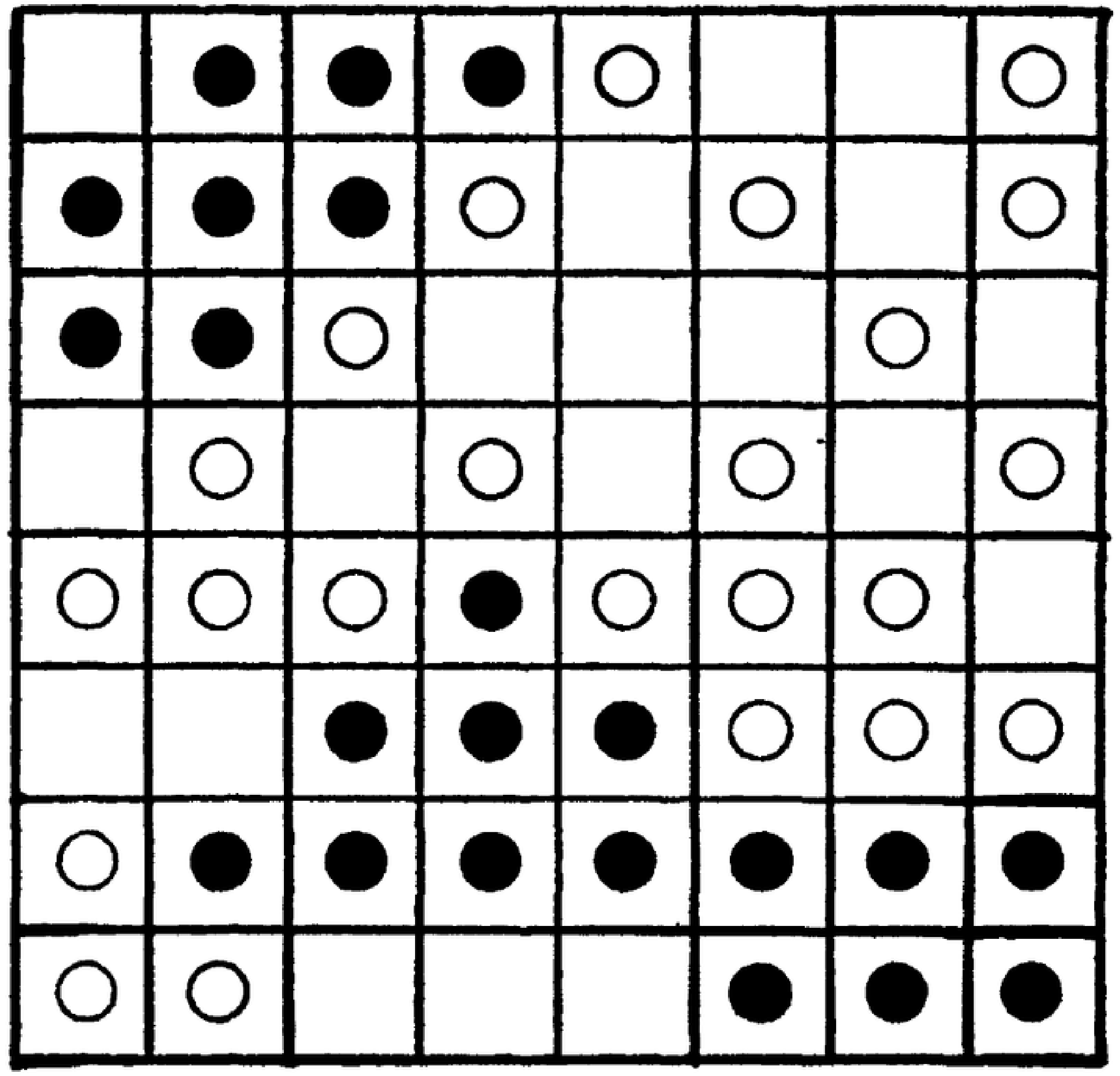
Relocation
Examples



- All "Unhappy" agents with f<F are marked in red
- Agents are selected and unhappy one are relocated
  - random selection
  - sequential selection
- Options about relocation:
  - random
  - random happy
  - random close
  - close and happy

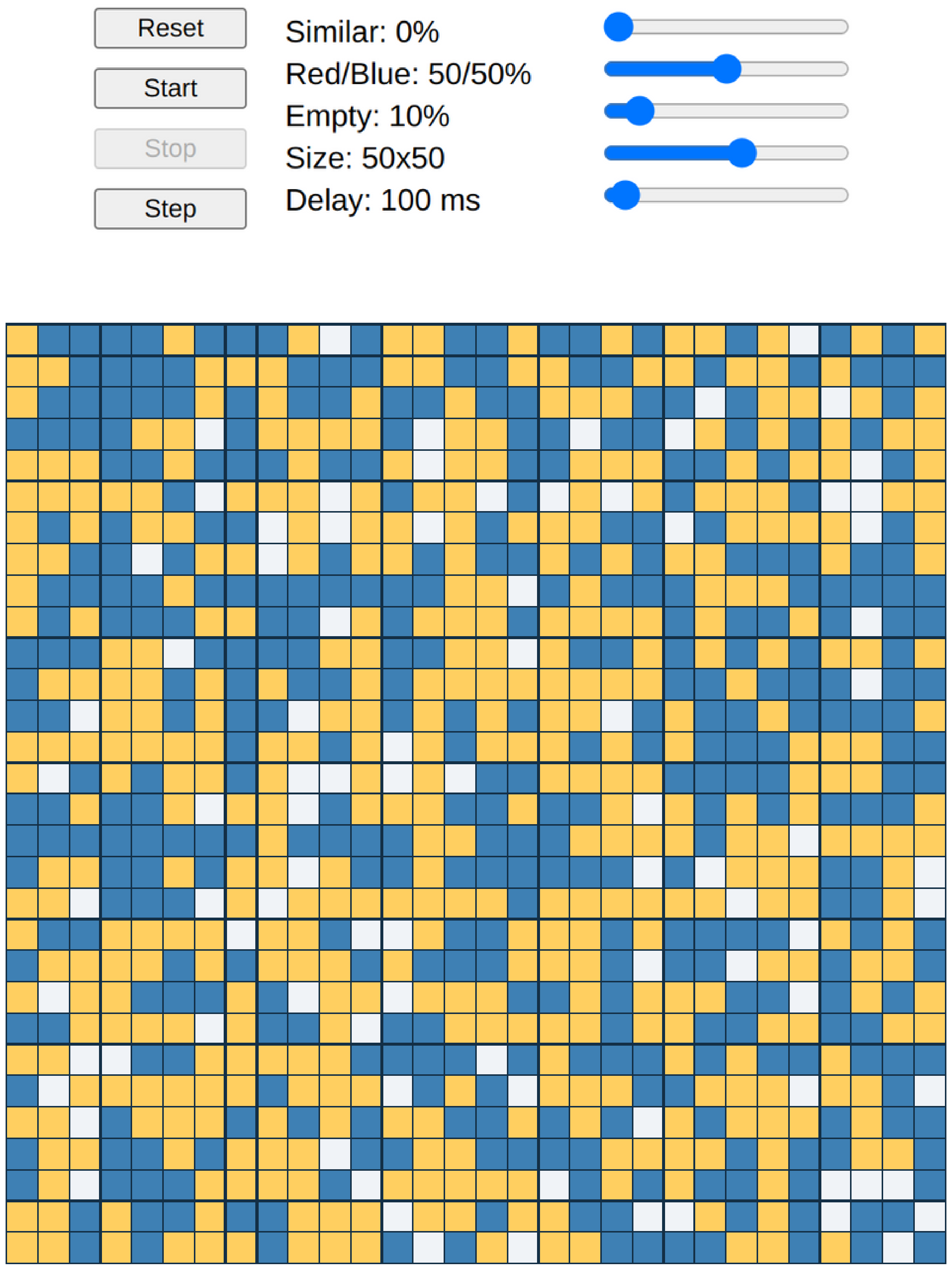
How many parameters does the model have?

Simulation **Result**



- State after running a few iterations
- All agents are satisfied (f≥F)
- No more relocations take place (simulation ends)
- Result looks segregated
- Two large black regions
- Rest formed by scattered white agents

Segregation emerges spontaneously even when living in minority is tolerated

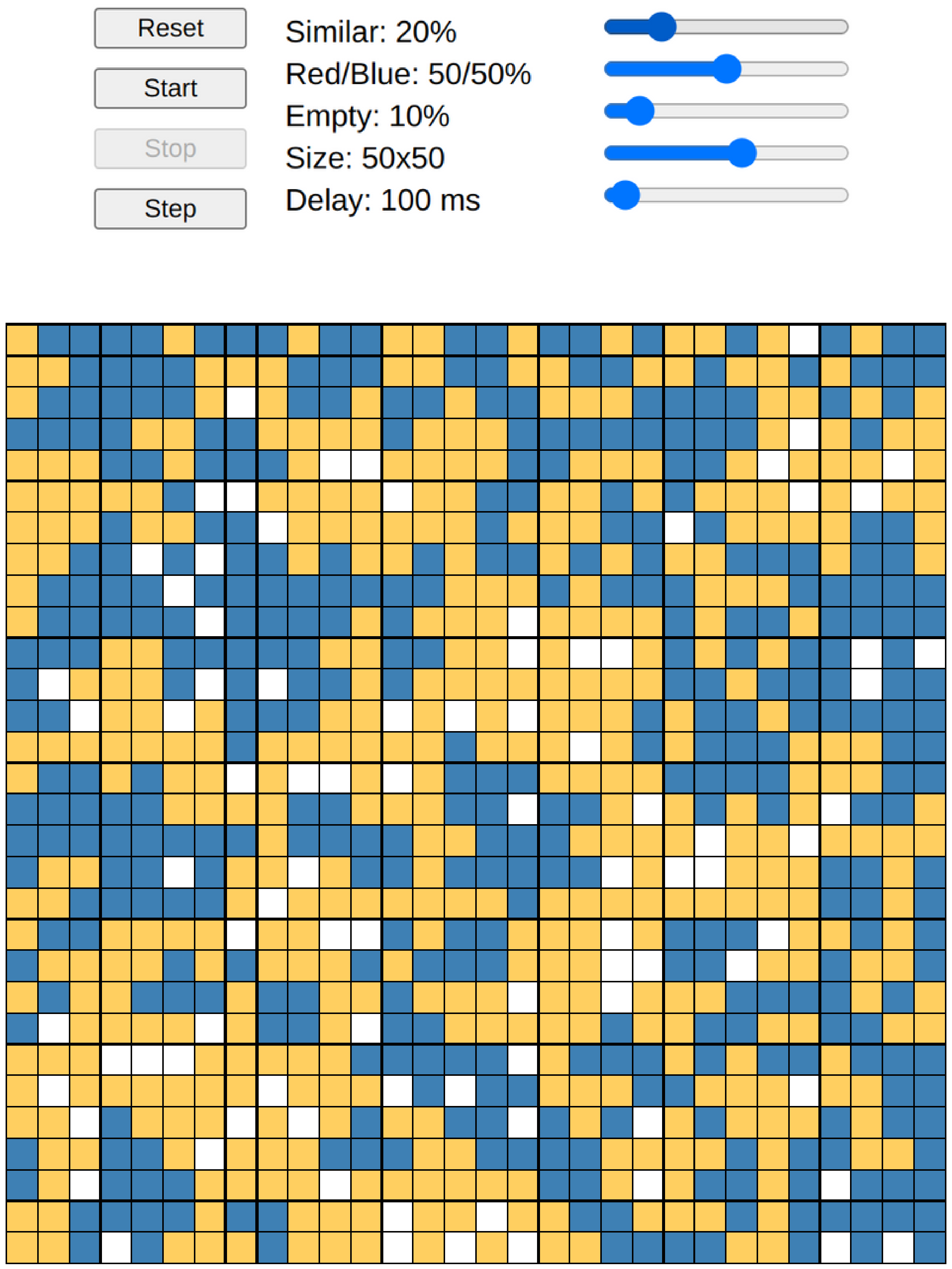


Bigger Simulation: Initial Condition

We want to investigate the role of F on the emergence of segregation

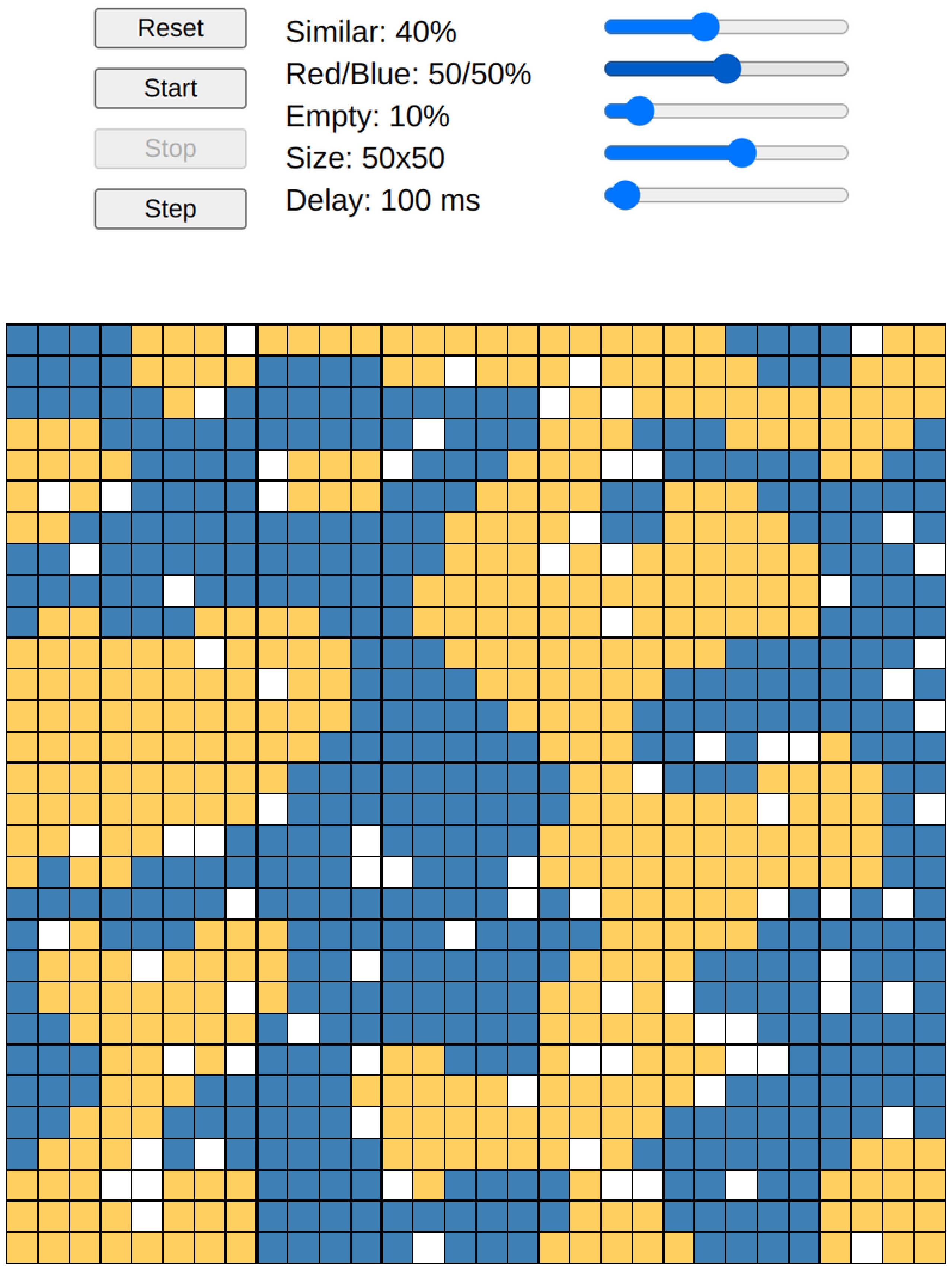
- We use random initial configuration
- Parameters can be tuned, we use a 50x50 grid, 10% empty cells and balance between groups
- The interface shows the percentage of satisfied as function of simulation round

