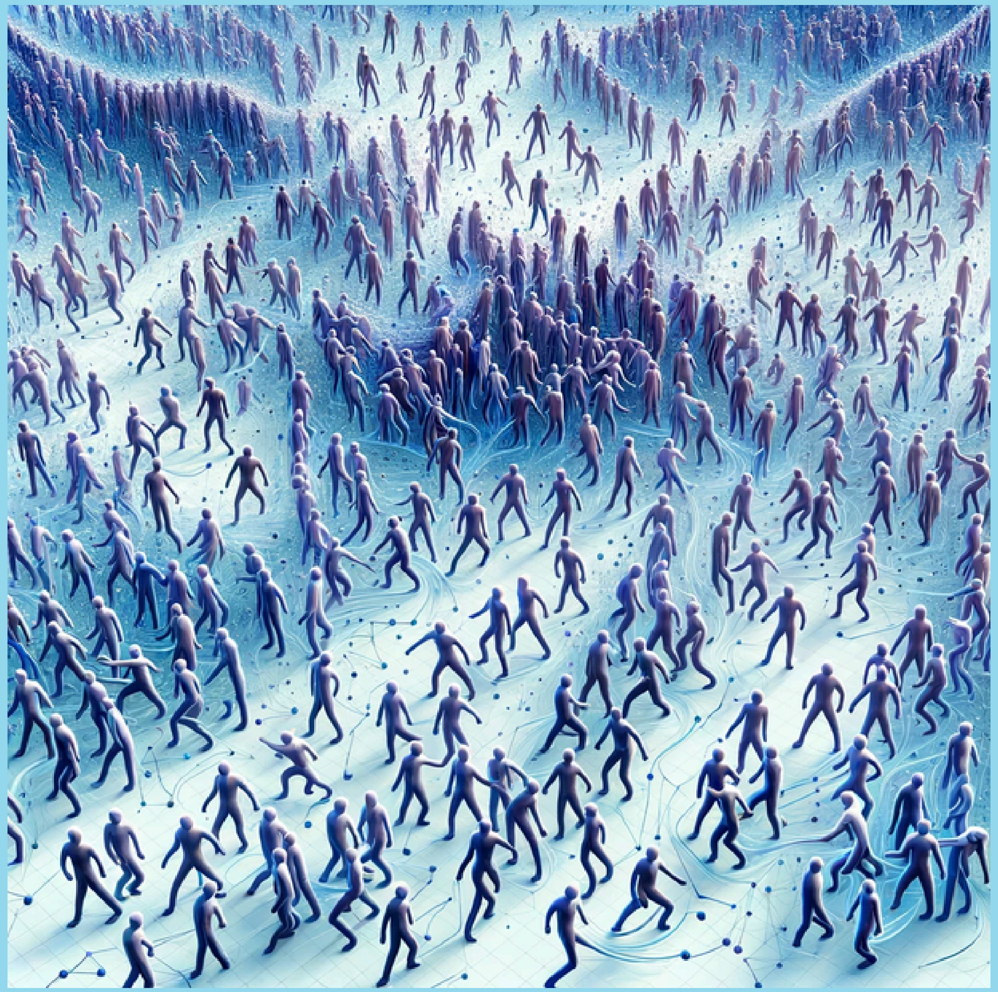

Universität Konstanz

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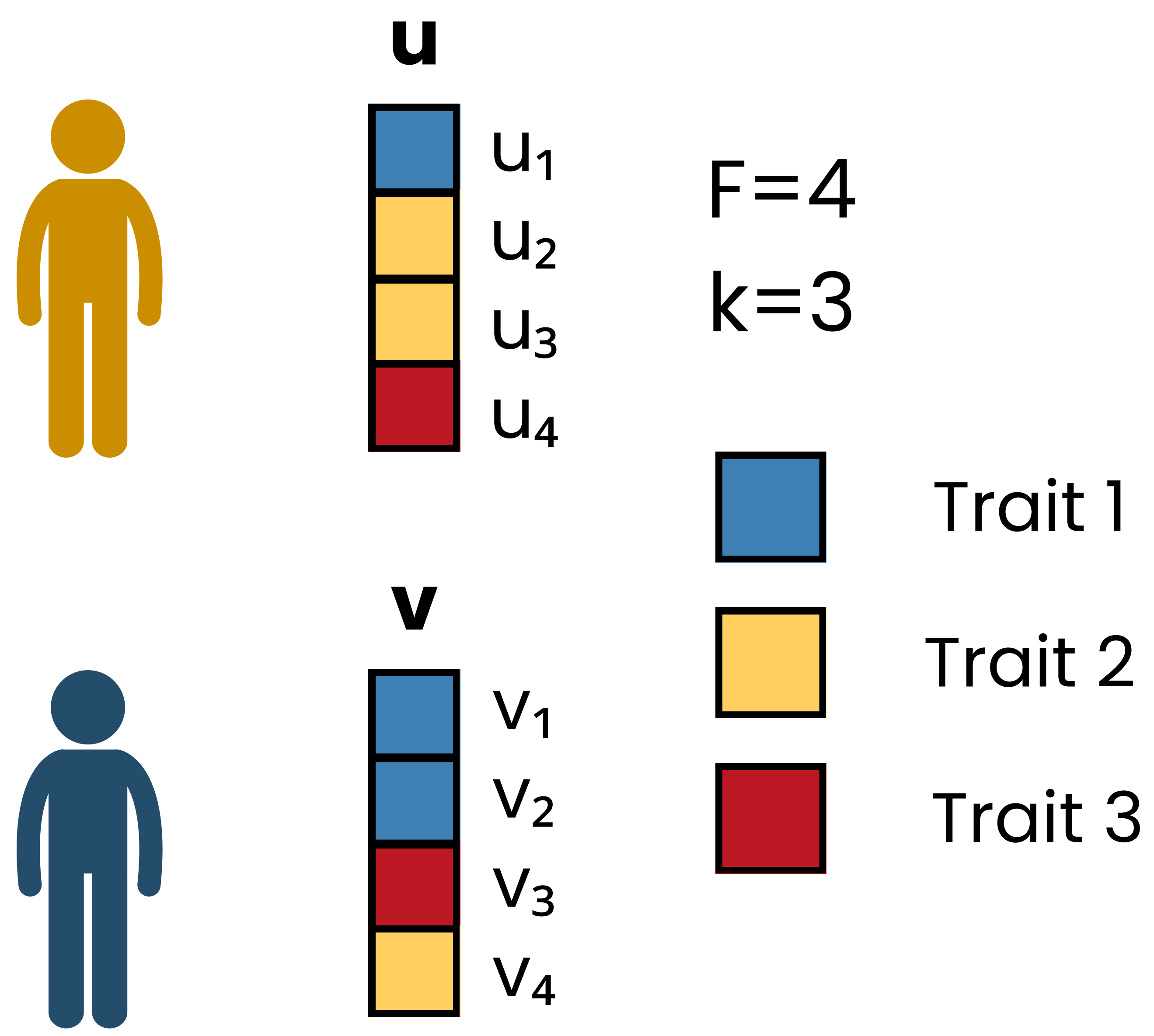
Diversity, Minorities and Granovetter's Model

Computational Modelling of Social Systems

Giordano De Marzo
Max Pellert

Recap



Culture and Language

How do culture and language form? How people manage to reach a consensus?

Axelrod's model

Simple model of culture that produces local consensus and global polarization.

The Naming Game

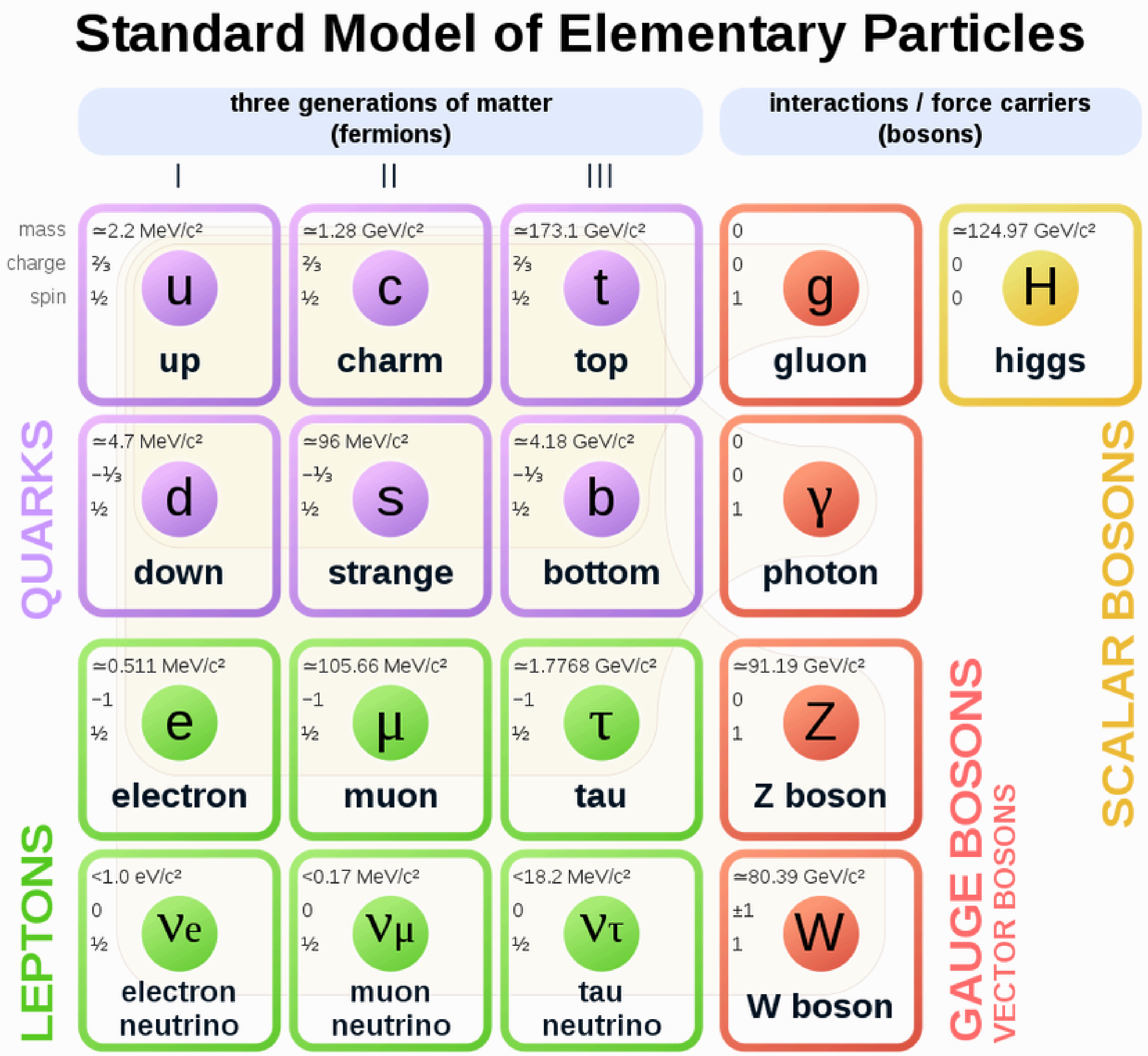
Models how a group of people reaches unanimity about how to name objects.

Outline

1. Collective Behavior and Complex Systems
2. Granovetter's Threshold Model
3. Analysis of Granovetter's Threshold Model
4. Can a Minority Win?

Collective Behavior and Complex Systems

More is **Different!**



"*The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe. [...] Psychology is not applied biology, nor is biology applied chemistry. [...] At each level of complexity entirely new properties appear.*" - Philip Anderson

[More is different: broken symmetry and the nature of the hierarchical structure of science. Philip Anderson, Science (1972)]

Complicated or Complex?