Bigger Simulation: F=0.2



Low Intolerance Regime F=0.2

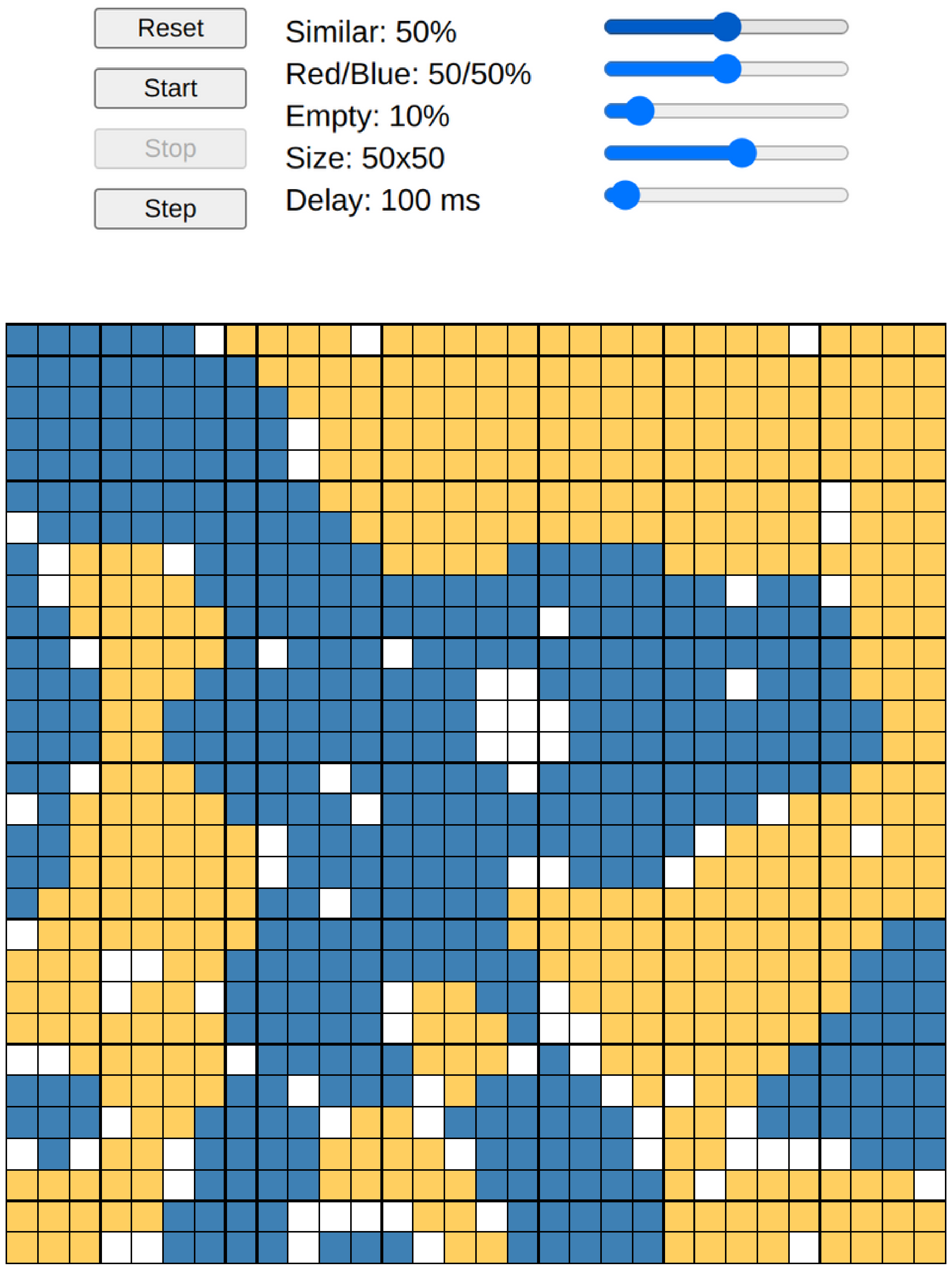
- Result very similar to initial configuration
- No apparent segregation
- All agents satisfied soon



Bigger Simulation: F=0.4

Medium Intolerance Regime F=0.4

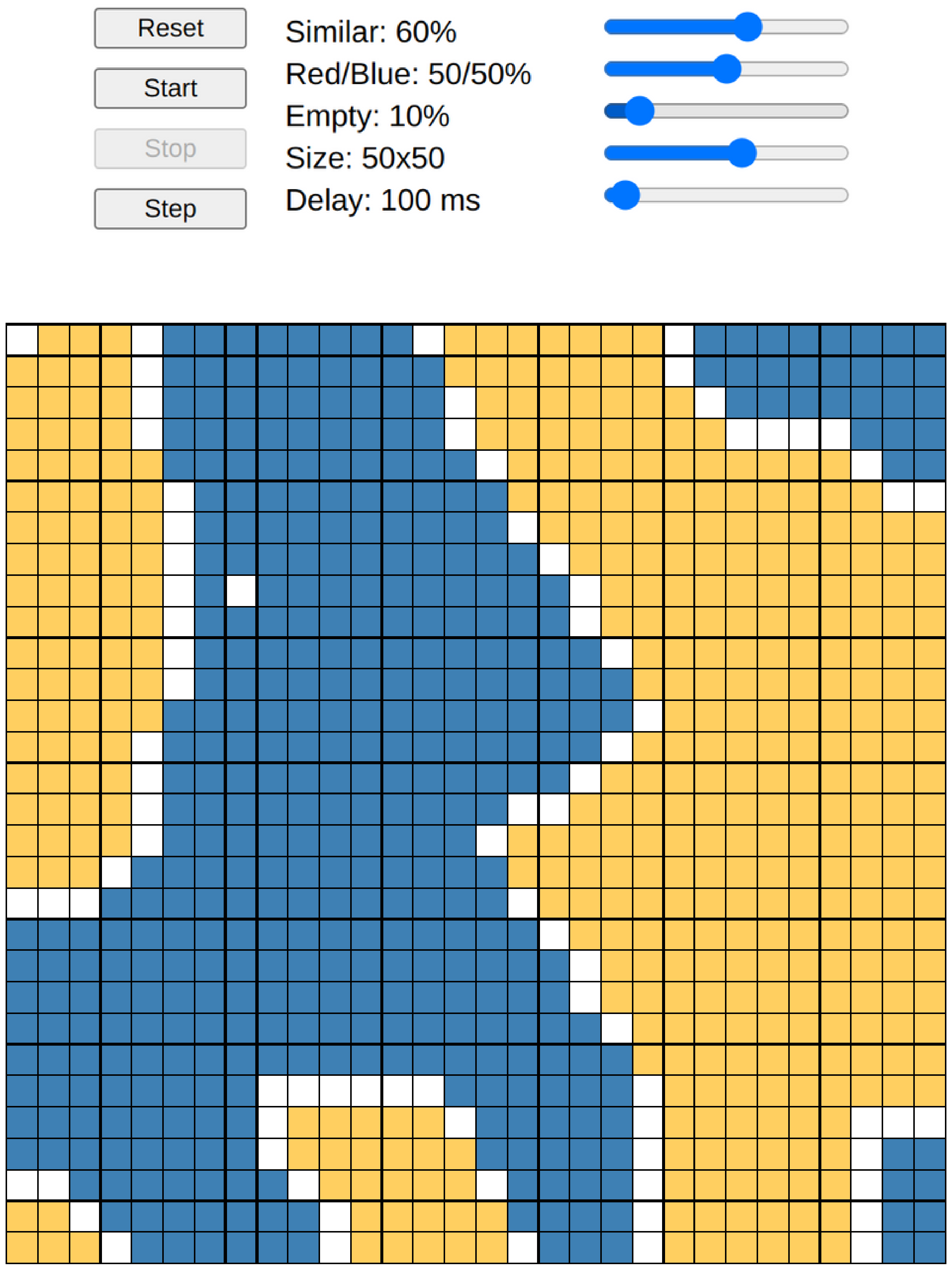
- Result is segregated
- Agents are OK living in minority!
- All agents can be satisfied



Bigger Simulation: F=0.5

High Intolerance Regime F=0.5

- Result is substantially segregated
- Empty cells form borders between clusters
- Several iterations are necessary for all agents to be satisfied



Bigger Simulation: F=0.6

Extreme Intolerance Regime F=0.6

- Result is very segregated
- Empty cells form borders between clusters
- Several iterations are necessary for all agents to be satisfied

Analyzing Schelling's model

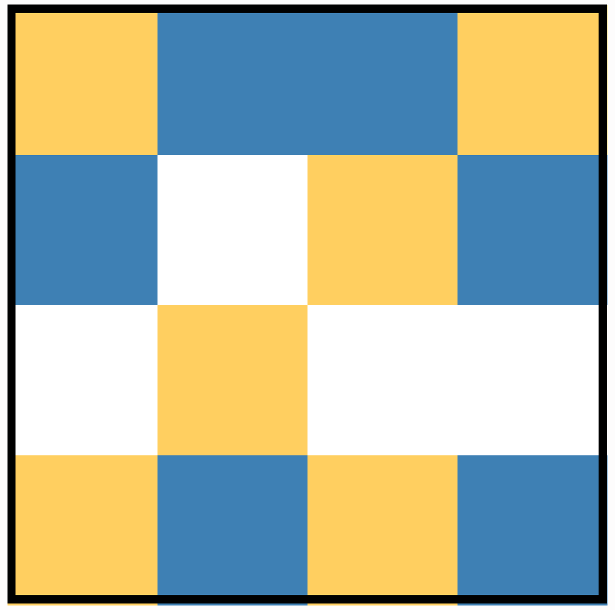
Measuring segregation: Moran's I

Moran’s index I is defined as

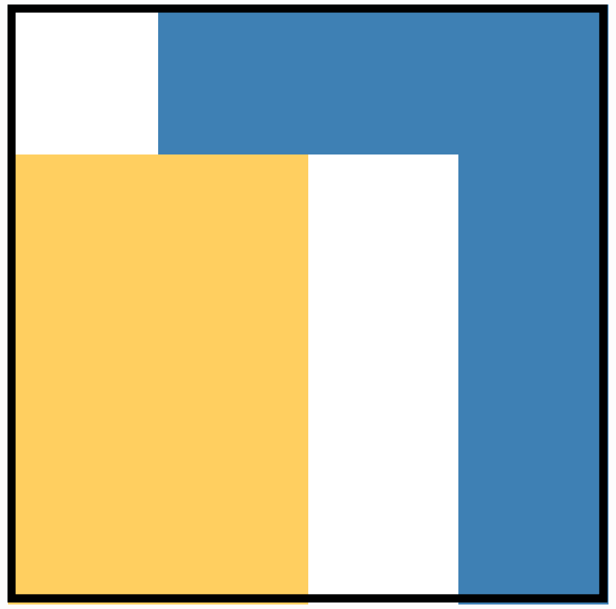
$$I = \frac{M \sum_i \sum_j w_{i,j}(x_i - \bar{x})(x_j - \bar{x})}{\left(\sum_i \sum_j w_{i,j}\right) \sum_i (x_i - \bar{x})^2}$$

- **M:** Number of occupied cells
- **$w_{i,j}$:** adjacency matrix of cells
  - $w_{i,j}=1$ if i is a neighbor of j, otherwise $w_{i,j}=0$
- **x_i:** color value of occupied cell i
  - $x_i=0$ if blue agent in it, $x_i=1$ if yellow agent in it
  - We ignore empty cells
- **$\bar{x}$:** mean value of x, i.e. fraction of blue agents