Complicated System

Example: a mechanical watch

- pieces have specific functions and well-defined relationships
- carefully engineered or designed
- it is easy to infer global behavior and understand outcomes of modifications

Complex System

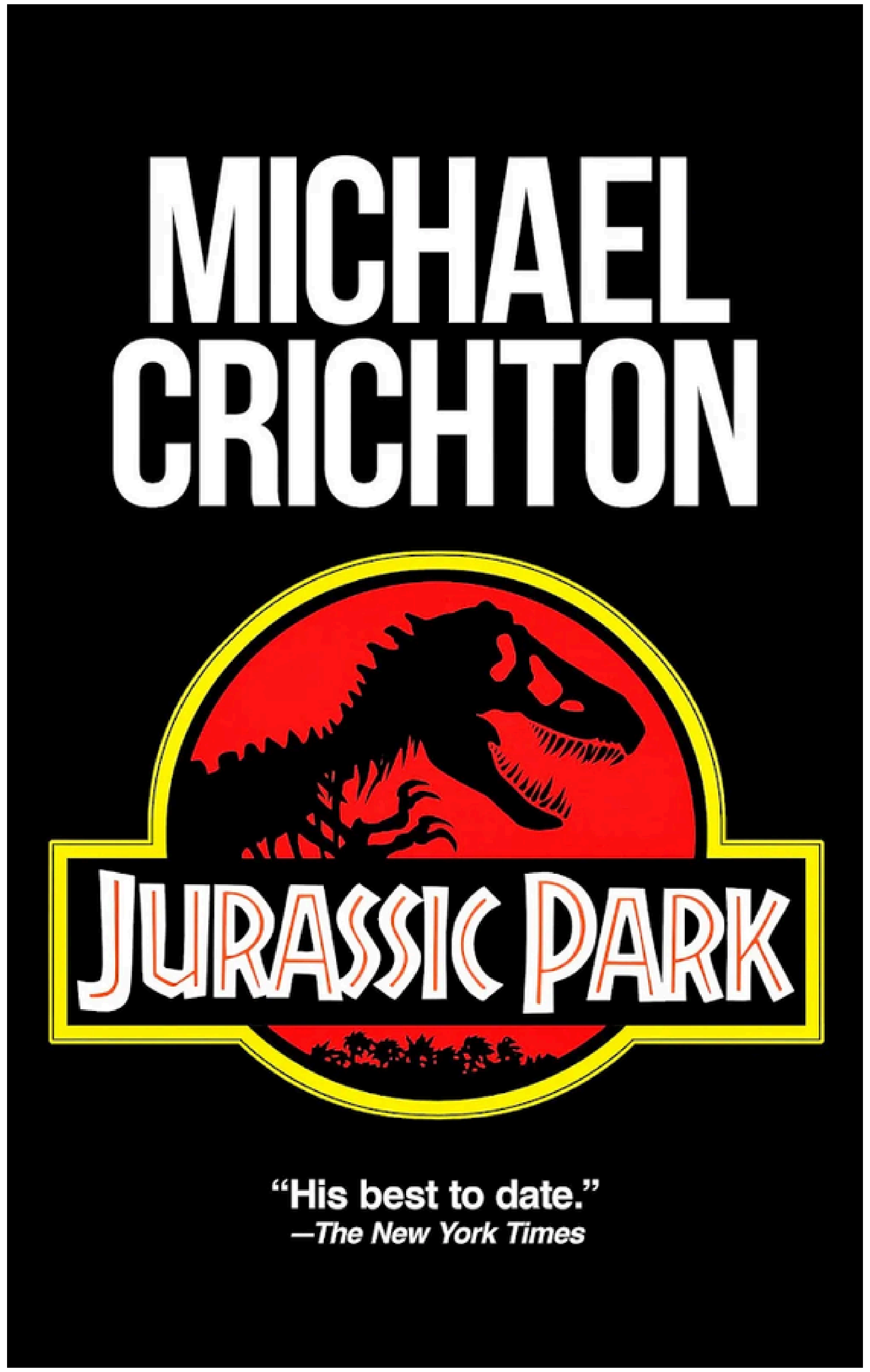
Example: a human cell

- pieces have unknown functions and relationships
- Self-organized, no external project
- it is hard to infer global behavior and understand outcomes of modifications

Jurassic Park, Chaos and Complexity

Jurassic Park is not a book about dinosaurs, it is a book about chaos and complex systems!

"Chaos theory throws it right out the window. It says that you can never predict certain phenomena at all. You can never predict the weather more than a few days away. All the money that has been spent on long-range forecasting-about half a billion dollars in the last few decades-is money wasted. It's a fool's errand. It's as pointless as trying to turn lead into gold."
- Dr Ian Malcolm, Jurassic Park



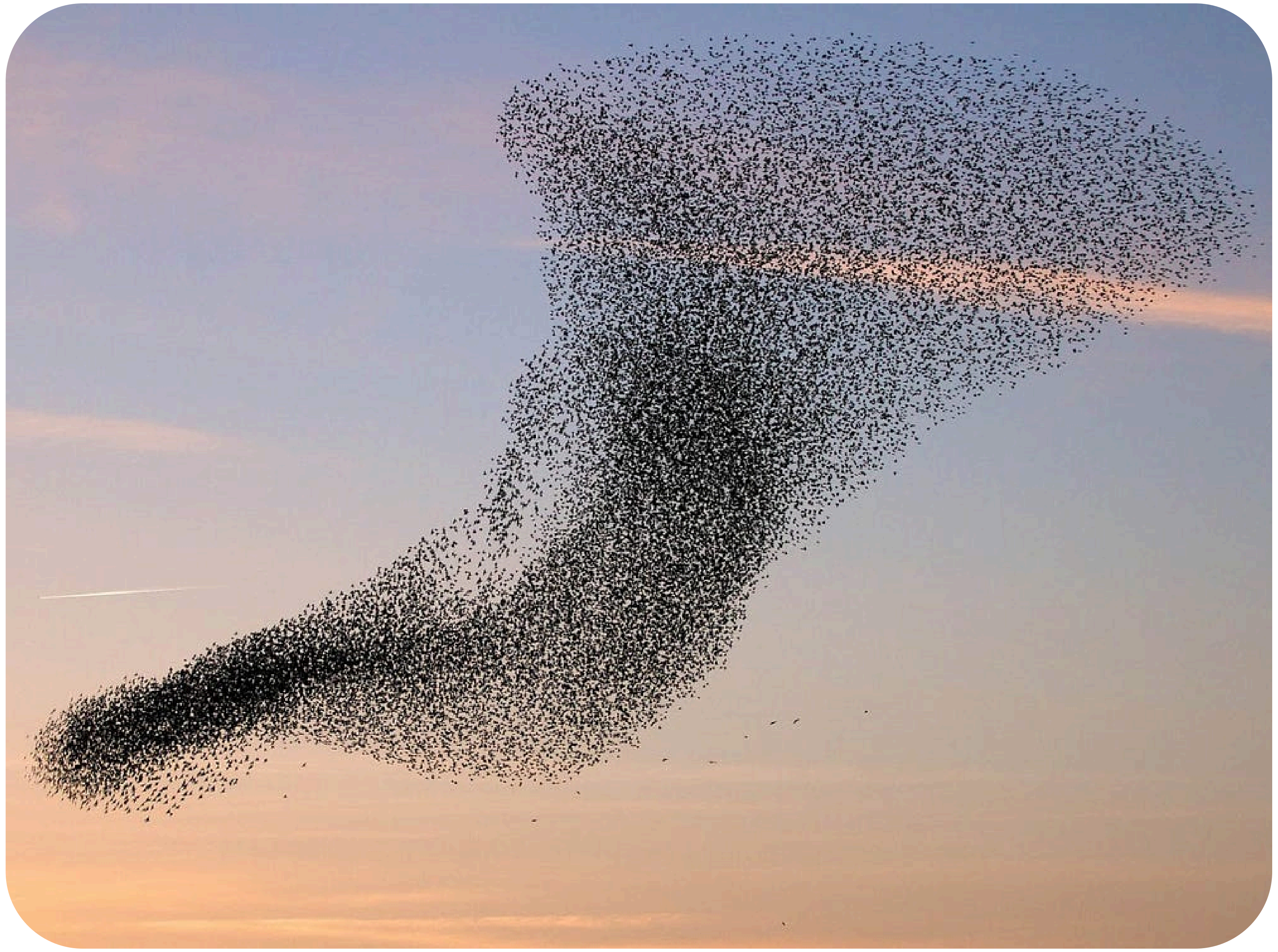
Collective Behavior Once Again

Complex Systems are characterized by emergent collective behaviors.

Nature is full of collective behavior examples:

- flocks of bird
- schools of fish
- ants and bees

Collective behavior can emerge even in very simple animals!



Diversity-Induced Collective Behavior

Interaction-induced collective behavior

The macro behavior depends on the **interaction** between individuals:

- Schelling's model: low tolerance triggers moves that lead to segregation
- Alxelrod's model: cultural exchange leads to larger cultures or supports coexistence of few cultures

Diversity-induced collective behavior

The macro behavior emerges from **differences** between individuals. Same interaction pattern can lead to very different outcomes

Granovetter's Threshold Model

The Riot Toy Example

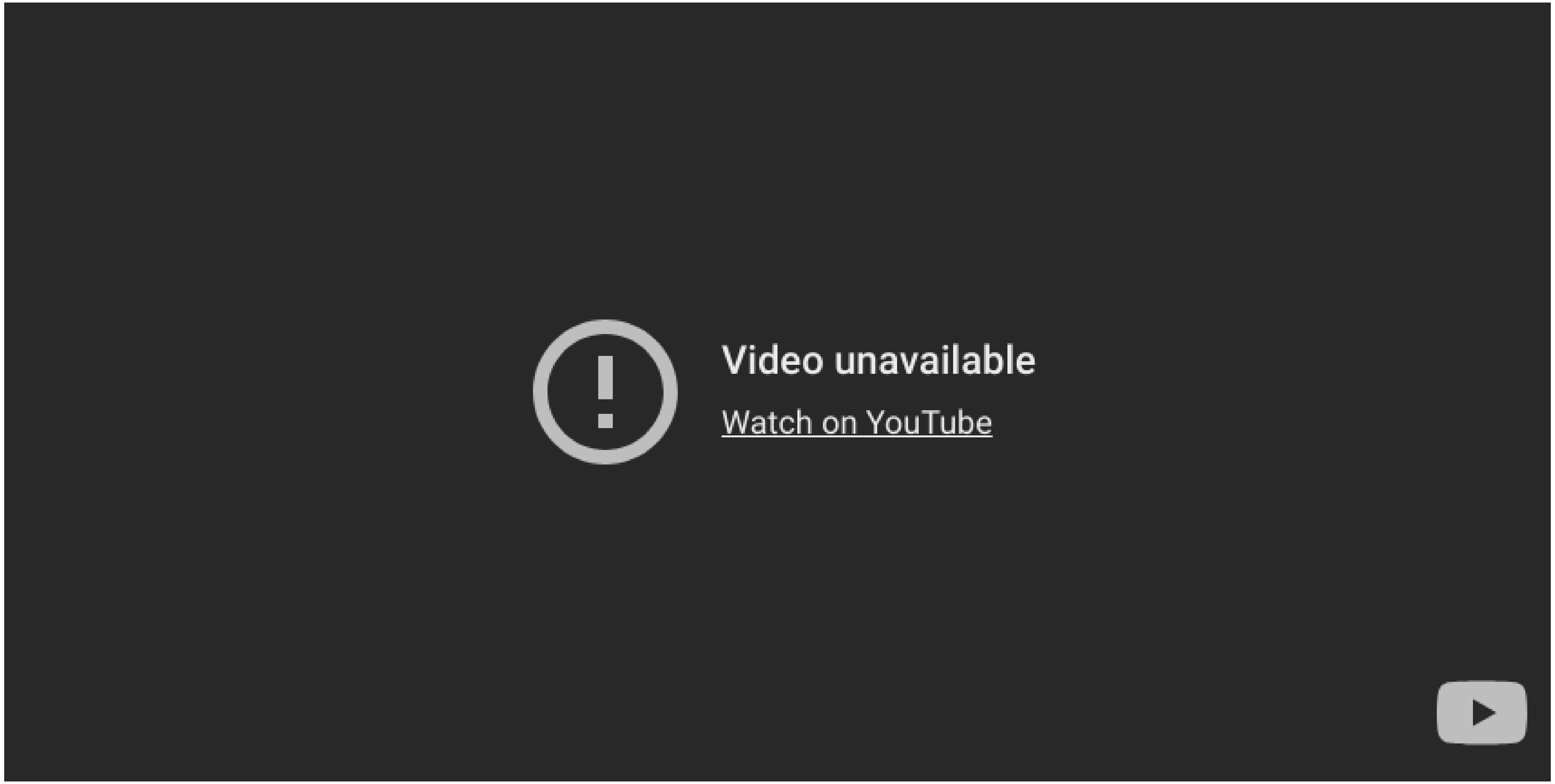
A group of individuals is part of a demonstration:

- Individuals have a **threshold** of how many others have to be rioting to join the riot
- If enough people are in the riot, individuals with lower threshold join too

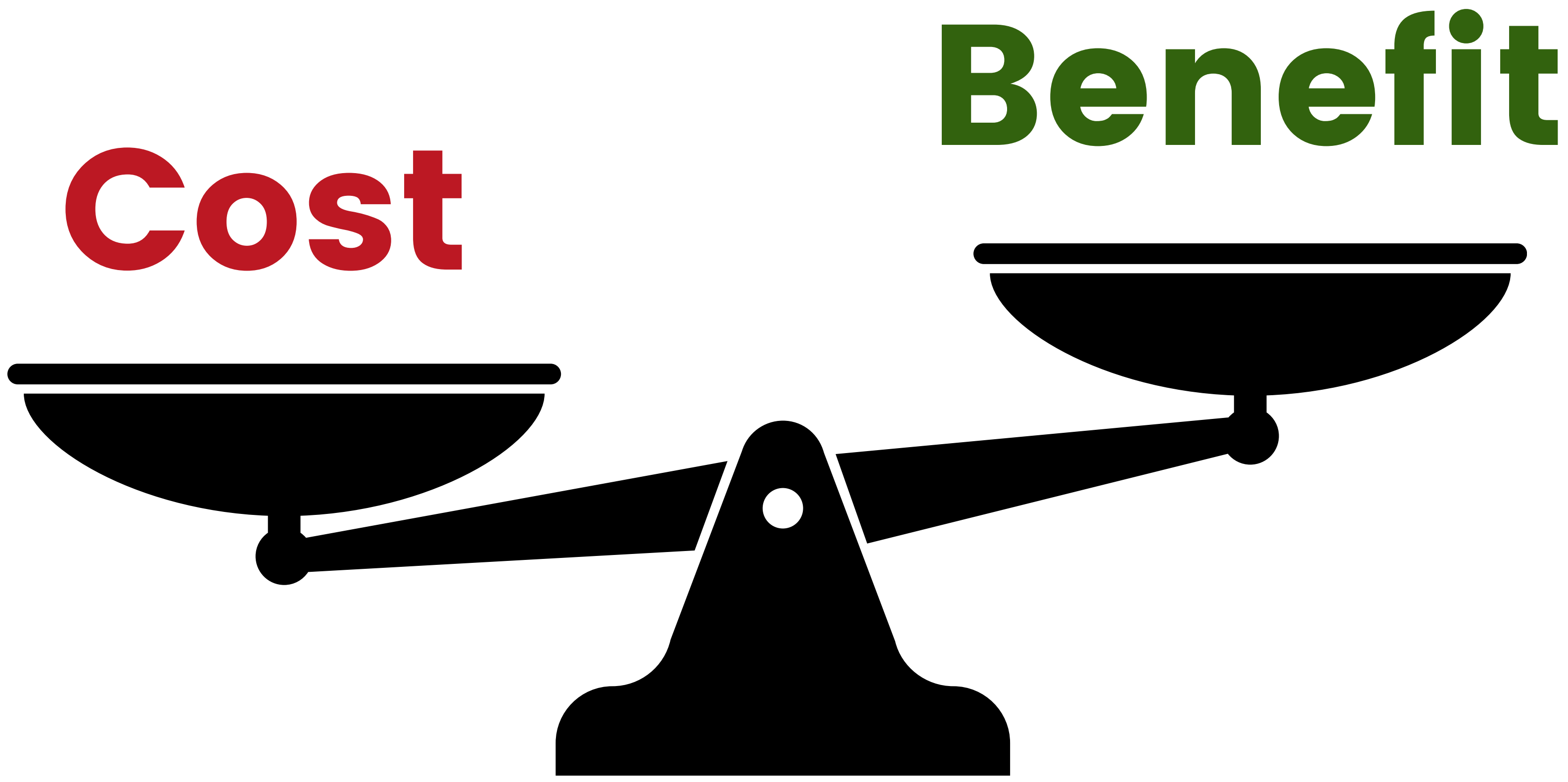
This is an example of **binary opinions**

- Proto-opinion: just participate / not participate
- Other examples with binary decisions depending on size: Diffusion of innovations, rumors, strikes, voting...

An Example of Spreading



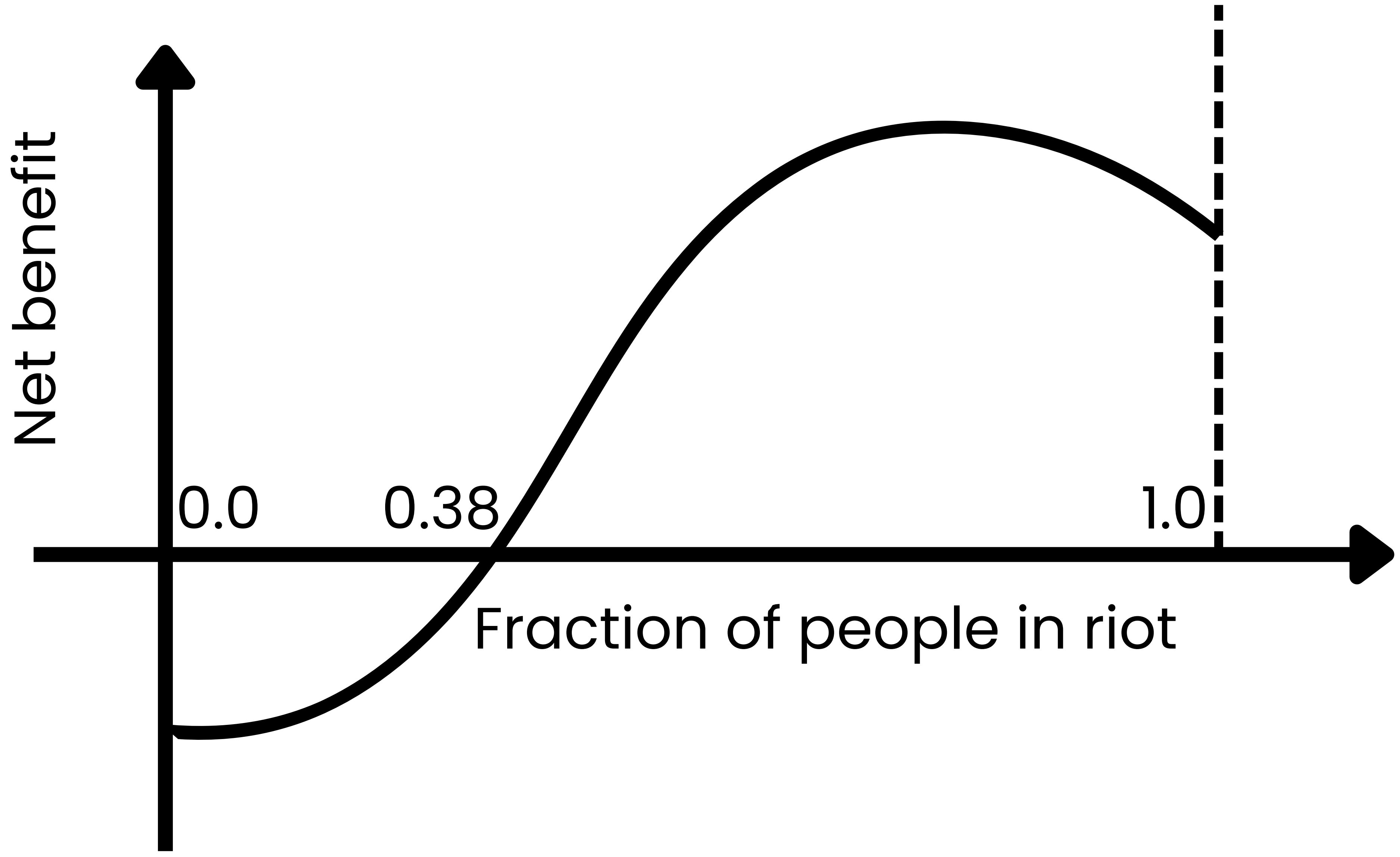
Rational Agents in Collective Actions



We assume agents to be **rational**, so the decision to join the collective action depends on:

- **Risk** or **cost** of participating.
  - Risk of being jailed in riot
  - Wage loss in strike
  - Cost of technology adoption
- The **benefit** (potential or sure) of the action taking place.
  - Political change after demonstration
  - Political party winning an election
  - Profit out of adopting innovation