I is also known as **spatial autocorrelation**



Low Morans’ I



High Morans’ I

How to calculate Moran's I

We rewrite Moran's I as

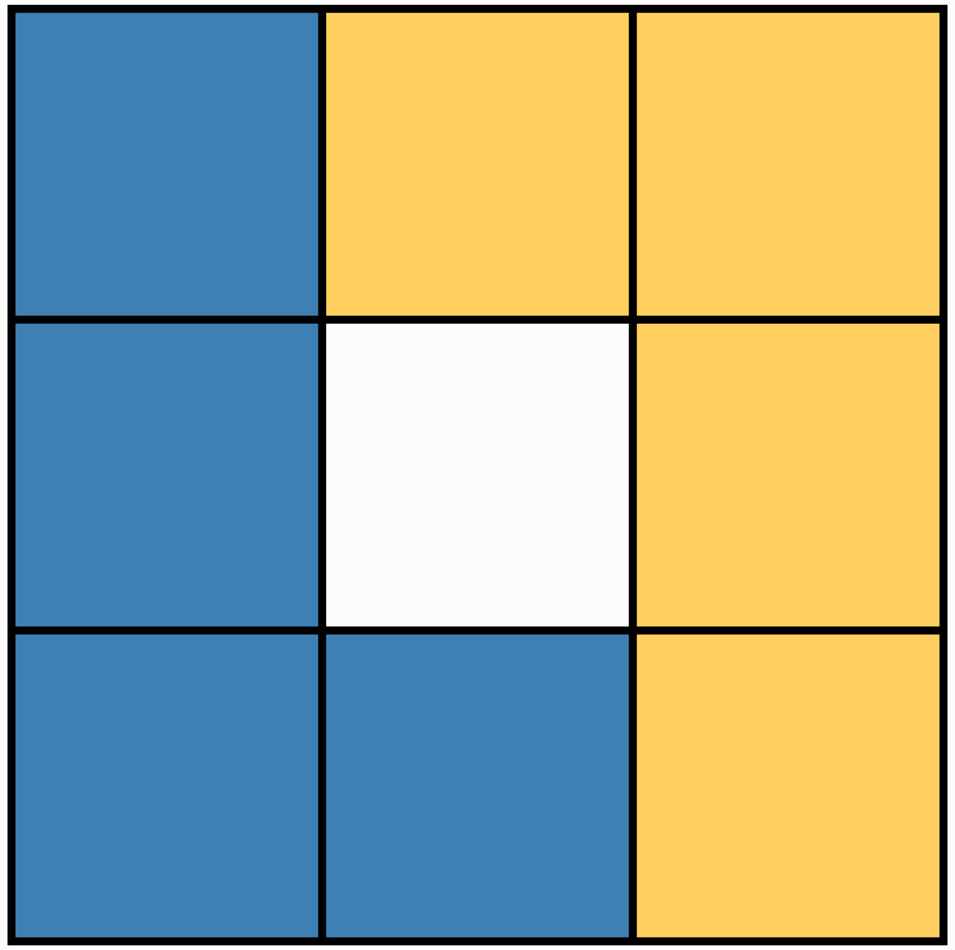
$$I = \frac{MC}{W\sigma}$$

Where these variables are defined as

$$C = \sum_i \sum_j w_{i,j}(x_i - \bar{x})(x_j - \bar{x})$$

$$W = \sum_i \sum_j w_{i,j}$$

$$\sigma = \sum_i (x_i - \bar{x})^2$$



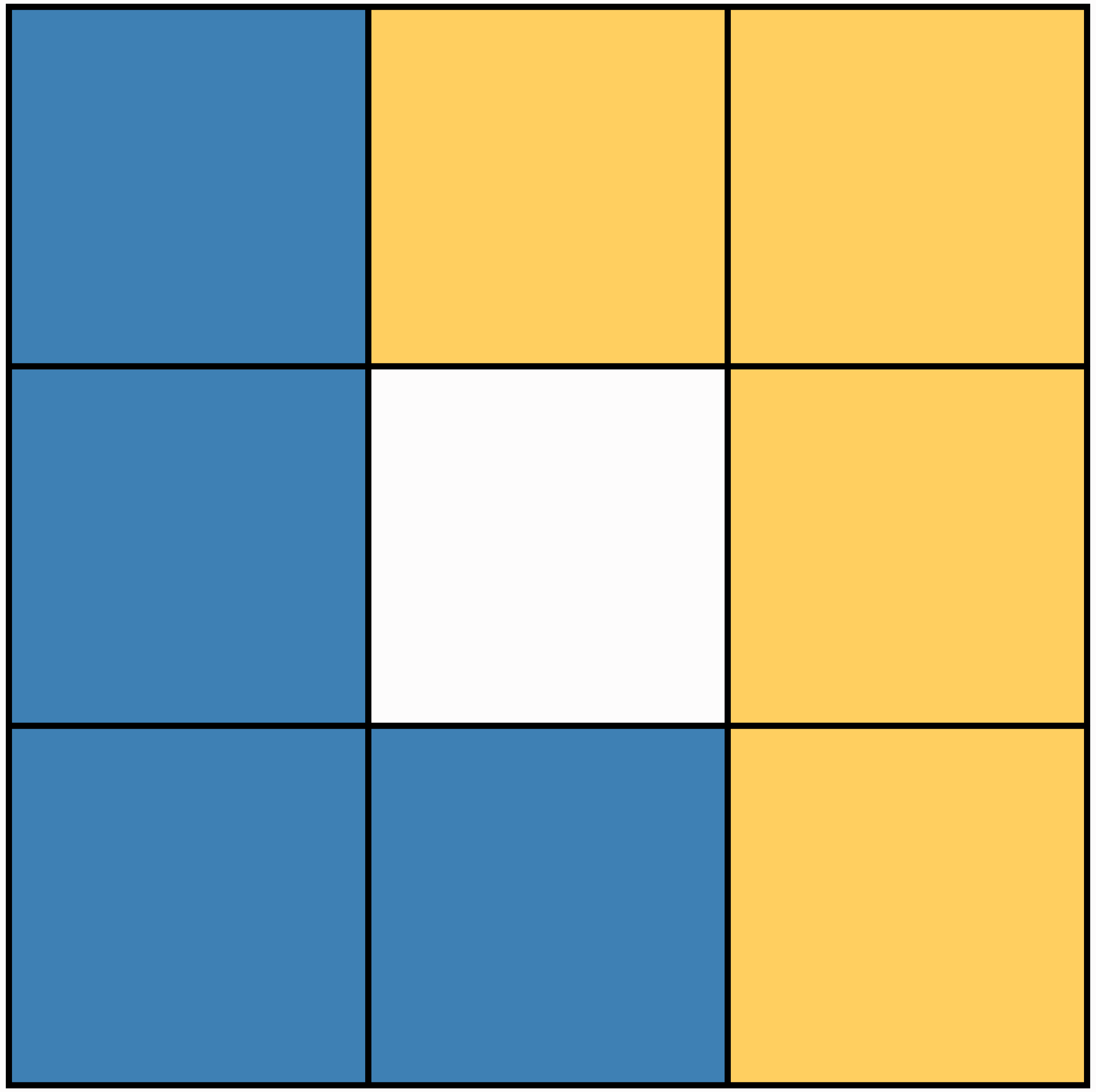
I=0.3

With a loop over the cells, we can iteratively calculate C, W and σ.

Moran's I Example

In our case we have

- 9 cells
- 3x3 neighborhood
- 4 red and 4 blue agents
    - $\mathbf{x_i}$=0 if i occupied by blue
    - $\mathbf{x_i}$=1 if i occupied by yellow
    - $\bar{\mathbf{x}}$=0.5
    - **M**=8



Moran's I Example

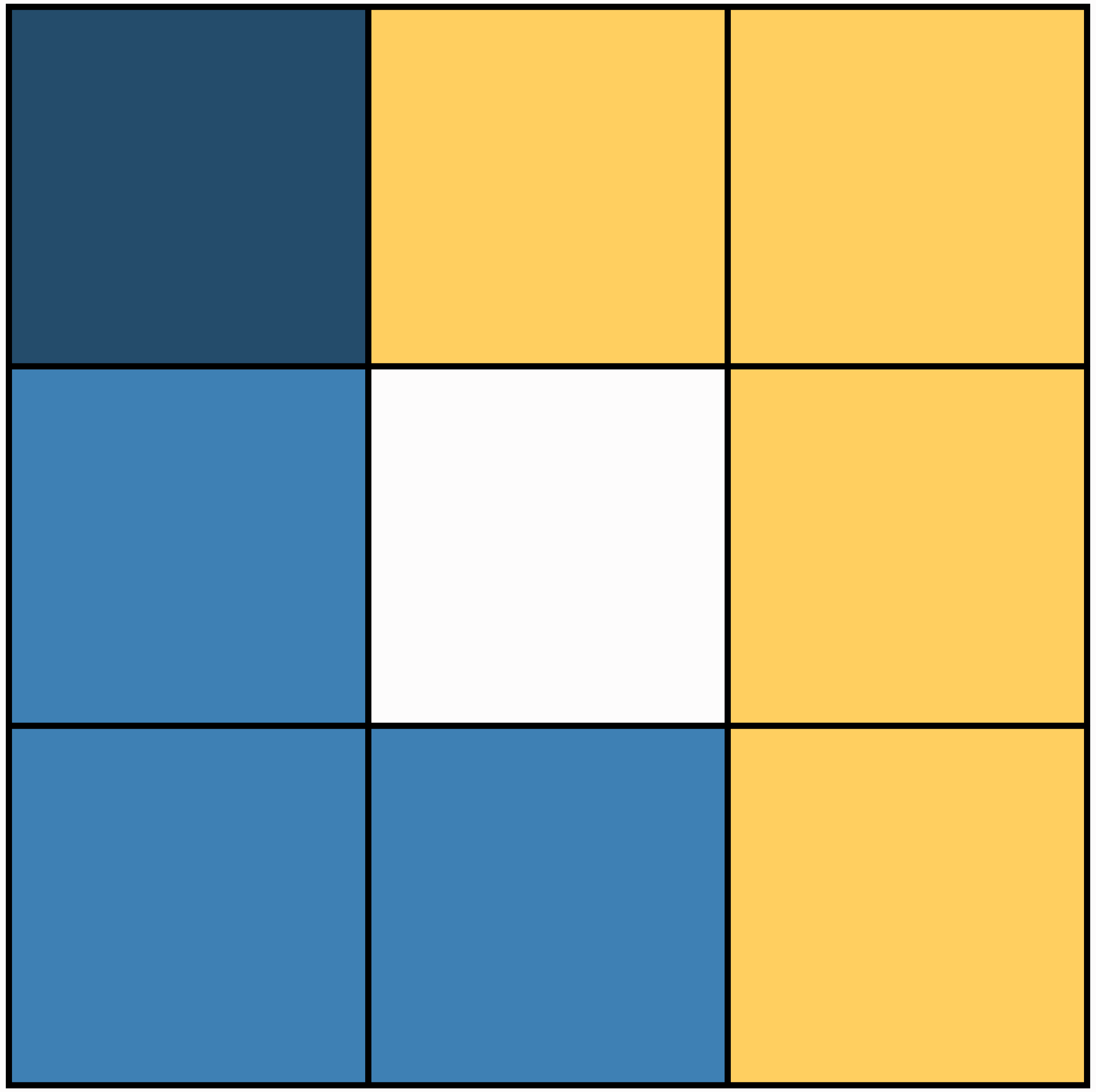
- First cell: blue (x_1=0)
- Contribution to σ:

$$\sigma \to \sigma + (x_1 - \bar{x})^2 = \sigma + (0 - 0.5)^2$$

- Contribution to W: 2 neighbors

$$W \to W + 2$$

Empty neighborhood cell doesn't count!



Moran's I Example

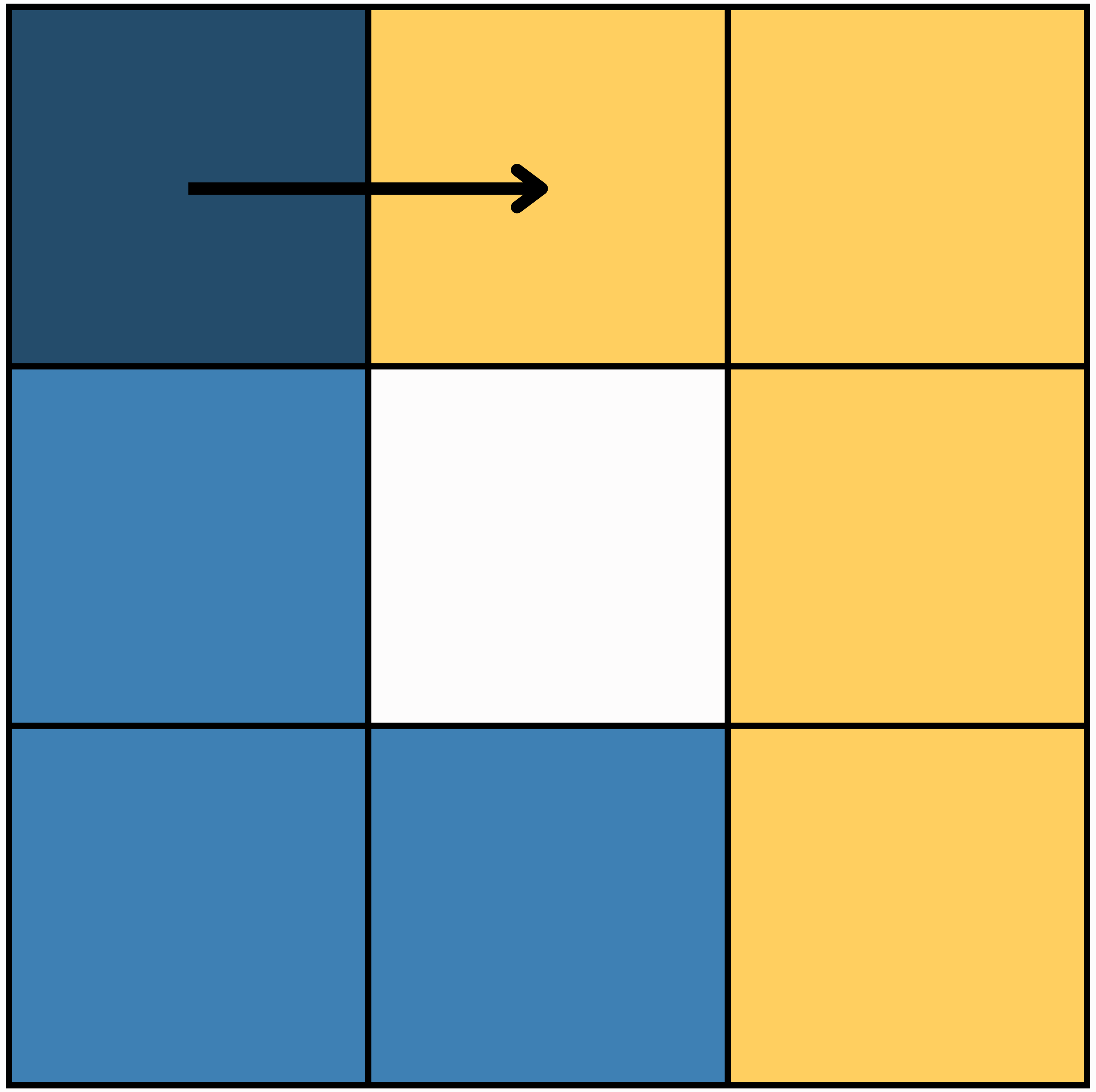
- First neighbor (cell 2) contribution to C

$$C \to C + w_{1,2}(x_1 - \bar{x})(x_2 - \bar{x})$$

- Because cells 1 and 2 are neighbors, $w_{1,2}=1$

$$C \to C + 1 \cdot (0 - 0.5)(1 - 0.5)$$
$$C \to C - 0.25$$

Different neighbors reduce I



Moran's I Example

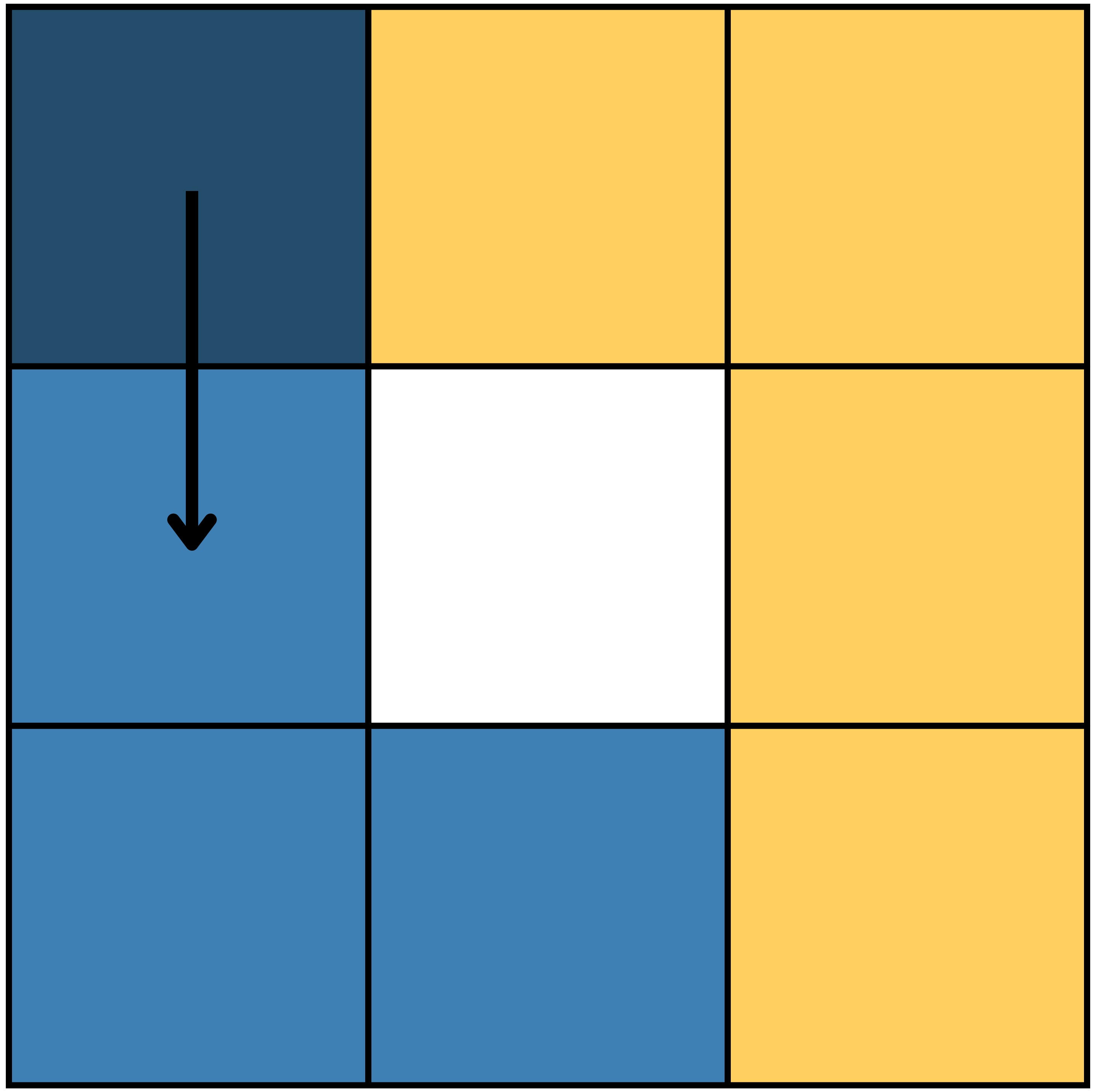
- First neighbor (cell 4) contribution to C

$$C \to C + w_{1,4}(x_1 - \bar{x})(x_4 - \bar{x})$$

- Because cells 1 and 2 are neighbors, $w_{1,2}=1$

$$C \to C + 1 \cdot (0 - 0.5)(0 - 0.5)$$
$$C \to C + 0.25$$

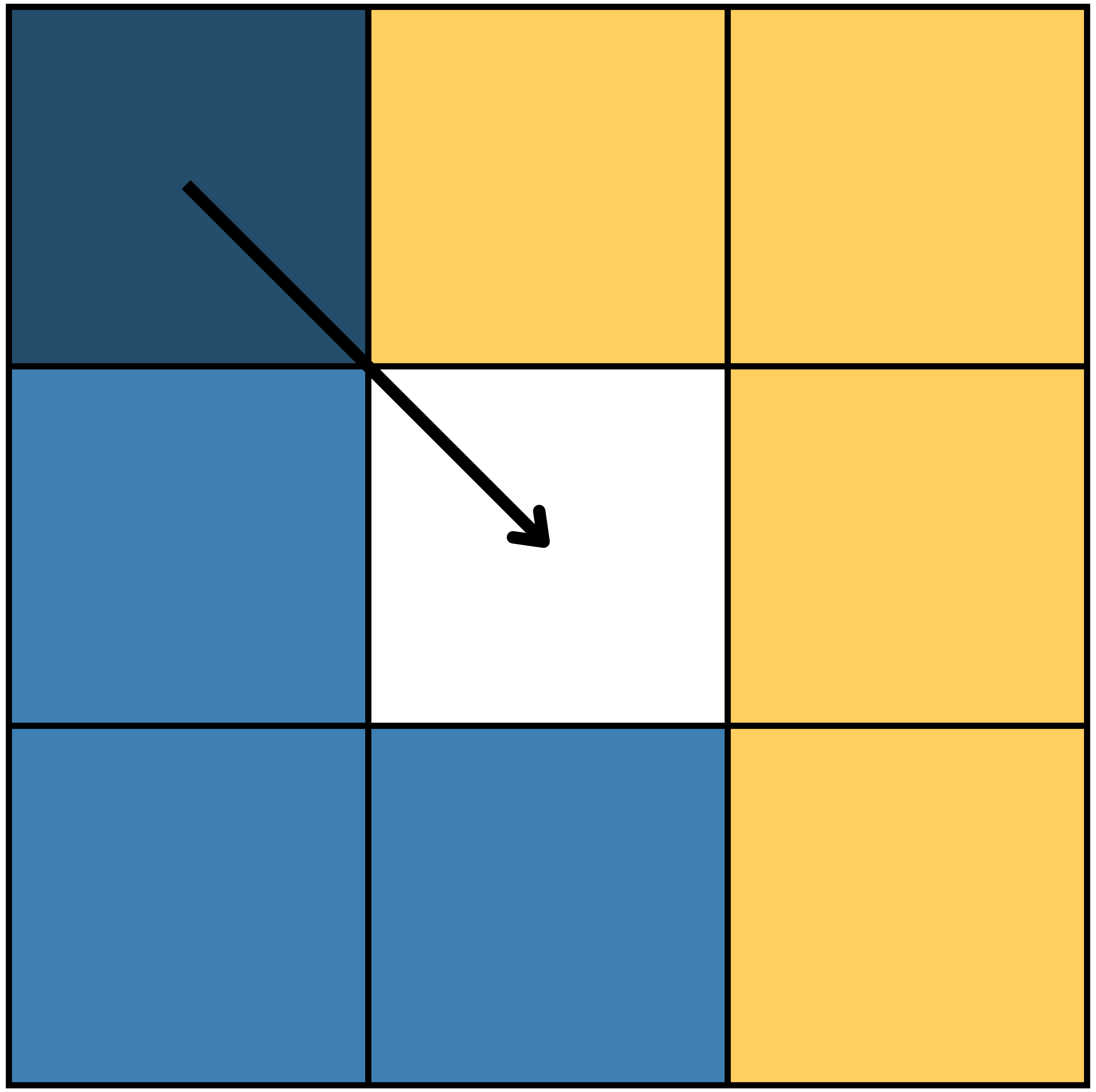
Equal neighbors increase I



Moran's I Example

- Second neighbor (cell 5) empty cell

No contribution to I



Moran's I Example

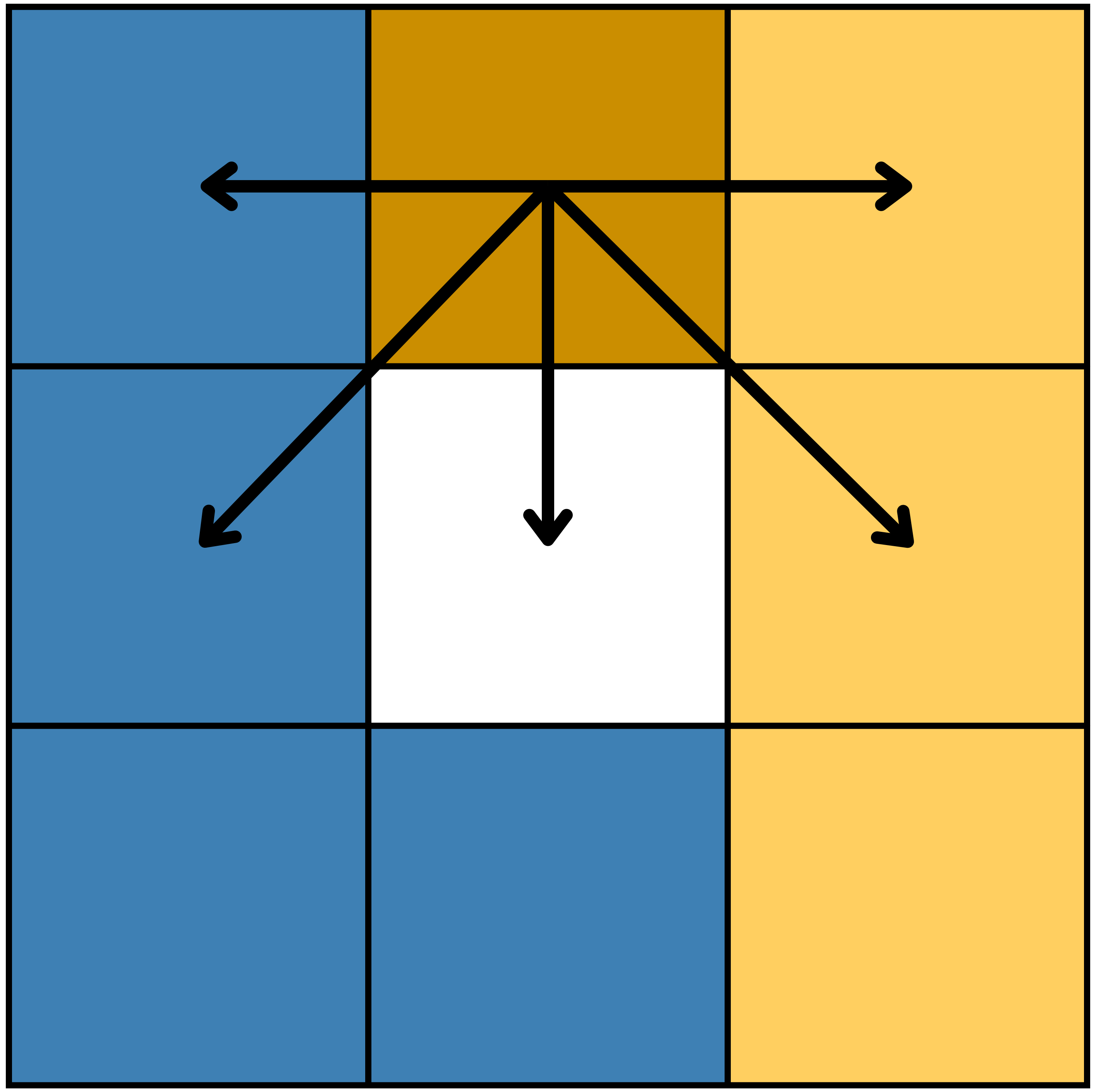
- Second cell: yellow (x_2=1)
- Contribution to σ and W

$$\sigma \to \sigma + (x_2 - \bar{x}^2) = \sigma + (1 - 0.5)^2$$

$$W \to W + 4$$

- There are four contributions to C

$$C \to C + 0.25 + 0.25 - 0.25 - 0.25$$



Moran's I Example

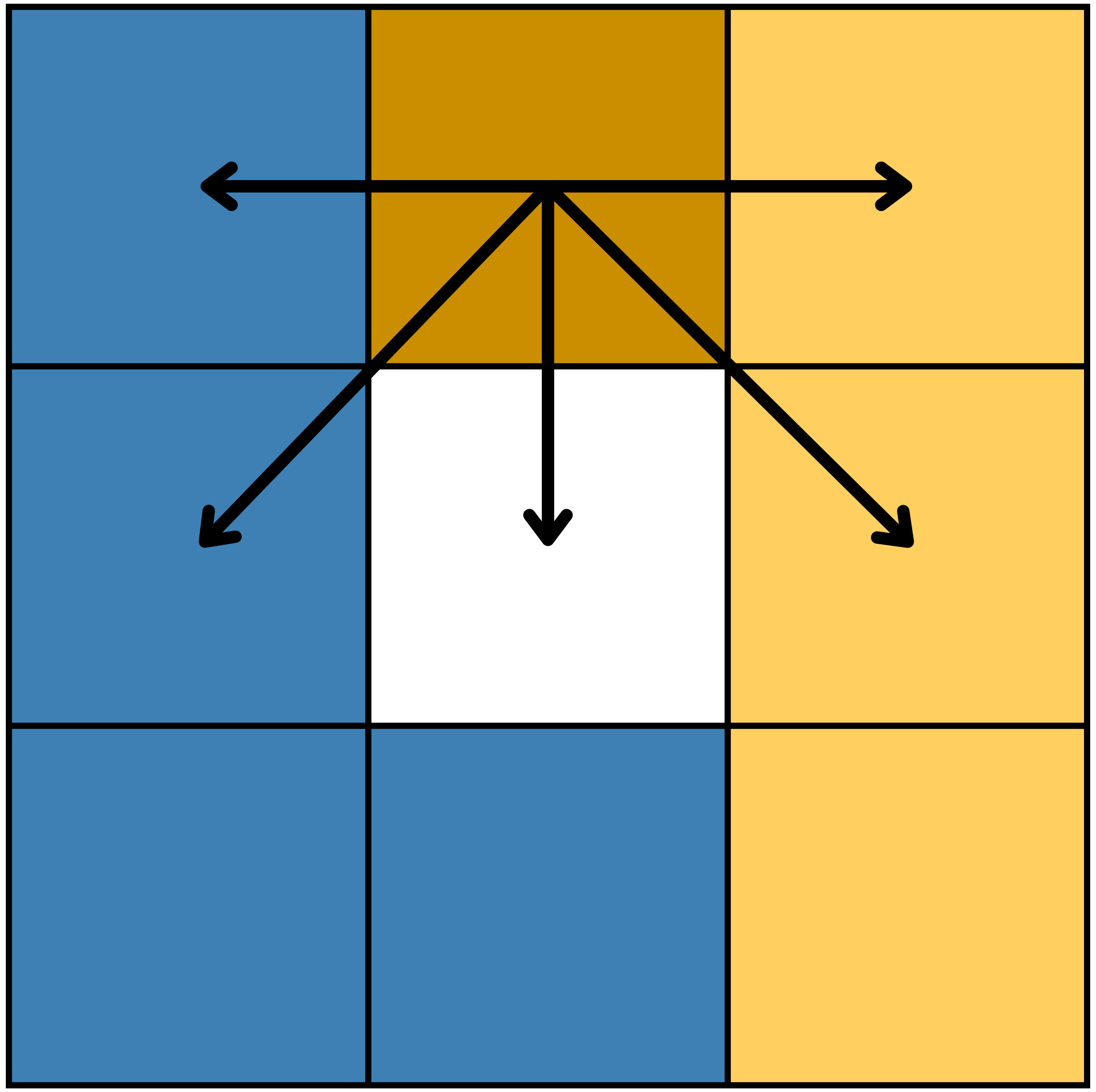
- Second cell: yellow (x_2=1)
- Contribution to σ and W