Net Benefit and Thresholds



Net benefit = benefit - costs

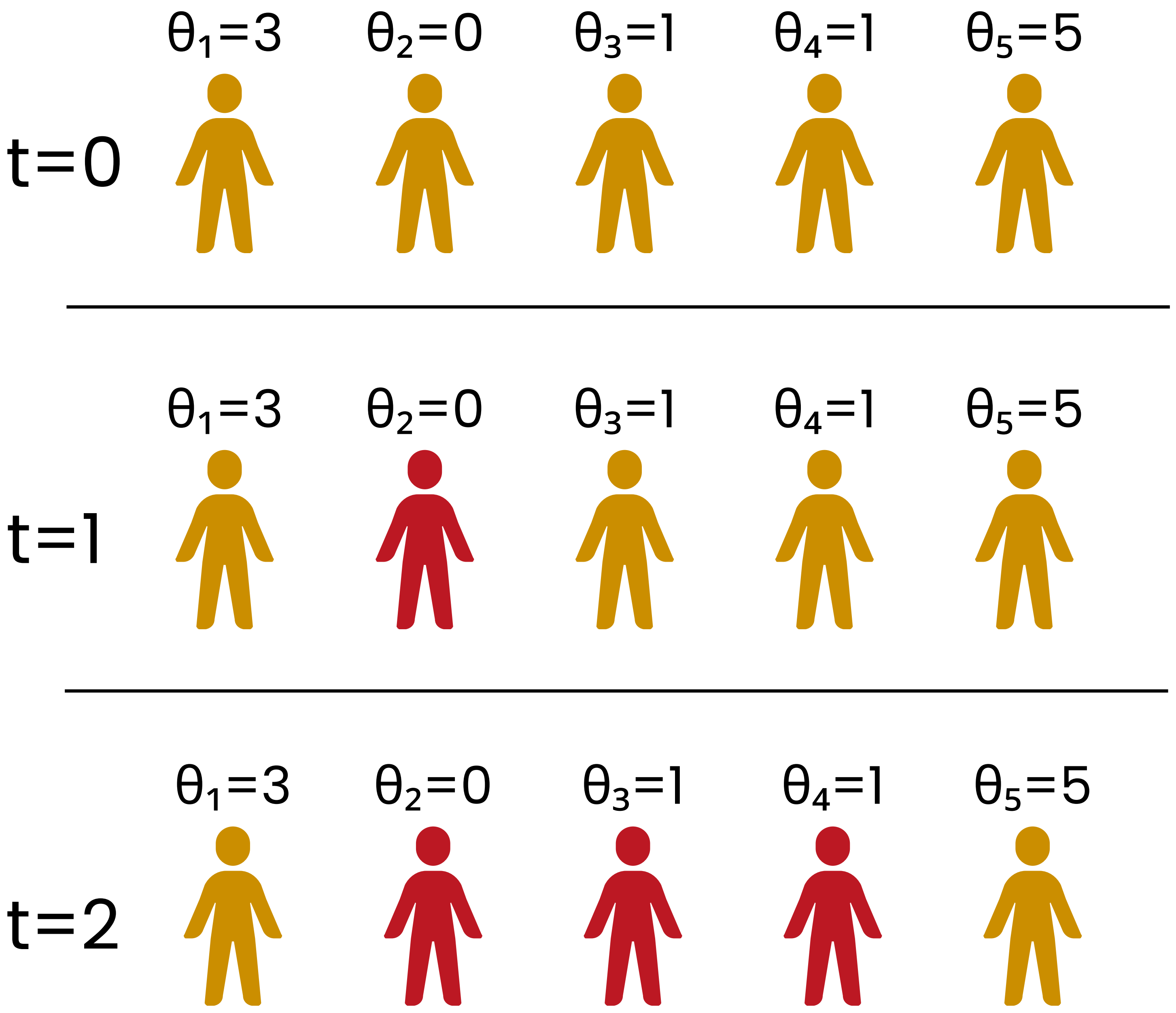
- Threshold to join: Net benefit must be positive (>0)
- benefits increase and costs decrease with more people in the action
  - group effect on social network
  - less possibility to be arrested in riot
  - economy of scale following technology adoption
- weaker assumption: there is only one crossing of zero in the function of net benefit vs people in action

Questions on Spreading and Diversity

We want to understand the role of **Diversity** in inducing the spreading of ideas or behaviors

- How does the distribution of preferences (thresholds) in a population affect its collective behavior?
- Knowing the preferences does not directly tell you how the population will behave, you need to analyze how the population behaves
- Aim: understanding groups beyond the representative "mean" member

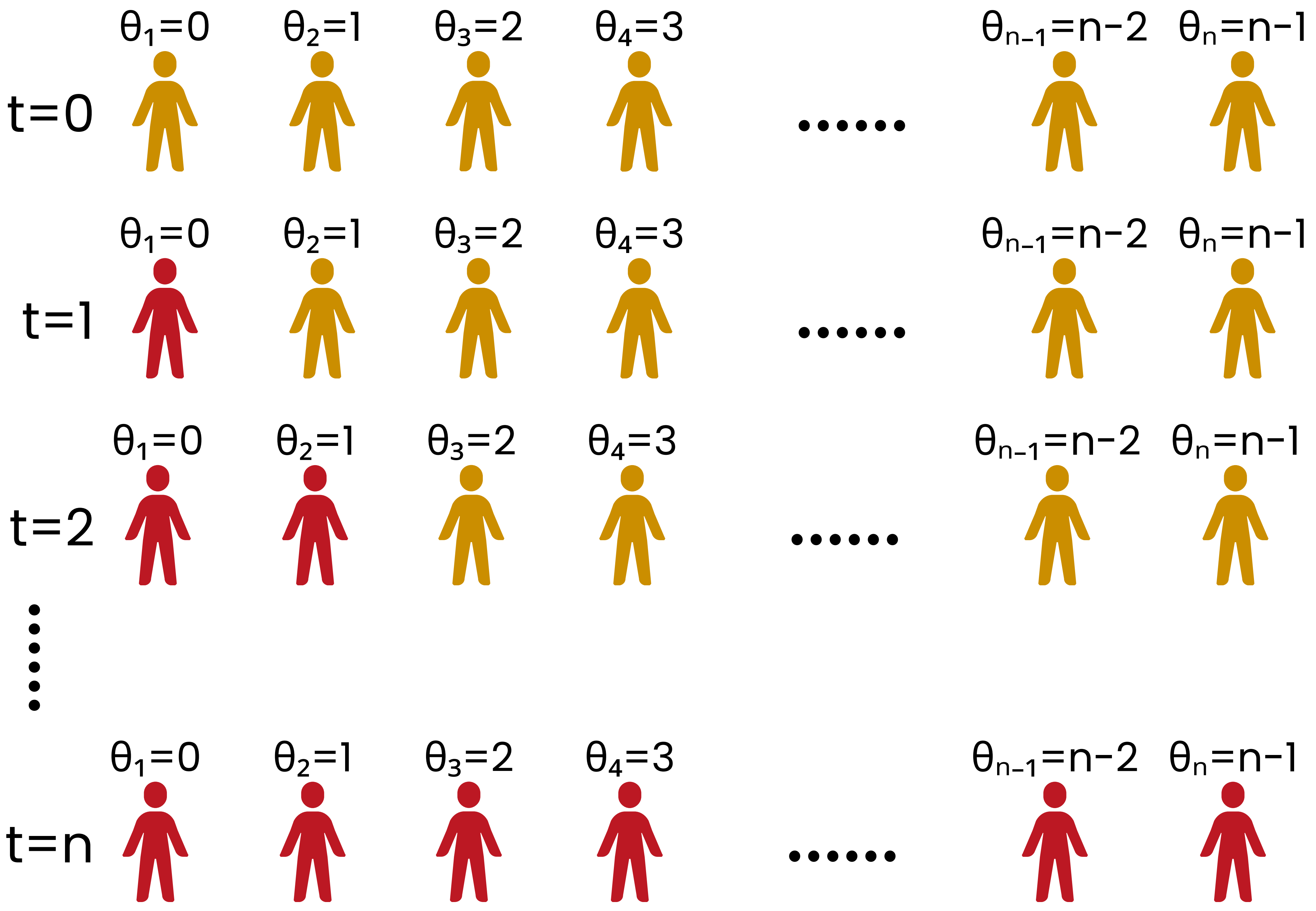
Granovetter's Model



Granovetter's model schematizes the process of joining a riot

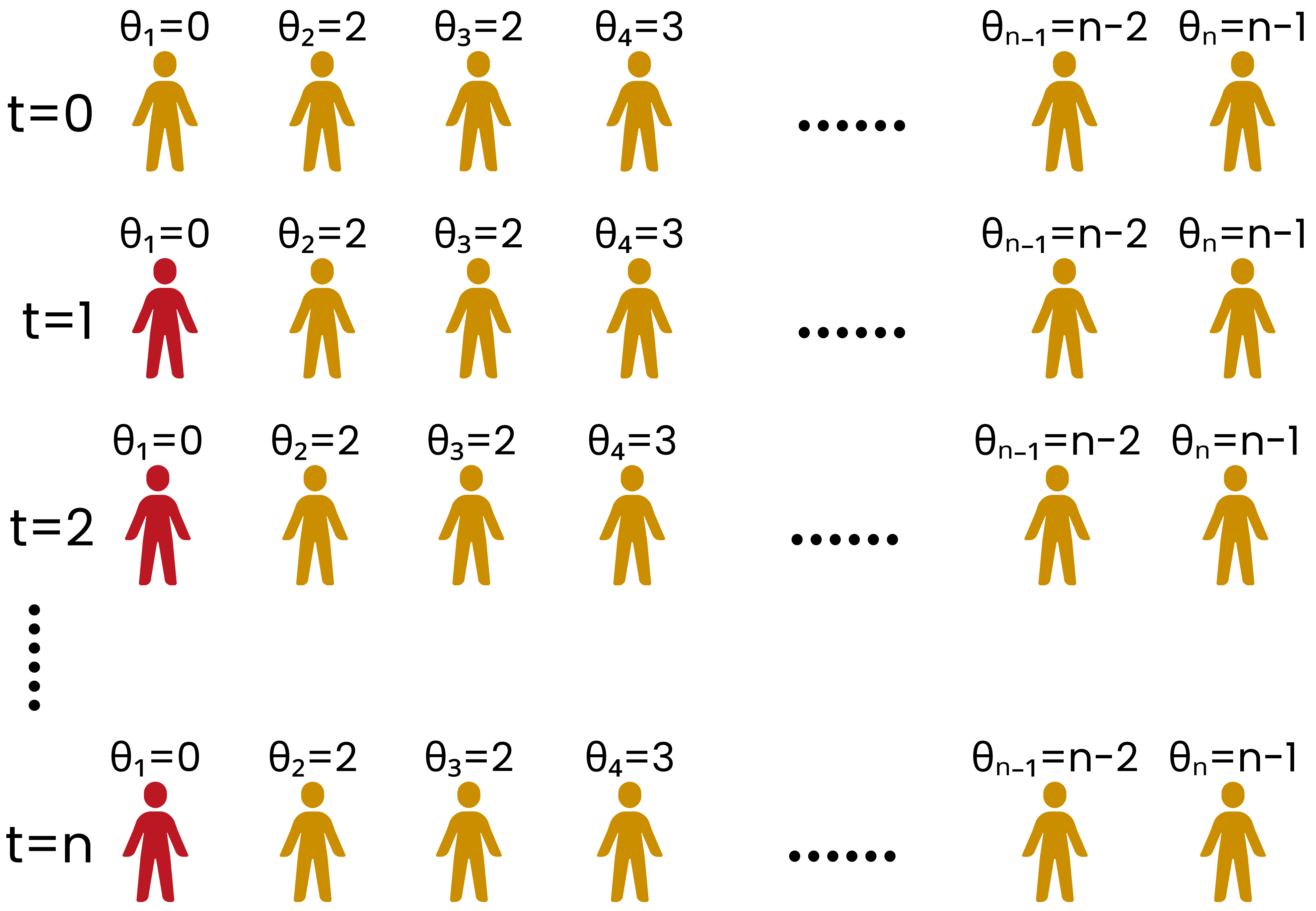
- there are ***n*** distinct agents
- each agent is characterized by a threshold $\boldsymbol{\theta_i}$
- an agent join the riot if and only if there is a number of agents larger or equal to its threshold in the riot
- we denote by $\boldsymbol{M(t)}$ the number of rioters at time t (and by $\boldsymbol{x(t)}$ the percentage)
- at the time step $t+1$ all agents with $\boldsymbol{\theta_i \leq M(t)}$ join the riot
- the simulation stops if all people are in the riot $M(t)=n$ or a stationary state $M(t+1)=M(t)$ is reached

One Example with Spreading



- *n* Agents
- Uniform sequence of thresholds with integer values *[0, n-1]*
- First agent activates, then second, and so on
- One agent joins per iteration and all agents are active in the end

One Example without Spreading

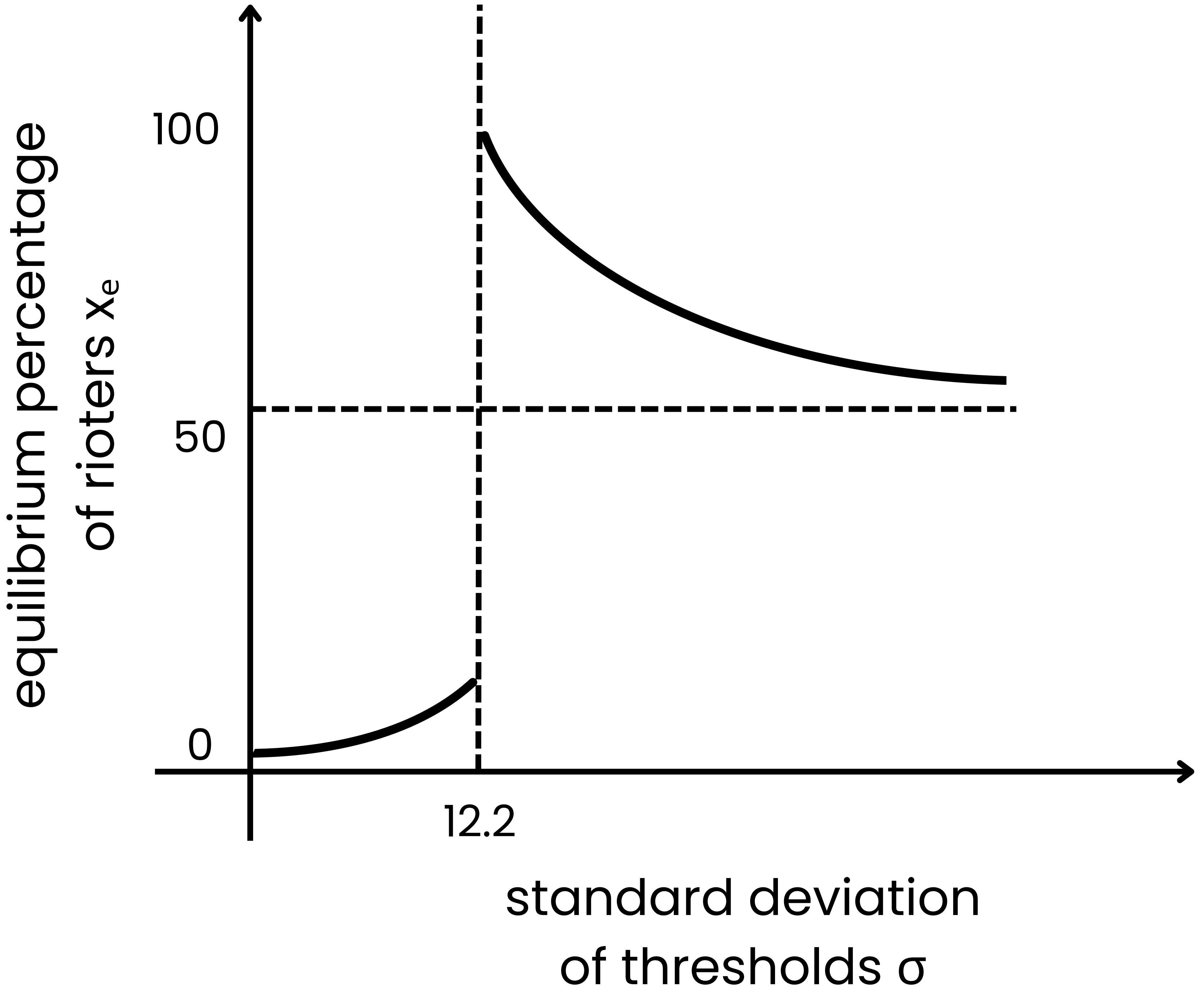


- Same example as before but agent with threshold 1 now has threshold 2
- First agent activates and simulation ends
- Radically different outcome for minimal change in thresholds!
- Deducing preference distributions from collective outcomes is risky

Gaussian Agents

In a real group of people there will be an **average behavior** with some **fluctuations** (very violent or very pacific people)

- Thresholds follow **normal distribution** with mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$
- we denote by $\boldsymbol{x_e}$ the equilibrium percentage of active agents
- Number of agent is $N=100$
- Mean value is constant $\mu=25$
- Sharp increase in x_e at a critical σ value: **discontinuous** or first order phase transition
- **Diversity-induced** collective behavior



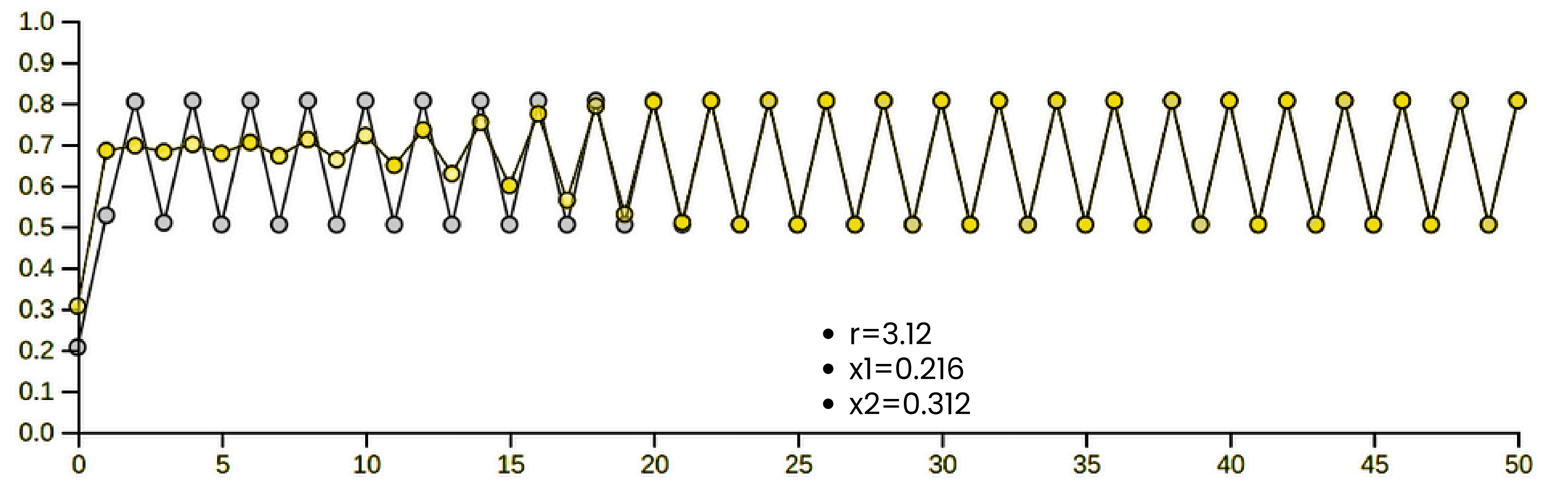
Analysis of Granovetter's Threshold Model

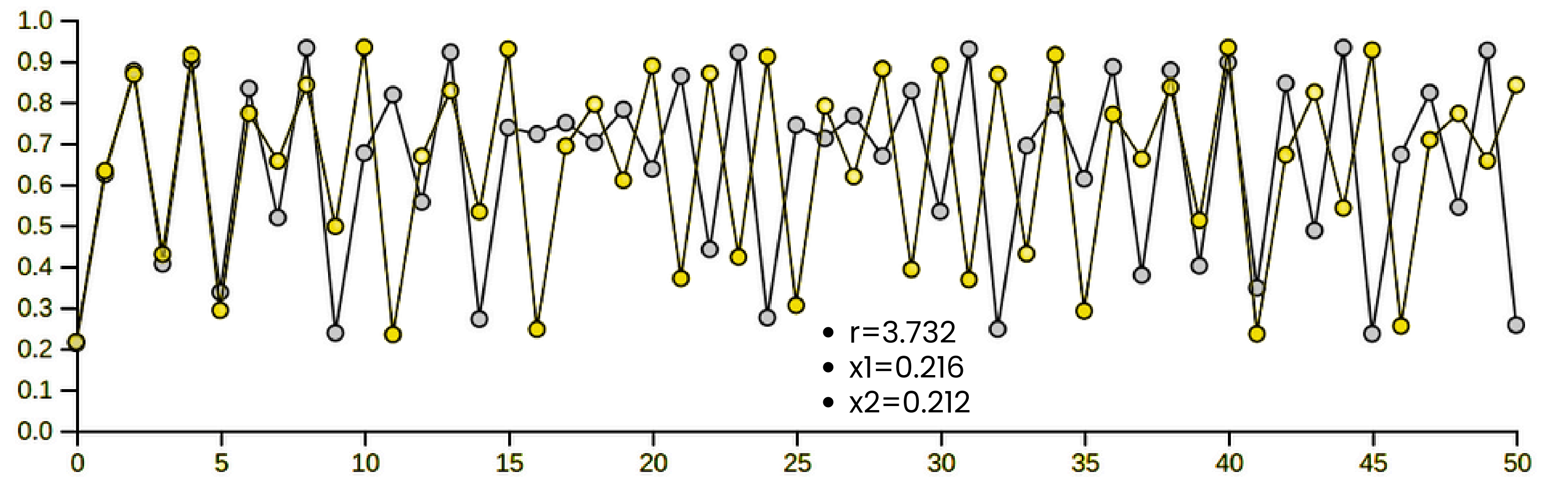
The Logistic Map Once Again

The Logistic Map is defined as:

- $x(t)$=ratio of existing population to the maximum possible population
  $x(t+1)=r\cdot x(t)\cdot[1-x(t)]$
- $0<r<4$ is the parameter of the model

We can visualize the evolution of x as function of t. When r grows the trajectories become first periodic and then chaotic.