$$\sigma \rightarrow \sigma + (x_2 - \bar{x}^2) = \sigma + (1 - 0.5)^2$$

$$W \rightarrow W + 4$$

- There are four contributions to C

$$C \rightarrow C + 0.25 + 0.25 - 0.25 - 0.25$$



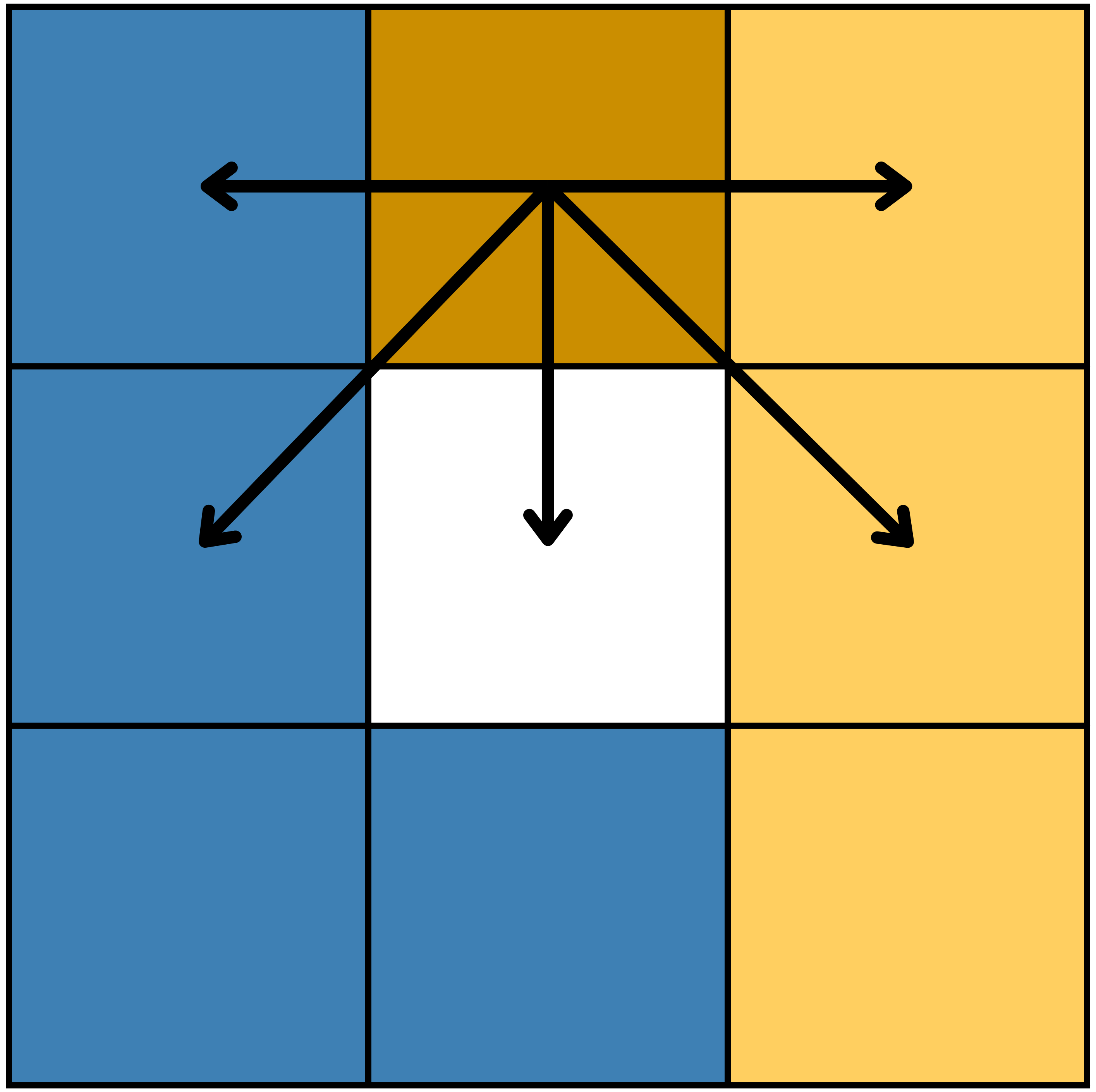
Moran's I Example

Putting all the pieces together

- **M**=8
- **C**=16·0.25−8·0.25=2
- **W**=24
- **σ**=0.25·8=2

$$I = \frac{MC}{W\sigma} = \frac{8 \cdot 2}{24 \cdot 2} = 0.3$$

https://

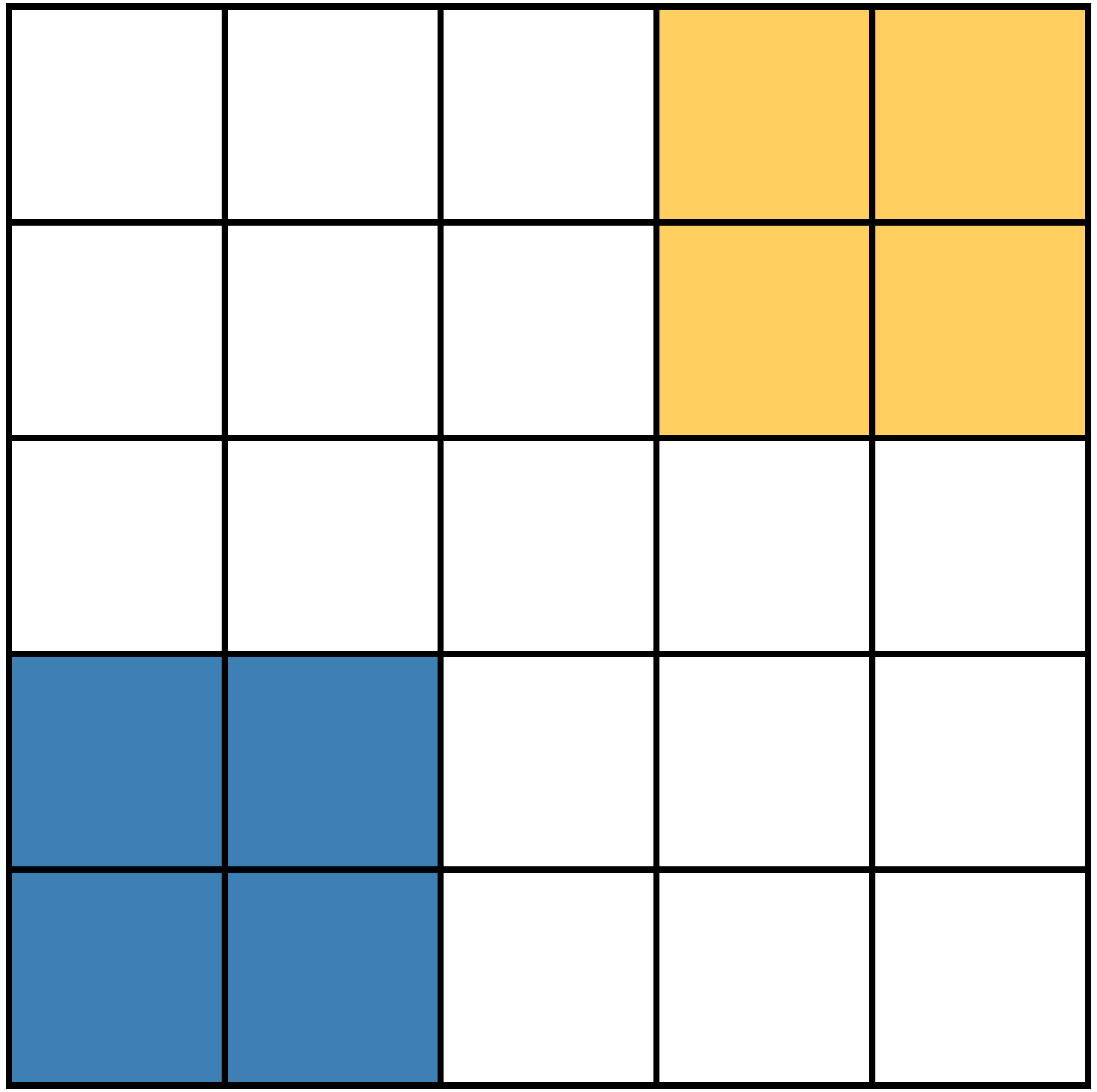


Max Moran's I Example

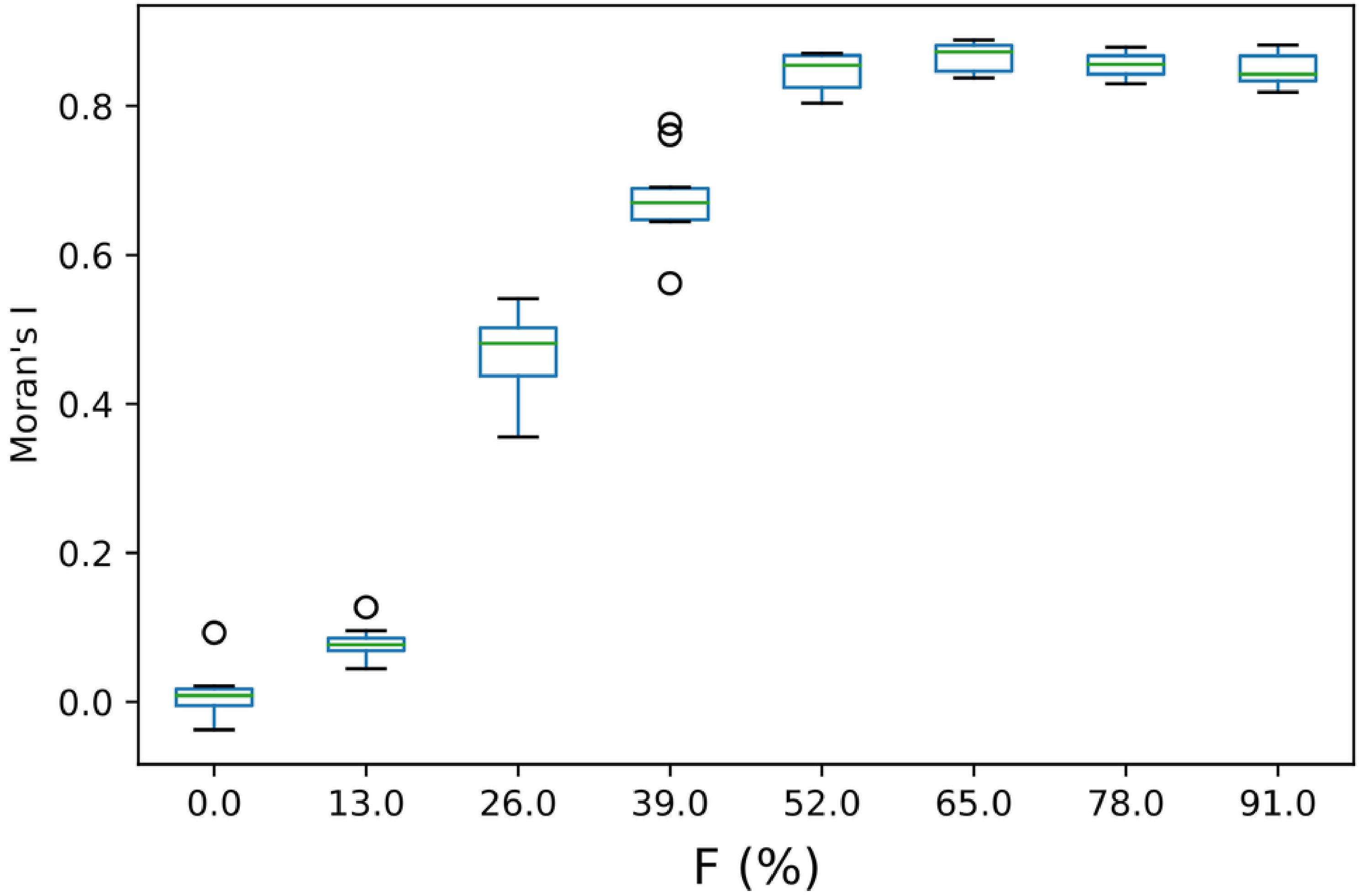
Putting all the pieces together

- **M**=8
- **C**=24·0.25=6
- **W**=24
- **σ**=0.25·8=2

$$I = \frac{MC}{W\sigma} = \frac{8 \cdot 6}{24 \cdot 2} = 1$$



Segregation vs Tolerance



- 3x3 neighborhood (up to 8 neighbors), torus edges (period boundary)
- Boxplots of I after convergence in several simulations
- Moran's I stays low for low F values
- Sharp increase above F=0.25 (2 different neighbors)
- Substantial segregation for F>0.33

There is spontaneous emergence of Segregation even if agents tolerate living in minority!

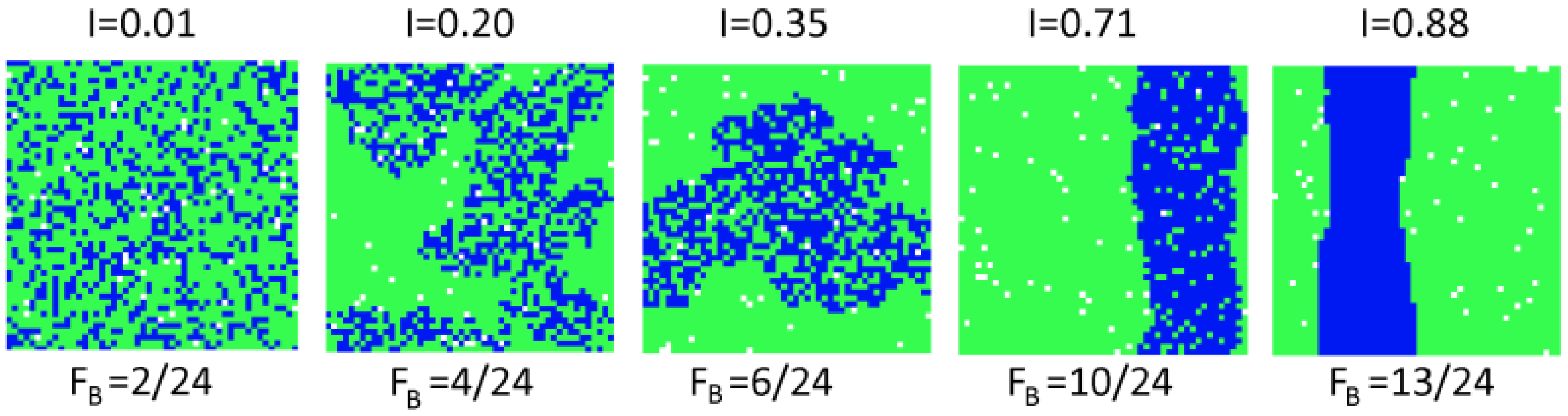
Possible Generalizations

We can generalize the model to study minorities

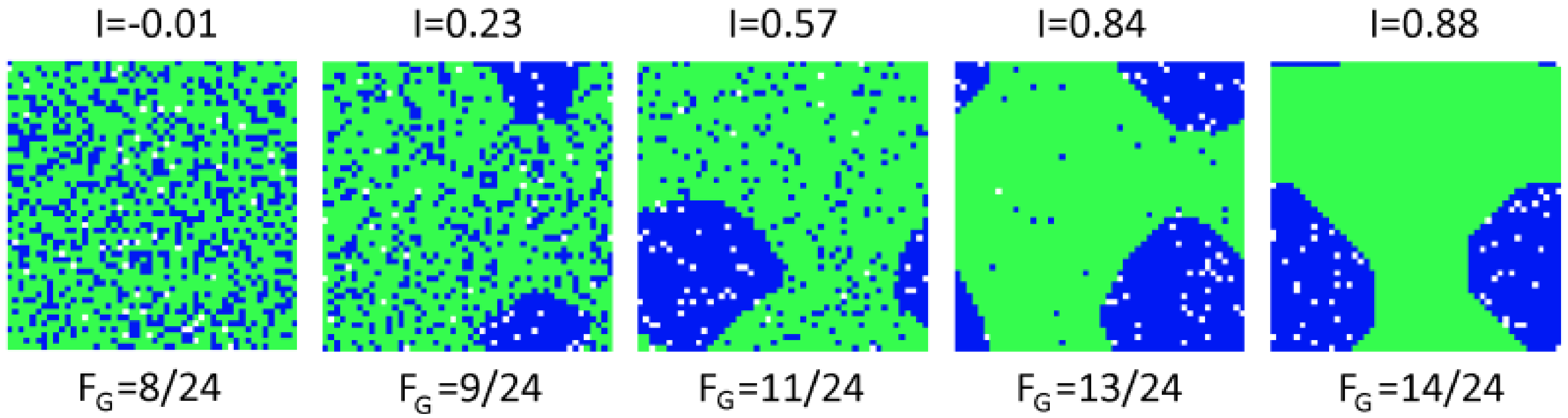
- different group sizes
- different tolerances

We can observe the coexistence of coherent regions with only one color and regions where the two types of agents are mixed.

[The Schelling Model of Ethnic Residential Dynamics: Beyond the Integrated - Segregated Dichotomy of Patterns. Erez Hatna and Itzhak Benenson. Journal of Artificial Societies and Social Simulation, 2012]



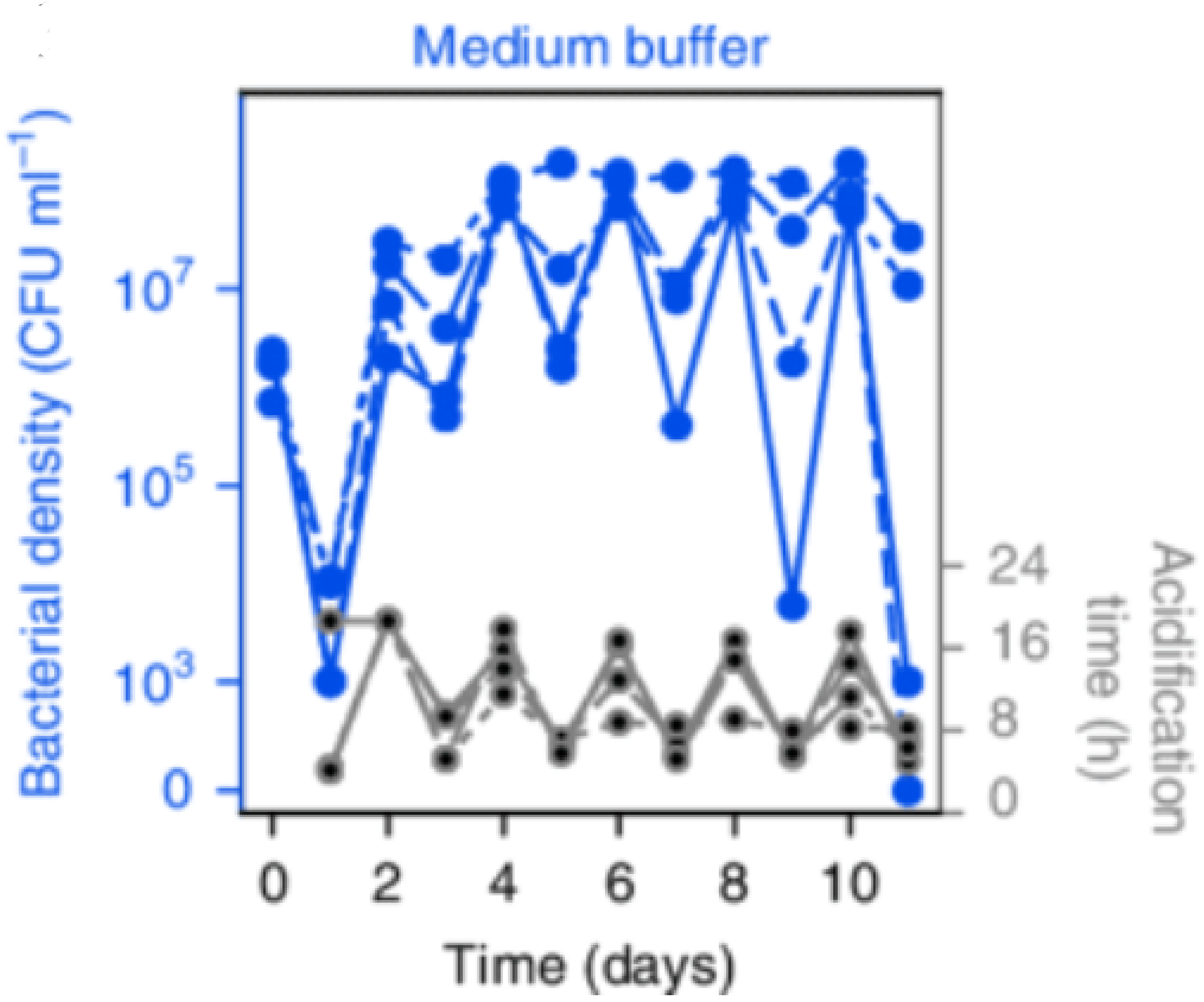
Completely tolerant Green majority (FG = 0)



Completely tolerant Blue minority (FB = 0)

Cellular Automata: The game of life

Deterministic Chaos



[Ecological suicide in microbes]

Chaotic is not random!

We model the evolution of a population of animals or bacteria with the **Logistic Map**:

- $x(t)$=ratio of existing population to the maximum possible population

$$x(t+1)=r \cdot x(t) \cdot [1-x(t)]$$

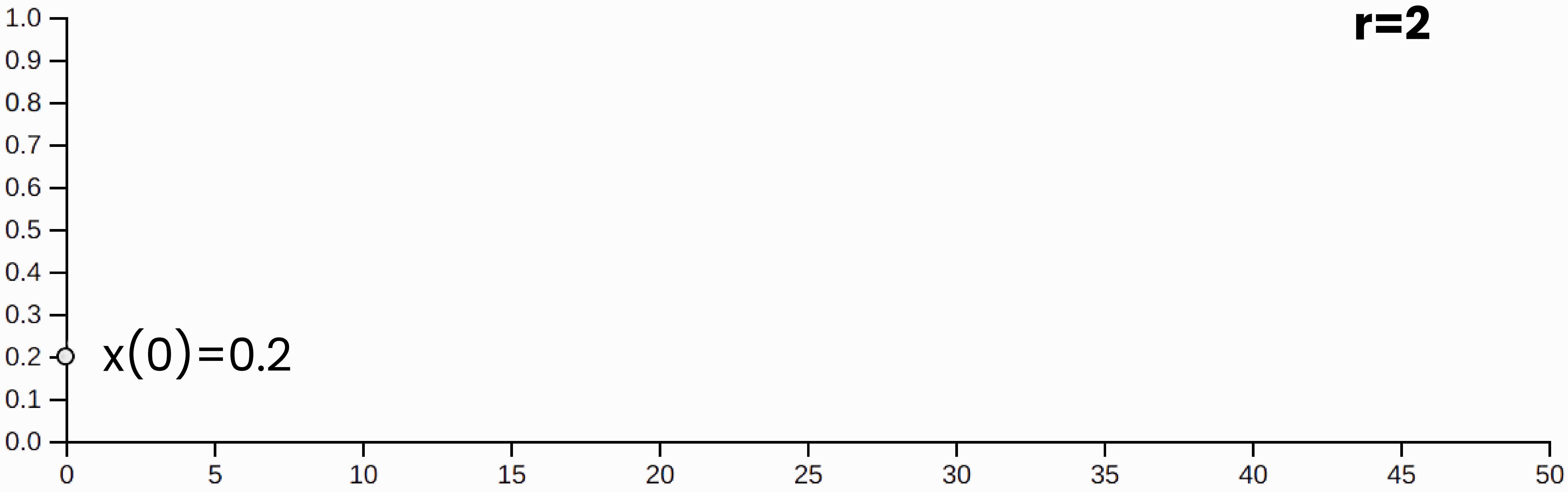
- $0<r<4$ is the parameter of the model

Two effects:

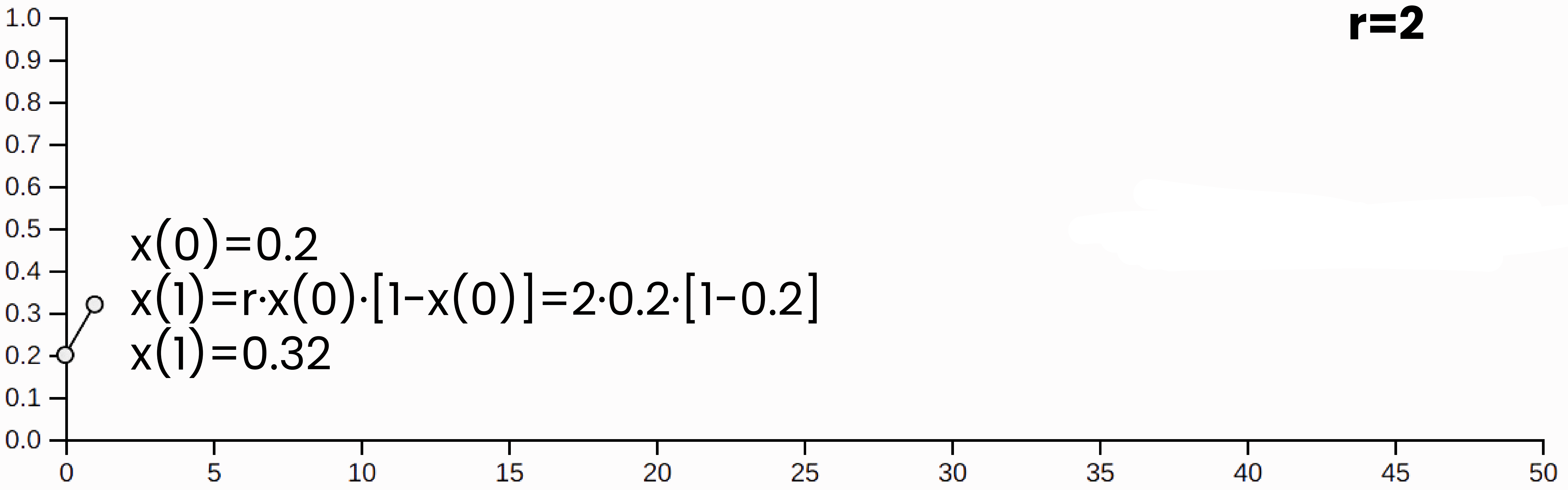
- **reproduction** when the population size is small the population grows
- **starvation** when the population size is large the population decreases

This model is completely deterministic!