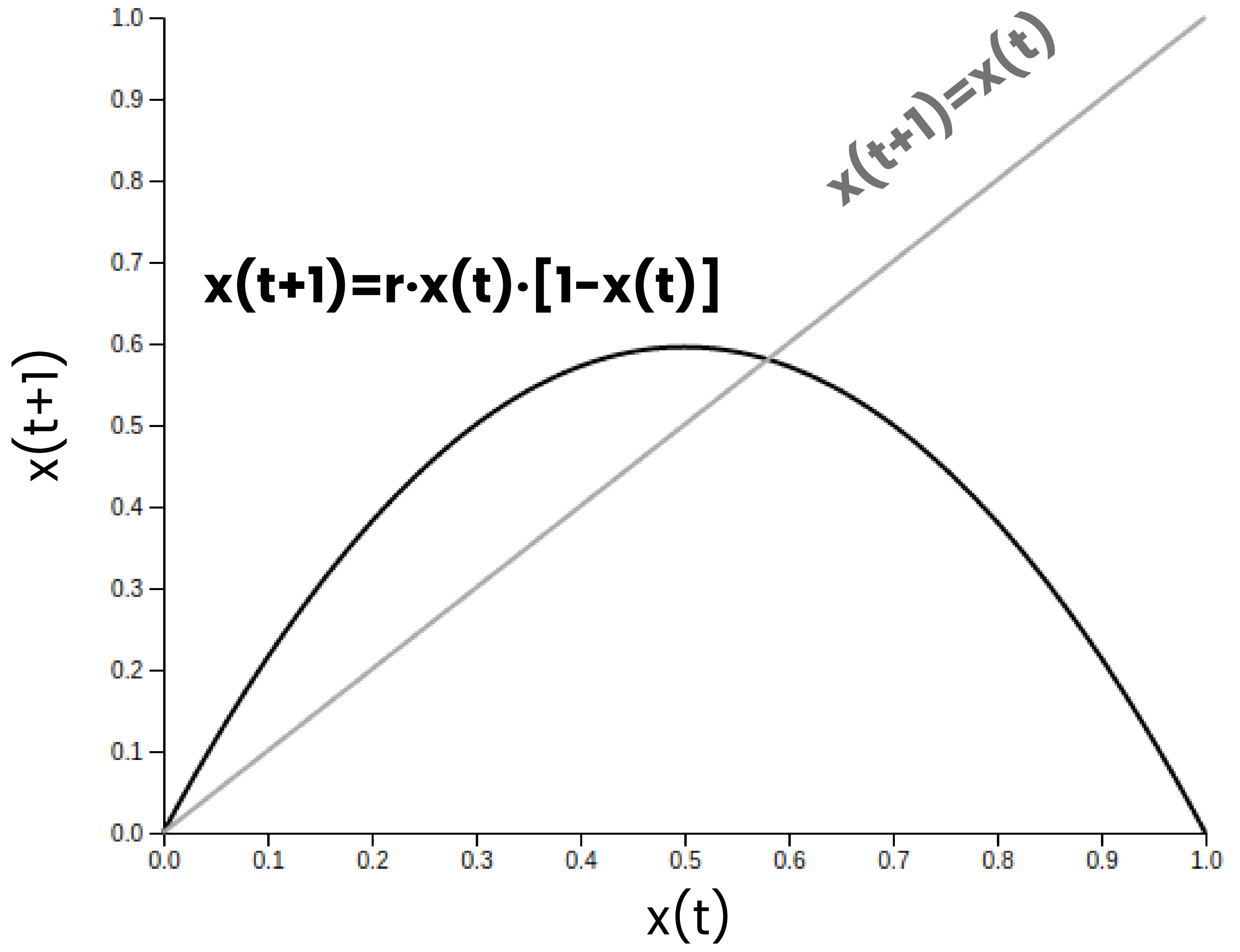




Another way to Visualize Maps

We can better understand maps by plotting *x(t+1)* as function of *x(t)*

- the x axis gives *x(t)*
- the y axis gives *x(t+1)*
- the straight line is *x(t+1)=x(t)*



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- the x axis gives $x(t)$
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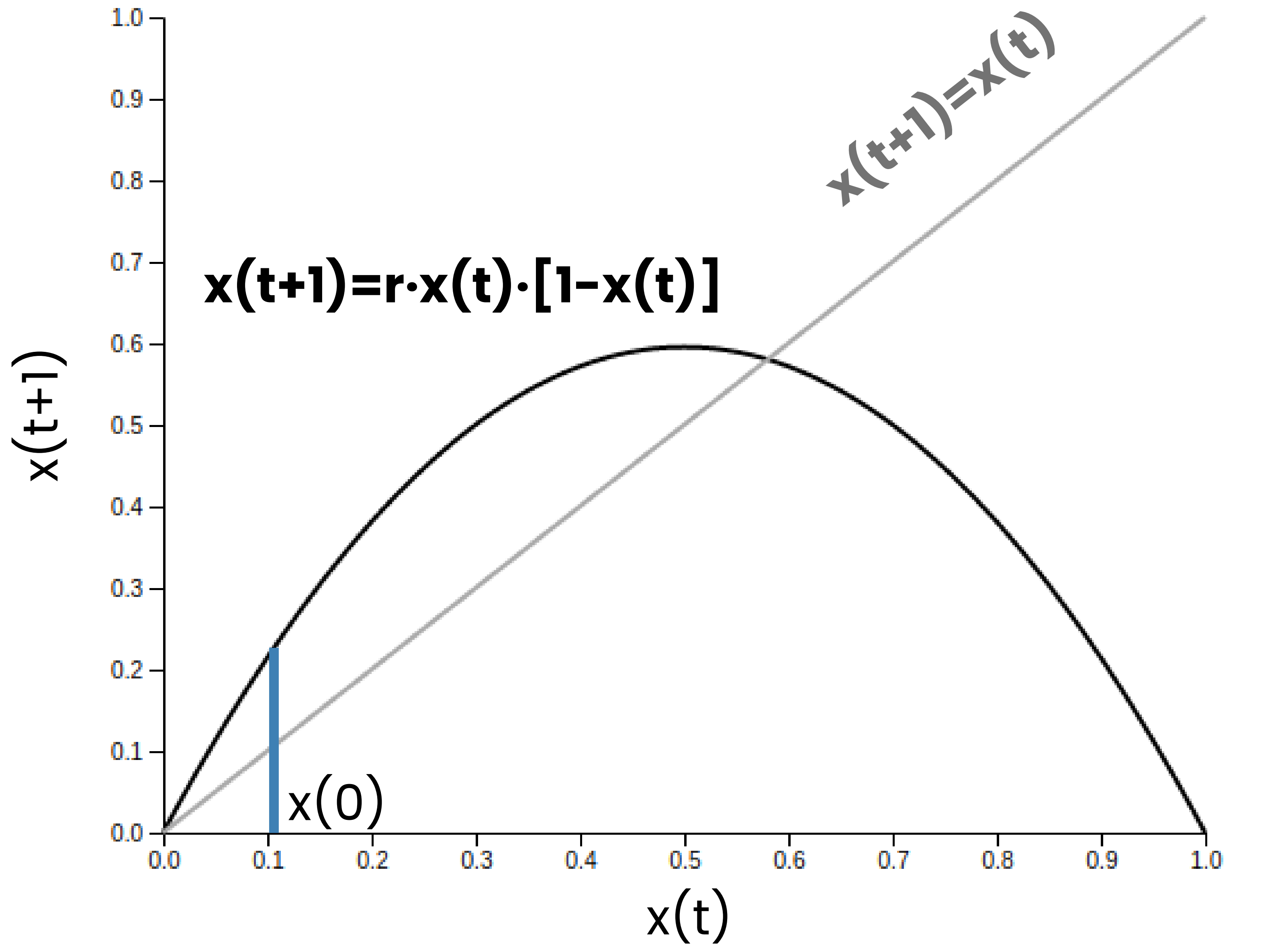
The recipe for evolving the system is very simple

1. start from $x(0)$ and move up until you intersect the curve



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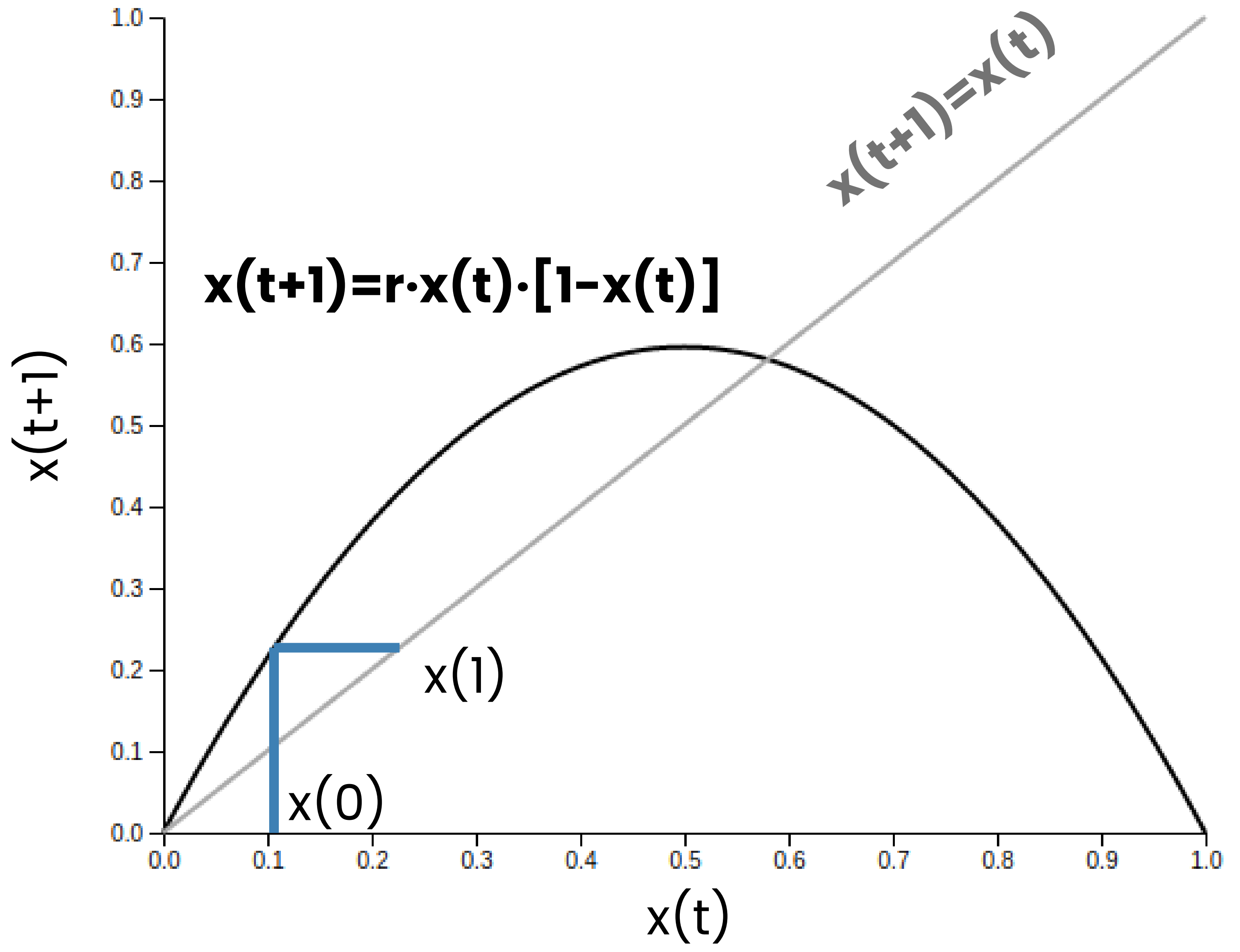
The recipe for evolving the system is very simple

1. start from $x(0)$ and move up until you intersect the curve
2. move horizontally until you intersect the straight line in $x(1)$