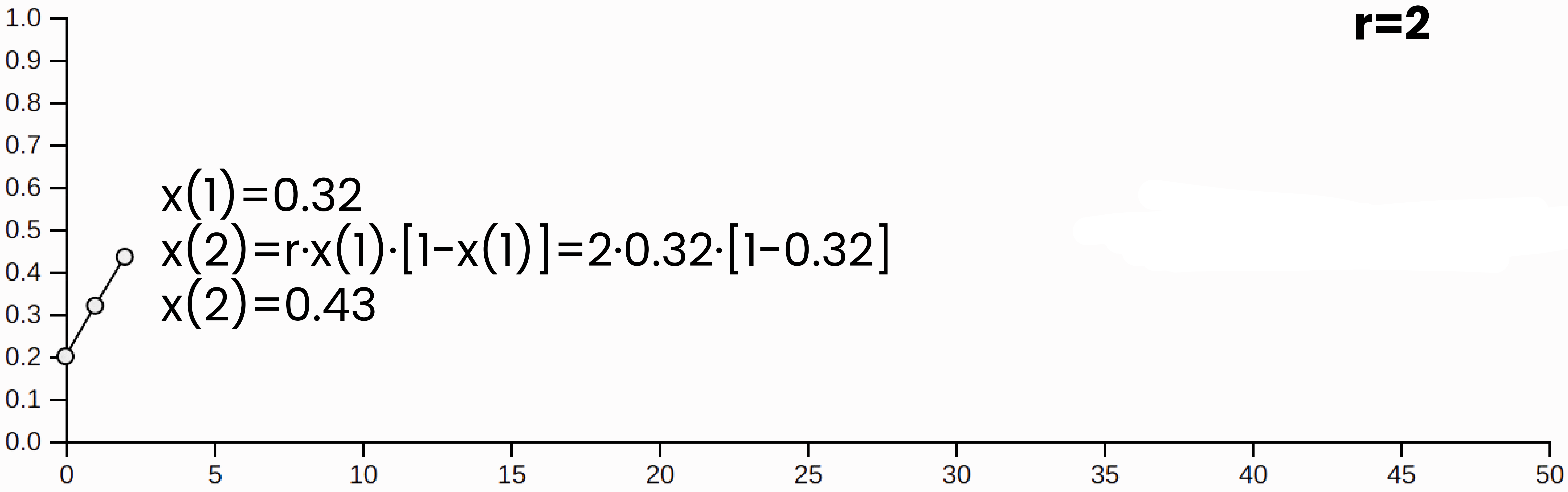
Example of Evolution



Example of **Evolution**

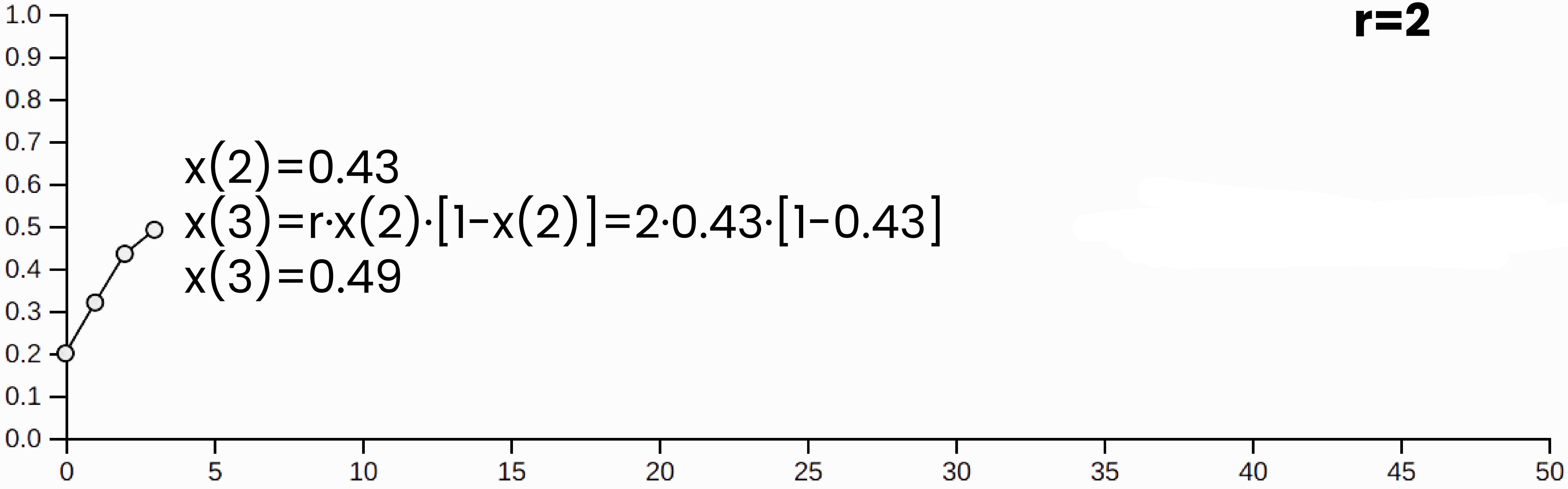


Example of Evolution



Example of Evolution

r=2

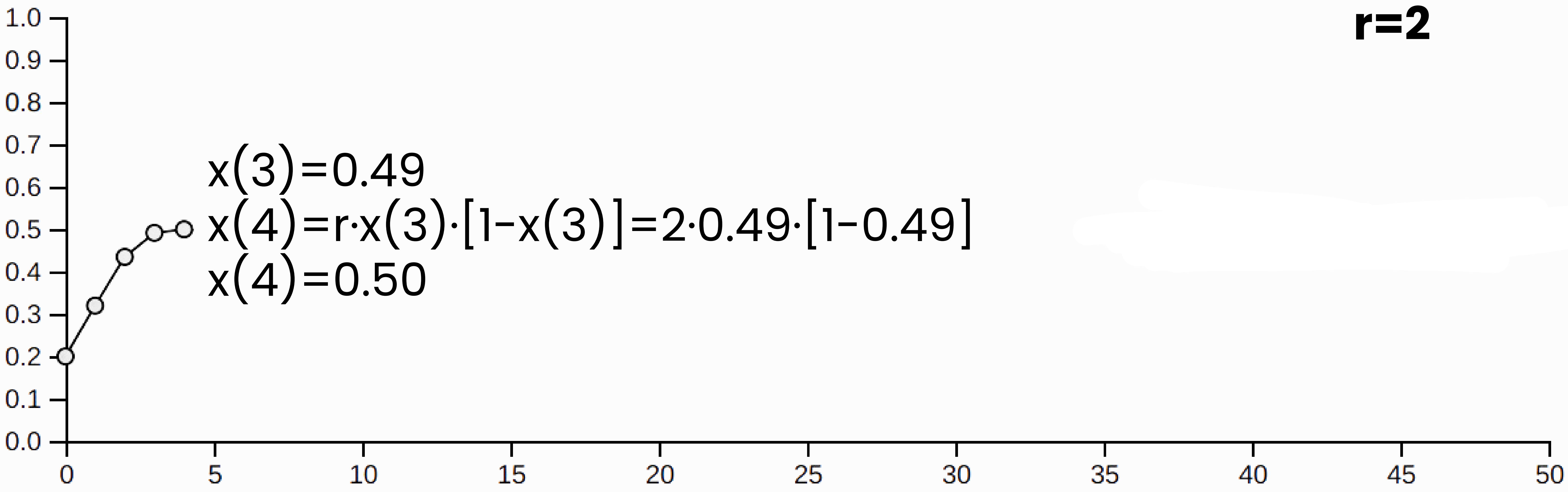


$x(2)=0.43$

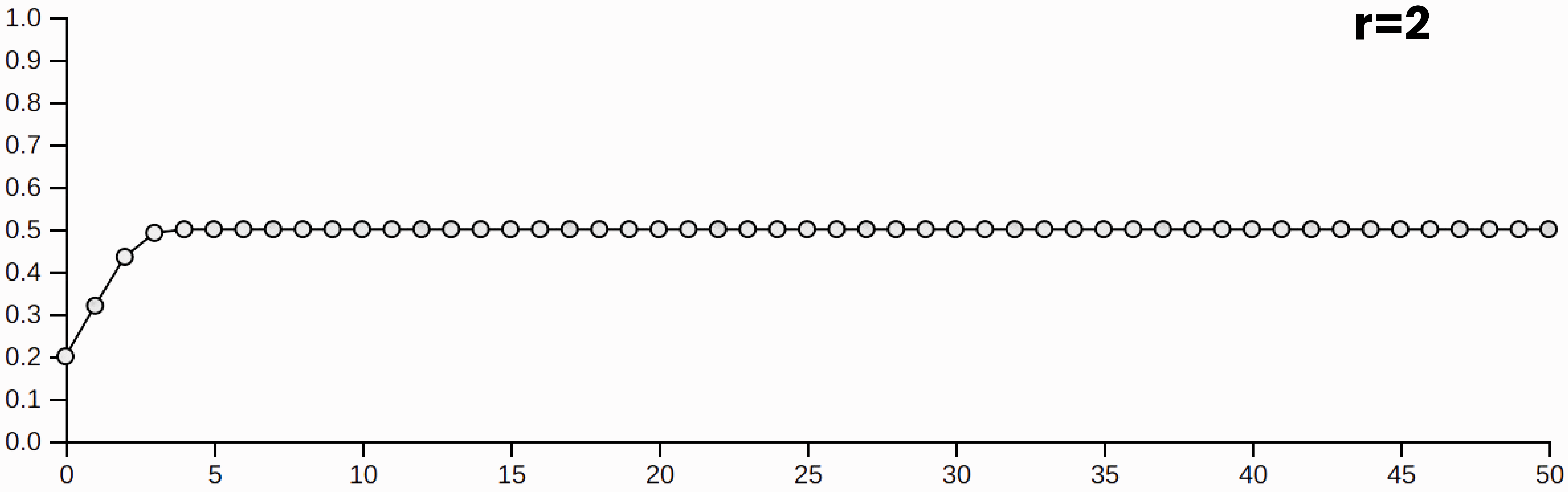
$x(3)=r \cdot x(2) \cdot [1-x(2)]=2 \cdot 0.43 \cdot [1-0.43]$

$x(3)=0.49$

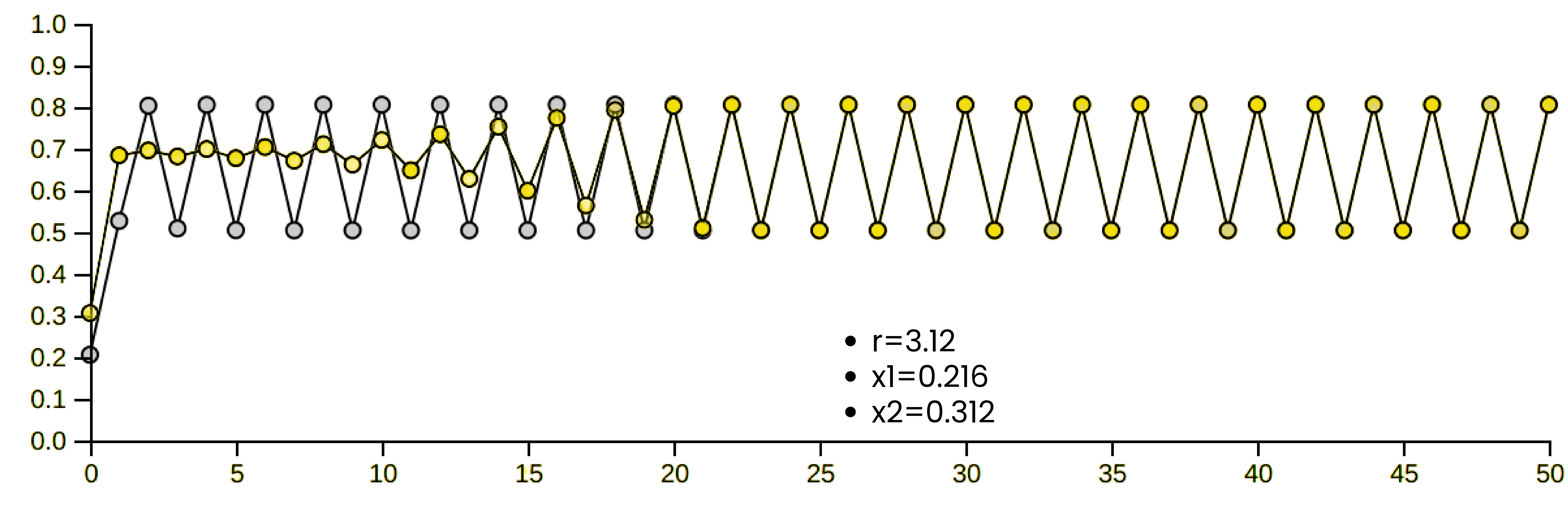
Example of Evolution



Example of Evolution

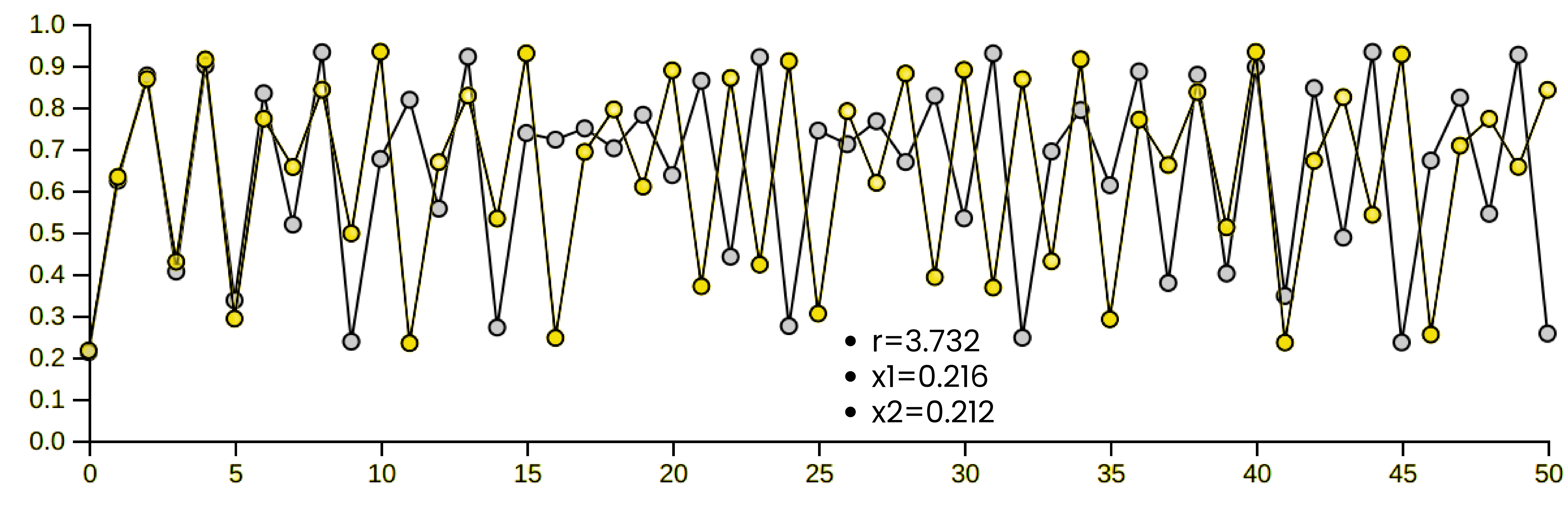


Non-Chaotic Trajectories



- The behavior is regular
- Two moderately far initial points results in very similar trajectories

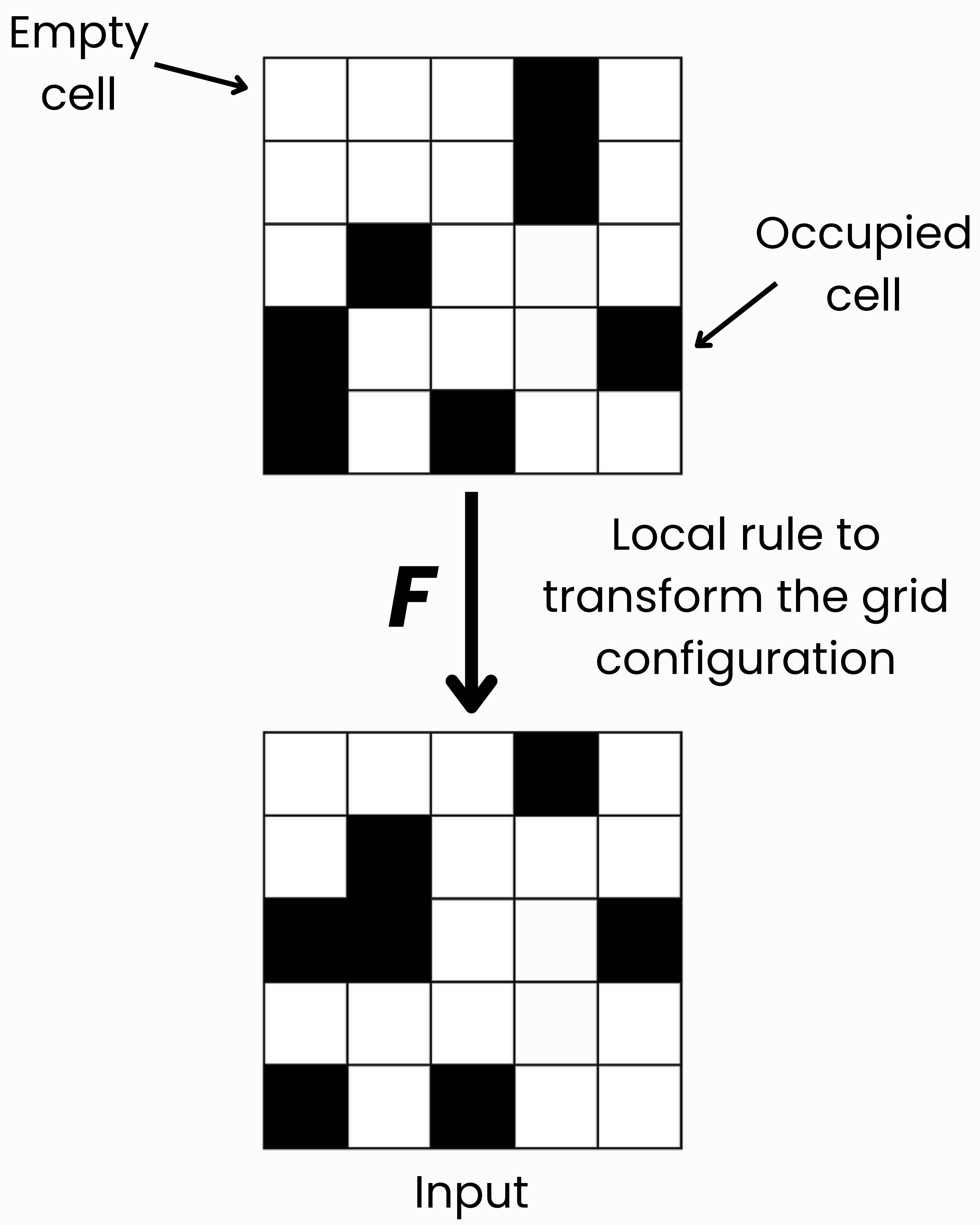
Chaotic Trajectories



- The behavior is apparently randomic
- Two very close initial points results in completely different trajectories