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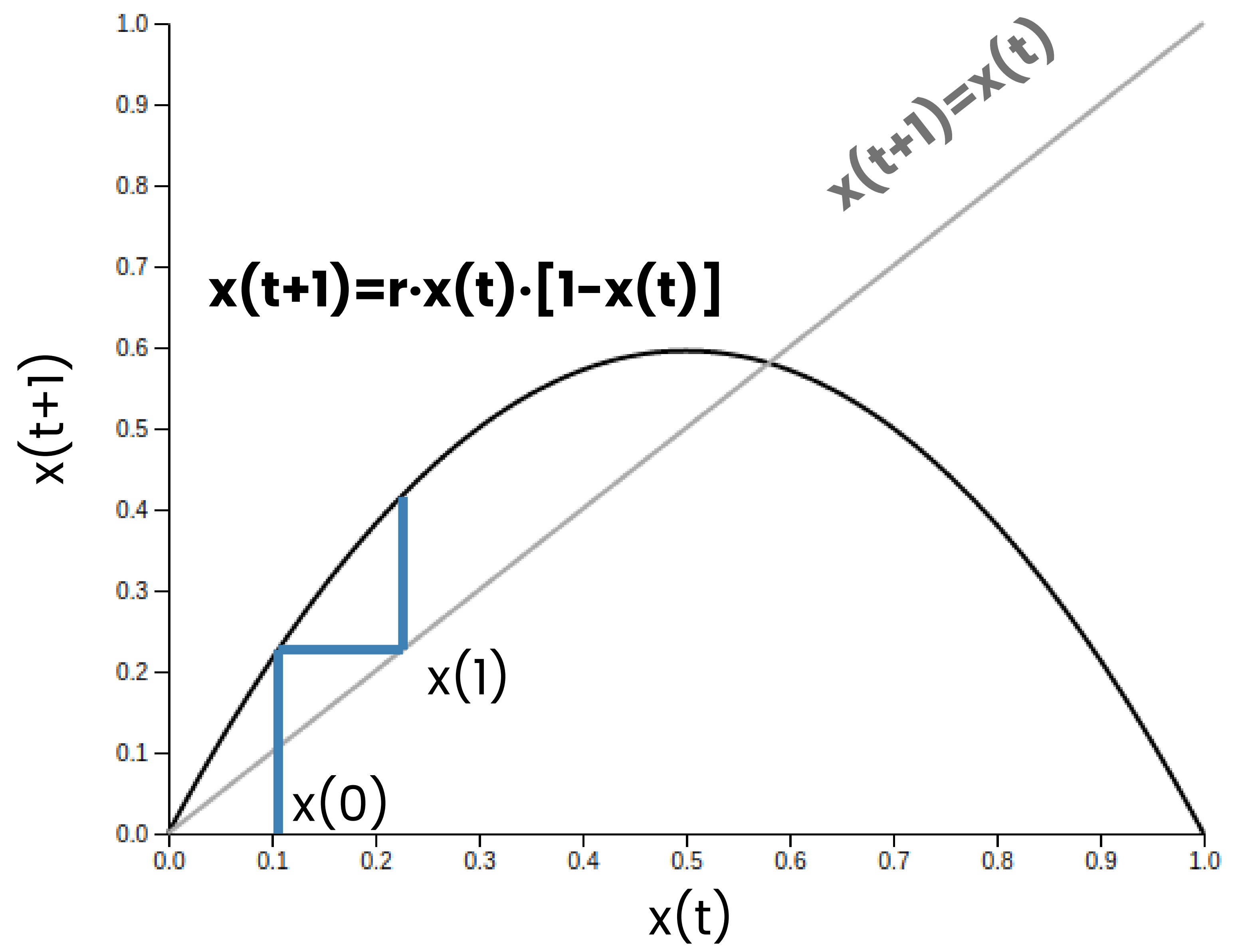
The recipe for evolving the system is very simple

1. start from $x(0)$ and move up until you intersect the curve
2. move horizontally until you intersect the straight line in $x(1)$
3. move vertically until you intersect the curve again



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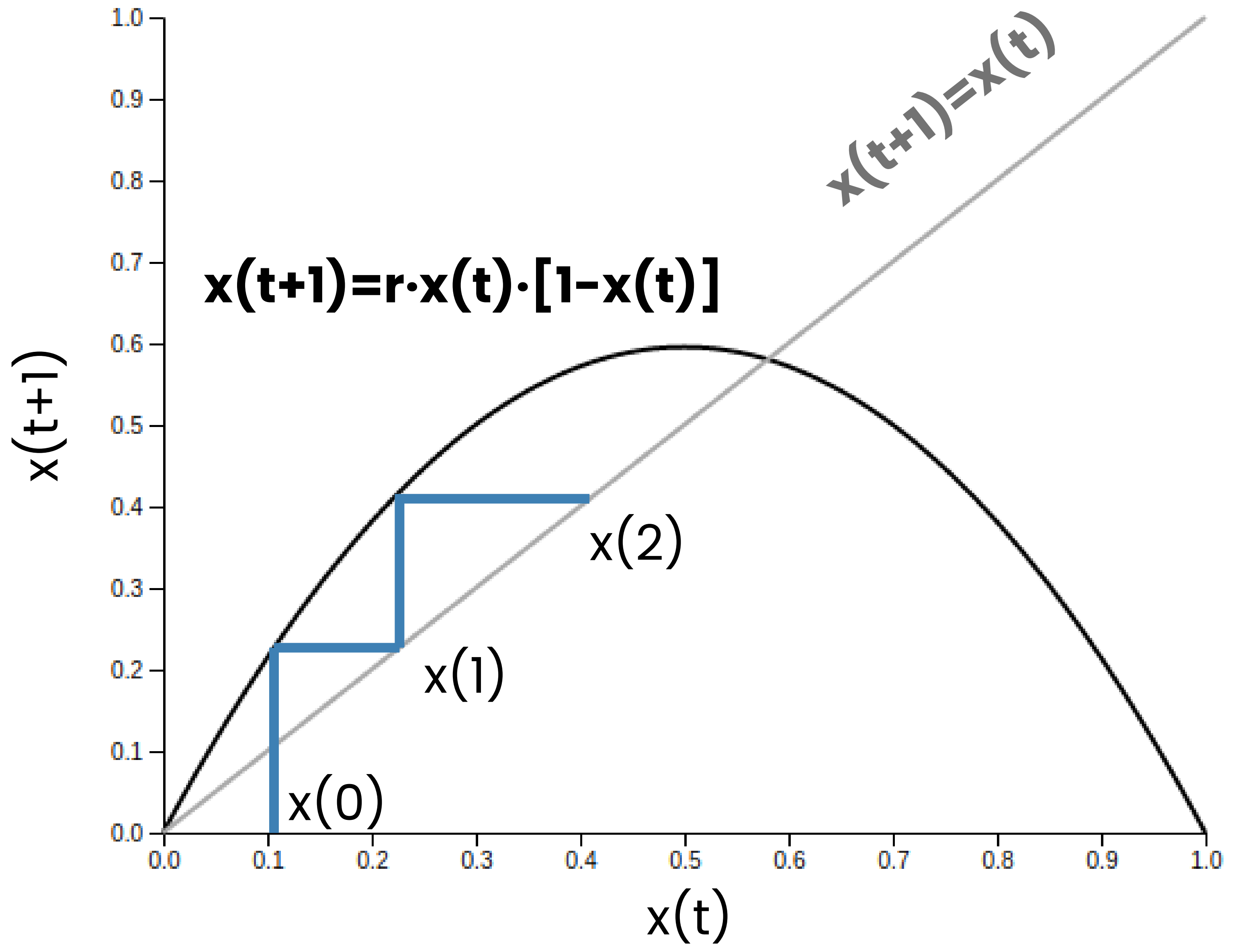
4. repeat from step 2



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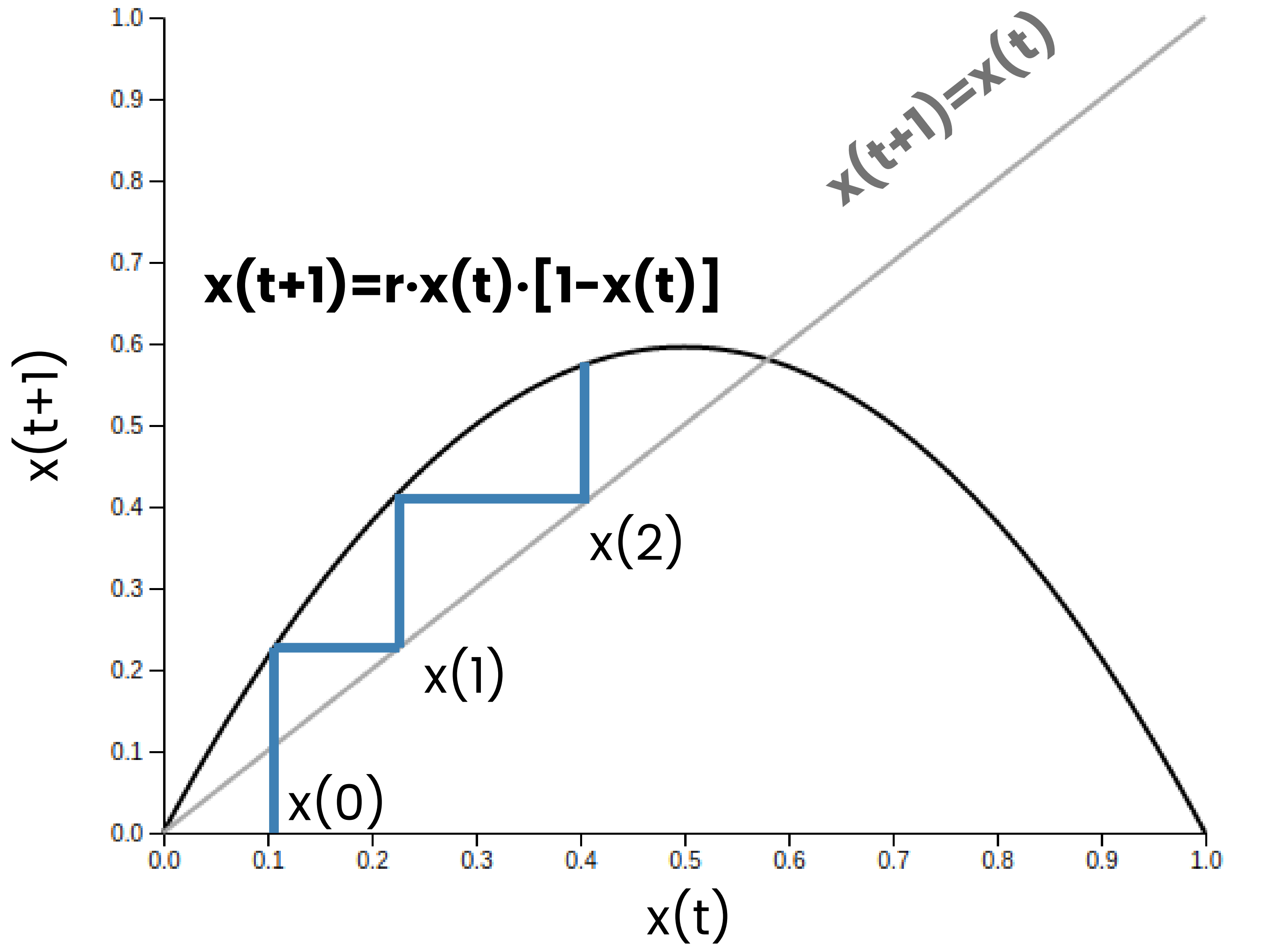