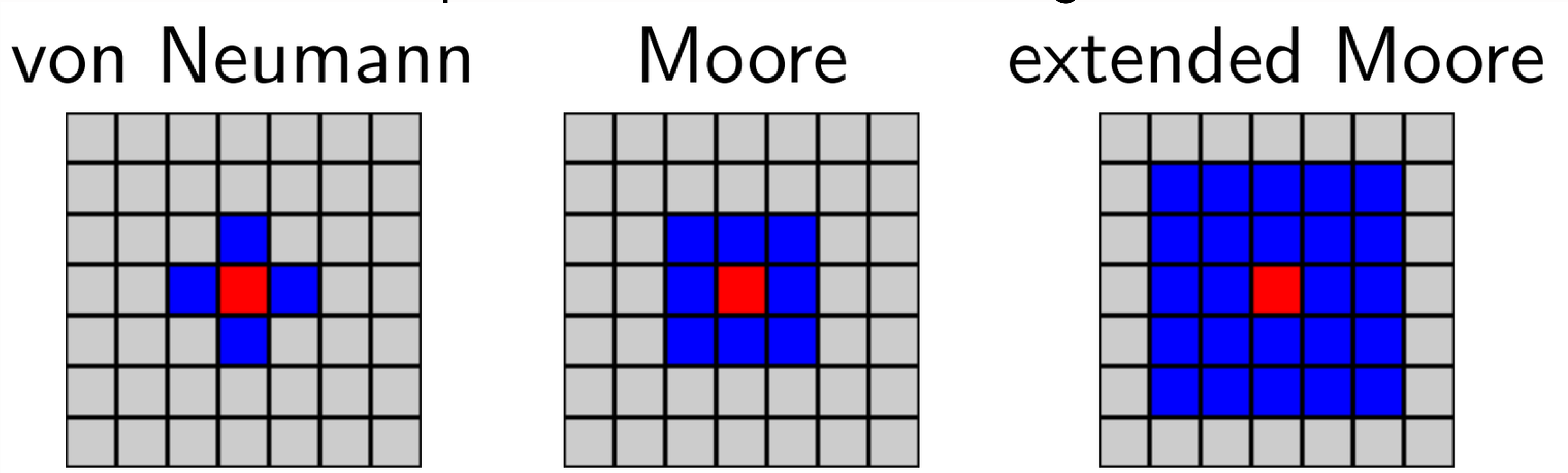
Cellular Automata

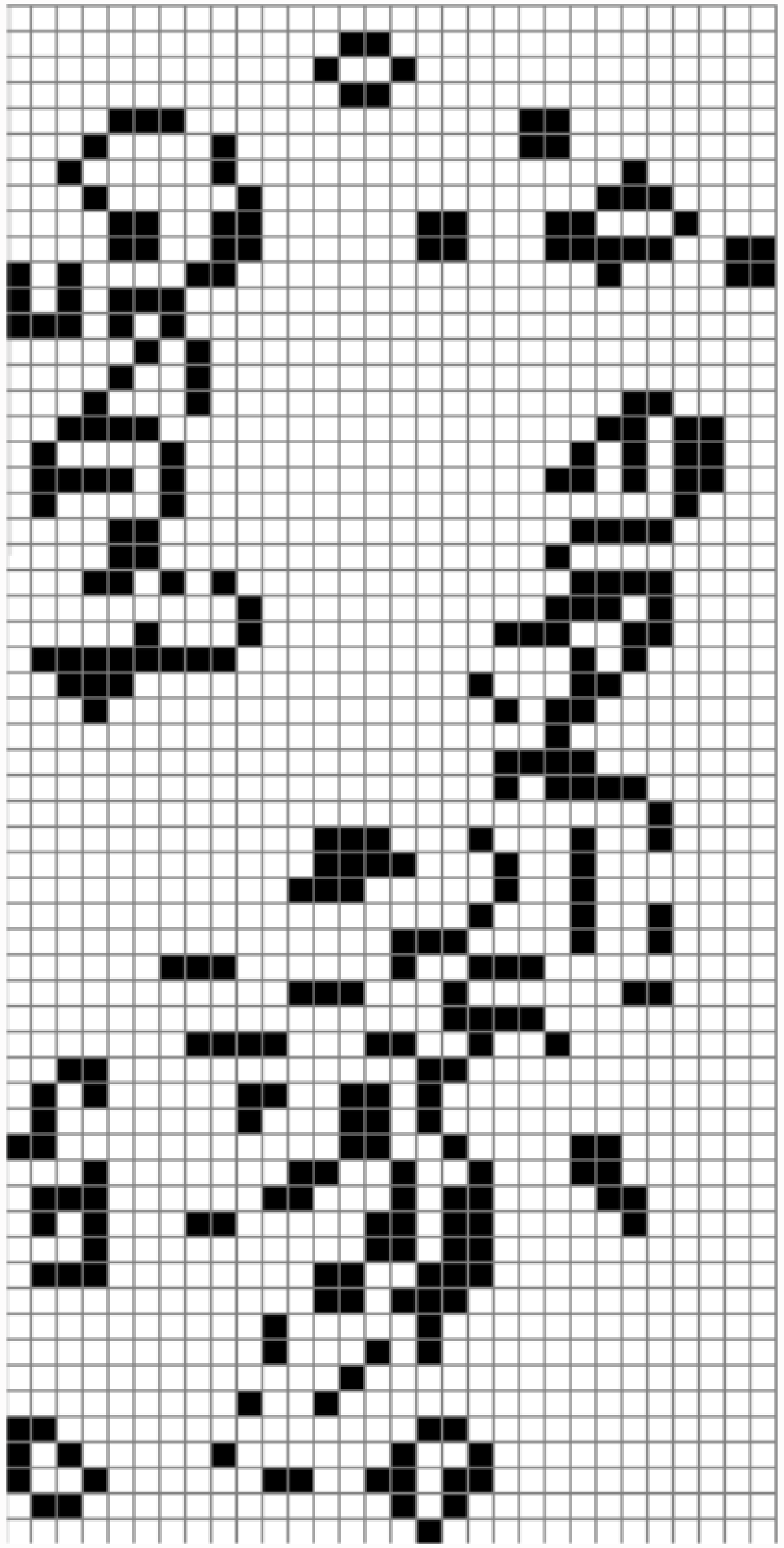
- Each agent is a cell of a lattice or grid
- formal structure of Cellular Automata
  - cell i internal state θi(t)=0/1 (empty/occupied)
  - neighborhood configuration: βi(t)
  - the state at time t+1 is function of the state and of the neighborhood at time t
    $$\theta_i(t+1) = F[\theta_i(t), \beta_i(t)]$$
- characteristics:
  - discrete space, time, and states
  - same transition rule for all cells
  - each cell is an agent (!)



Lattices and Neighbourhoods

- In computers we have finite memory, so finite cell space with borders
  - option 1: mirror border cells
  - option 2: periodic boundary condition: A Torus (donuts)
- Not only squares, but also triangular, hexagonal and even irregular tilings possible (e.g. Voronoi tesselations)
- We also have different possible definition of neighborhood





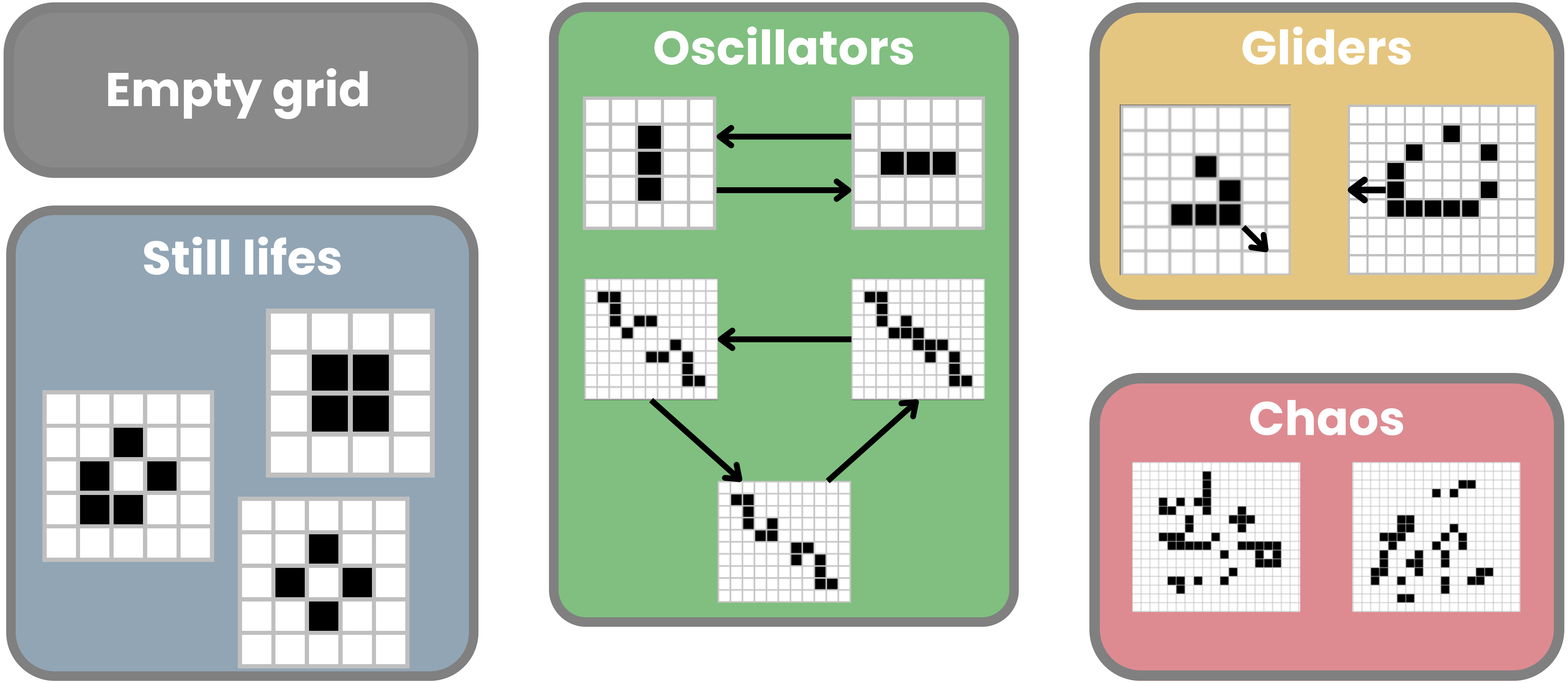
Conway's Game of Life

Simple cellular automaton with the following rules:

- 2D rectangular infinite grid with "dead" (white) and "alive" (black) cells
- a dead cell becomes alive if 3 neighbours are alive - reproduction
- an alive cell dies if less than 2 neighbours are alive - underpopulation
- an alive cell dies if more than 3 neighbours are alive - overcrowding

Complex behavior emerges from simple deterministic rules for agents!

Asymptotic Behavior



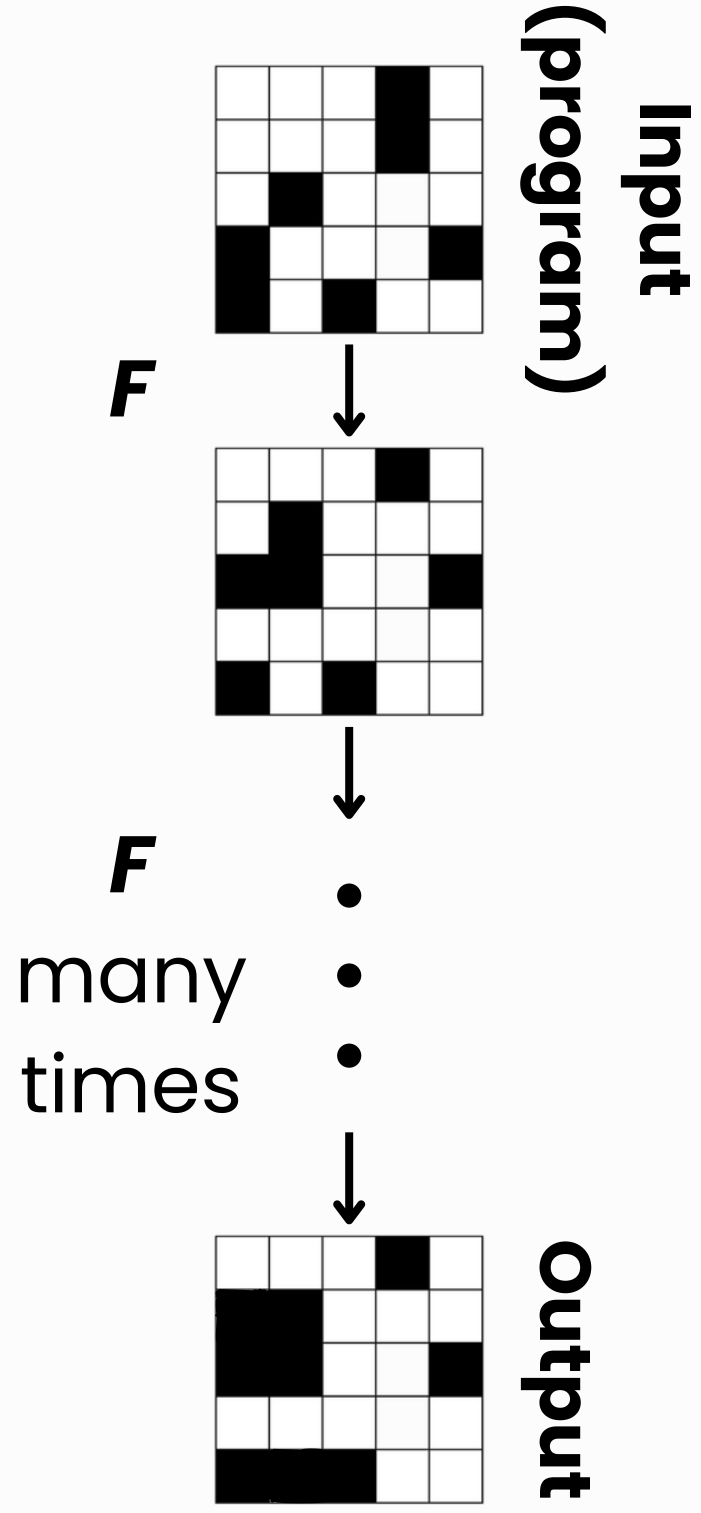
Chaos and Turing Completeness

The Game of Life is Turing Complete!

- you can implement NOT, AND, OR
- the Game of Life is equivalent to your smartphone or laptop
- you can run Life in Life

The “program” you execute is the initial configuration and the output is the final configuration

- a program ends when an empty grid, a still life, an oscillator or a glider (or combination of these) is obtained
- it is impossible to say if a program will ever finish to run (halting problem)
- complex chaotic behavior emerges



Conclusions

Spatial Segregation

Can it emerge without resource limits or central discrimination/laws?

Schelling's model of segregation

Agents in a lattice that relocate when their neighborhood similarity is below a threshold (Schelling, 1971)

Analyzing Schelling's model with Morans' I

Segregation emerges below 0.5, as low as 0.33 (Hatna & Benenson, 2012)

Game of Life and Logistic Map

Chaos and complex behavior can emerge from simple deterministic rules (Gardner, 1970)

Play Yourself to Understand!

Schelling's Model
http://nifty.stanford.edu/2014/mccown-schelling-model-segregation/

Logistic Map
https://kylepaulsen.com/stuff/game_of_life.html

The Game of Life
https://kylepaulsen.com/stuff/game_of_life.html