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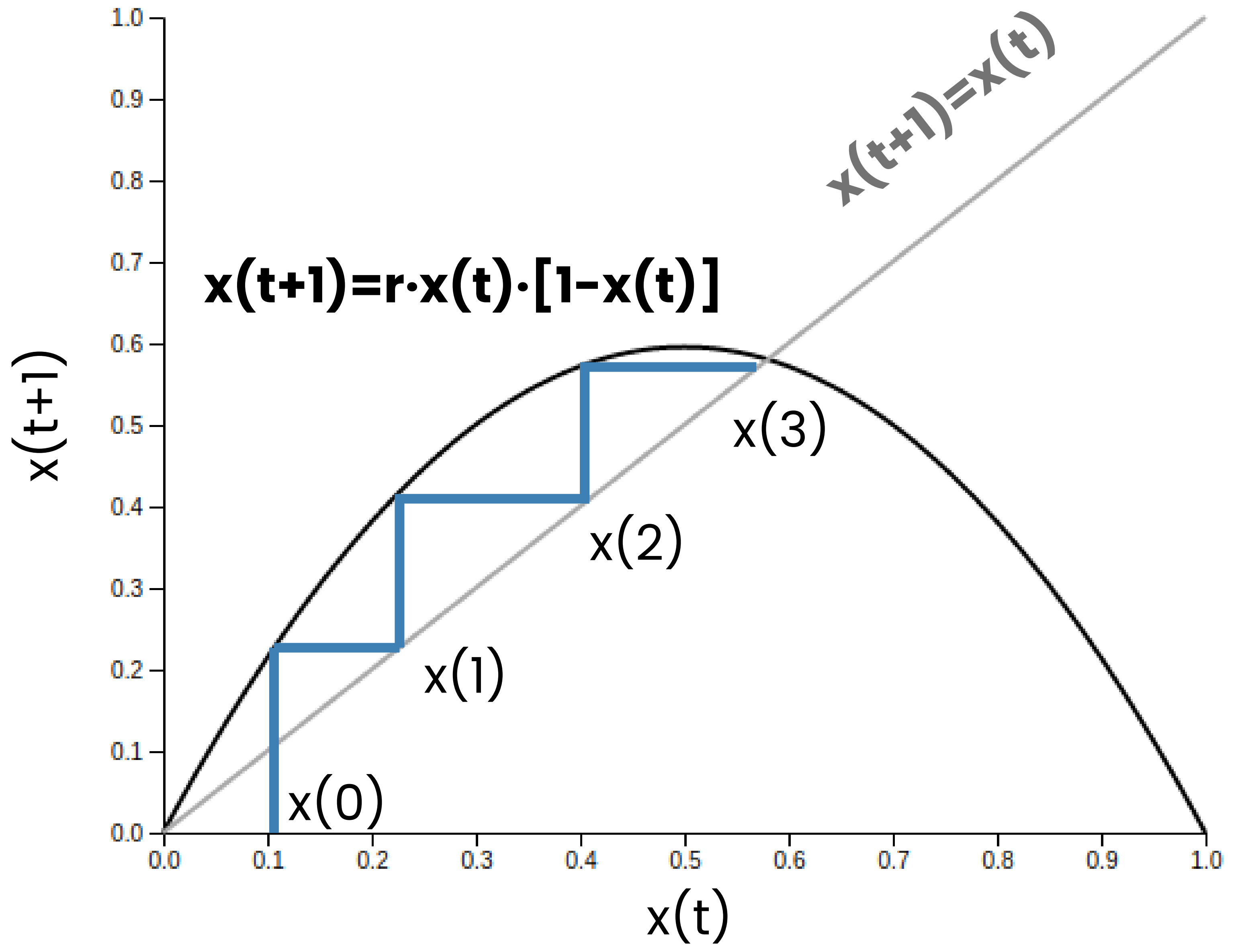
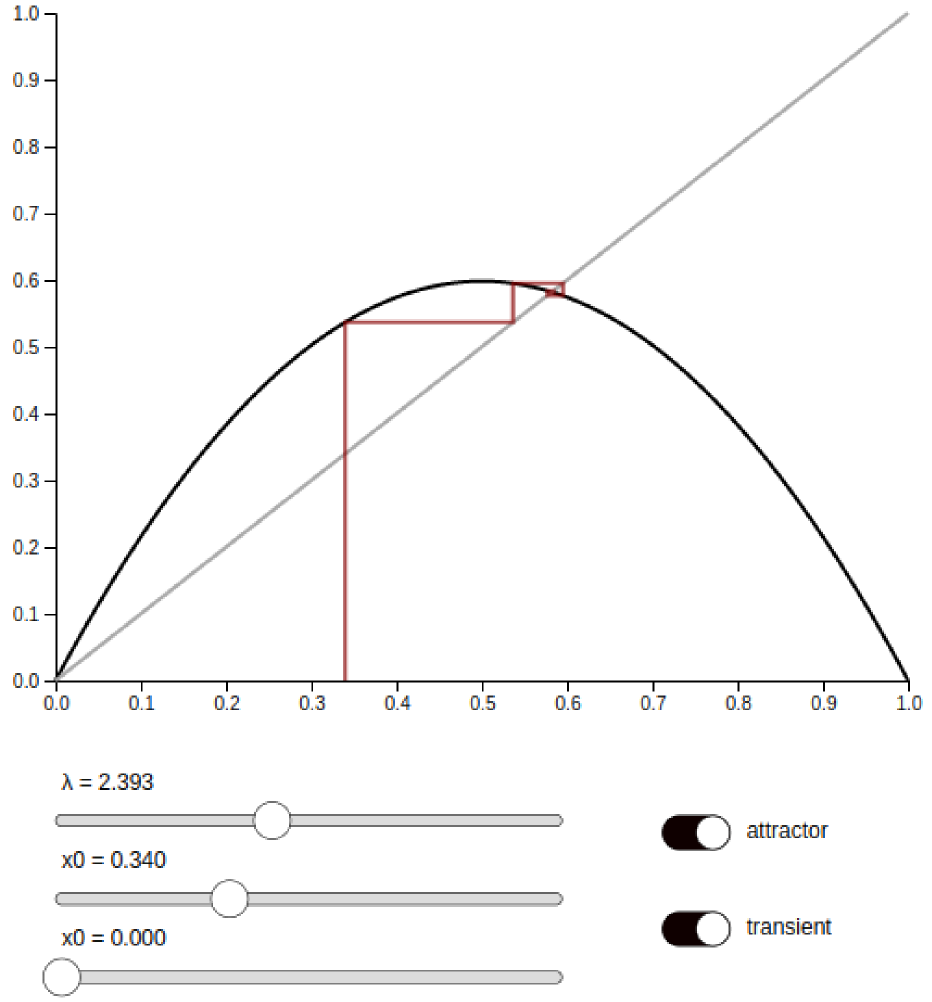
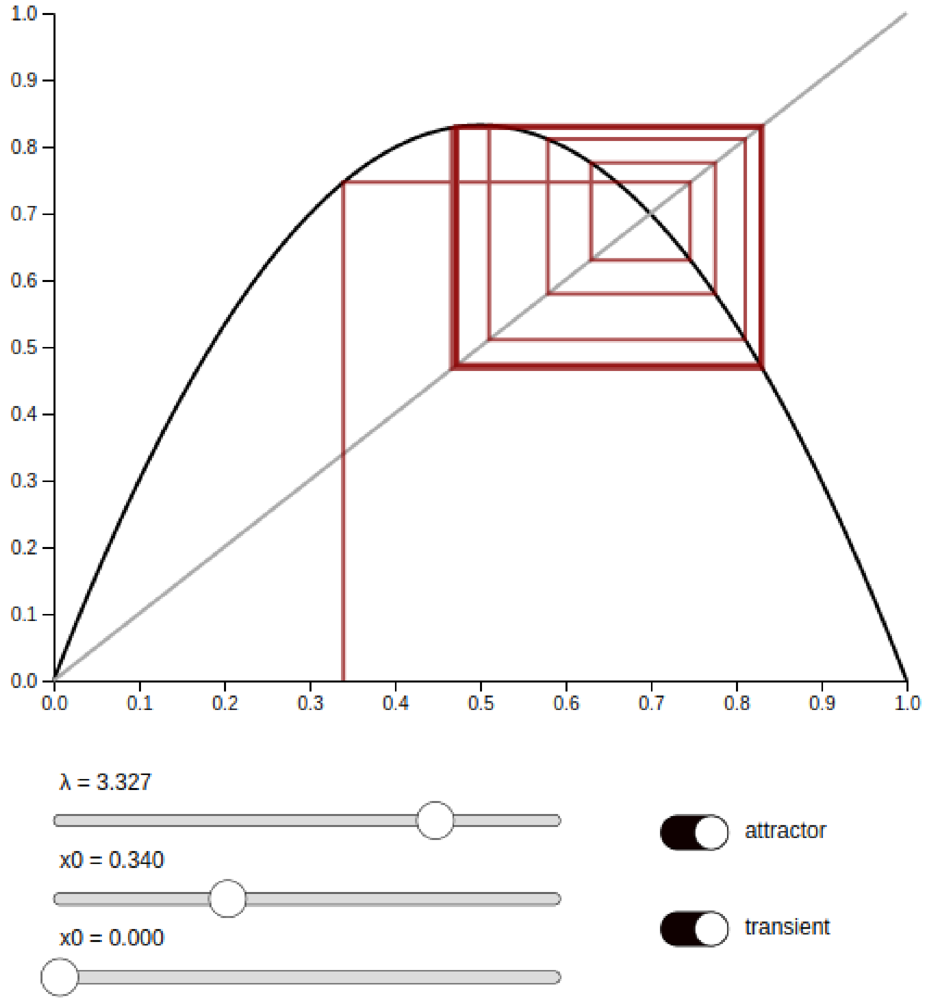


Periodic and Chaotic Trajectories

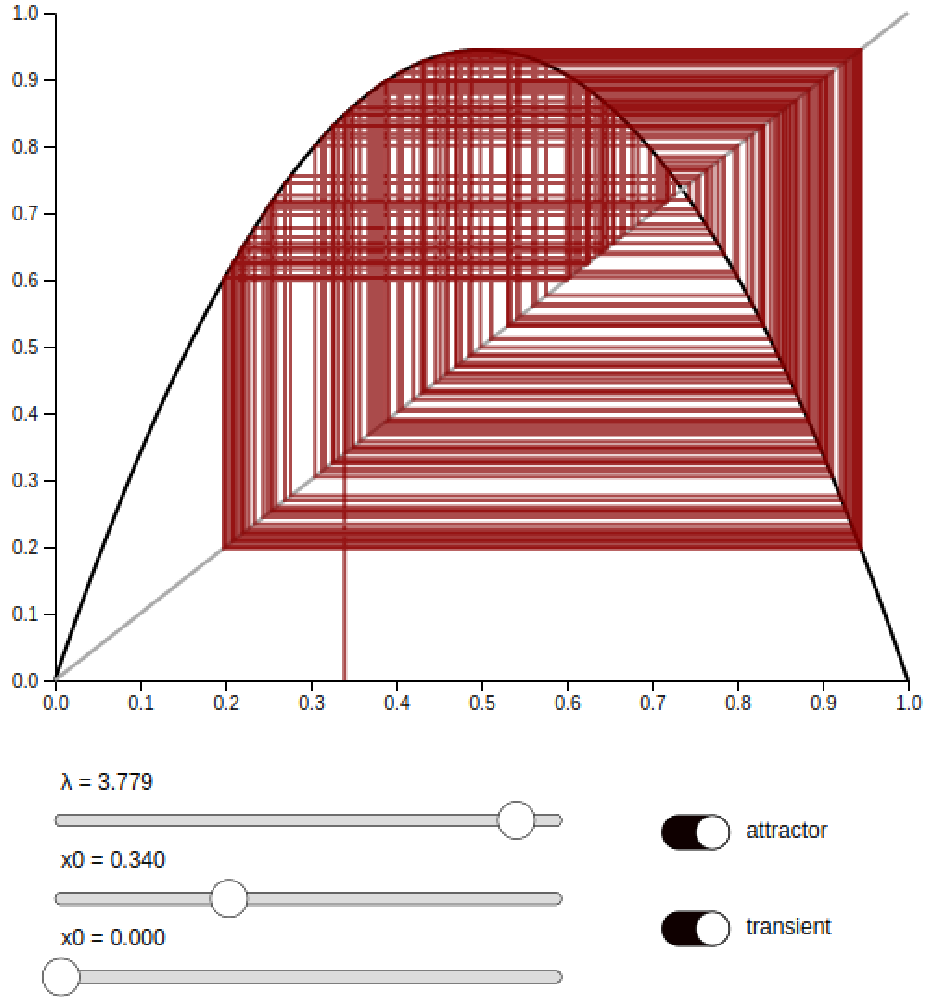
Fixed point (Period = 1)



Limit Cicle (Period = 2)



Chaos (Period = ∞)



https://www.complexity-explorables.org/flongs/logistic/

Granovetter's Model as a Map

Granovetter's threshold model is just a map!

- $x(t)$ is the fraction of rioters at time t
- $x(t+1)$ only depend on the thresholds (fixed) and $x(t)$
- the process is deterministic
- we have to understand the function that makes $x(t)$ evolve into $x(t+1)$

The evolution of the number of rioters is

$$M(t+1)=N[\theta \leq M(t)]$$

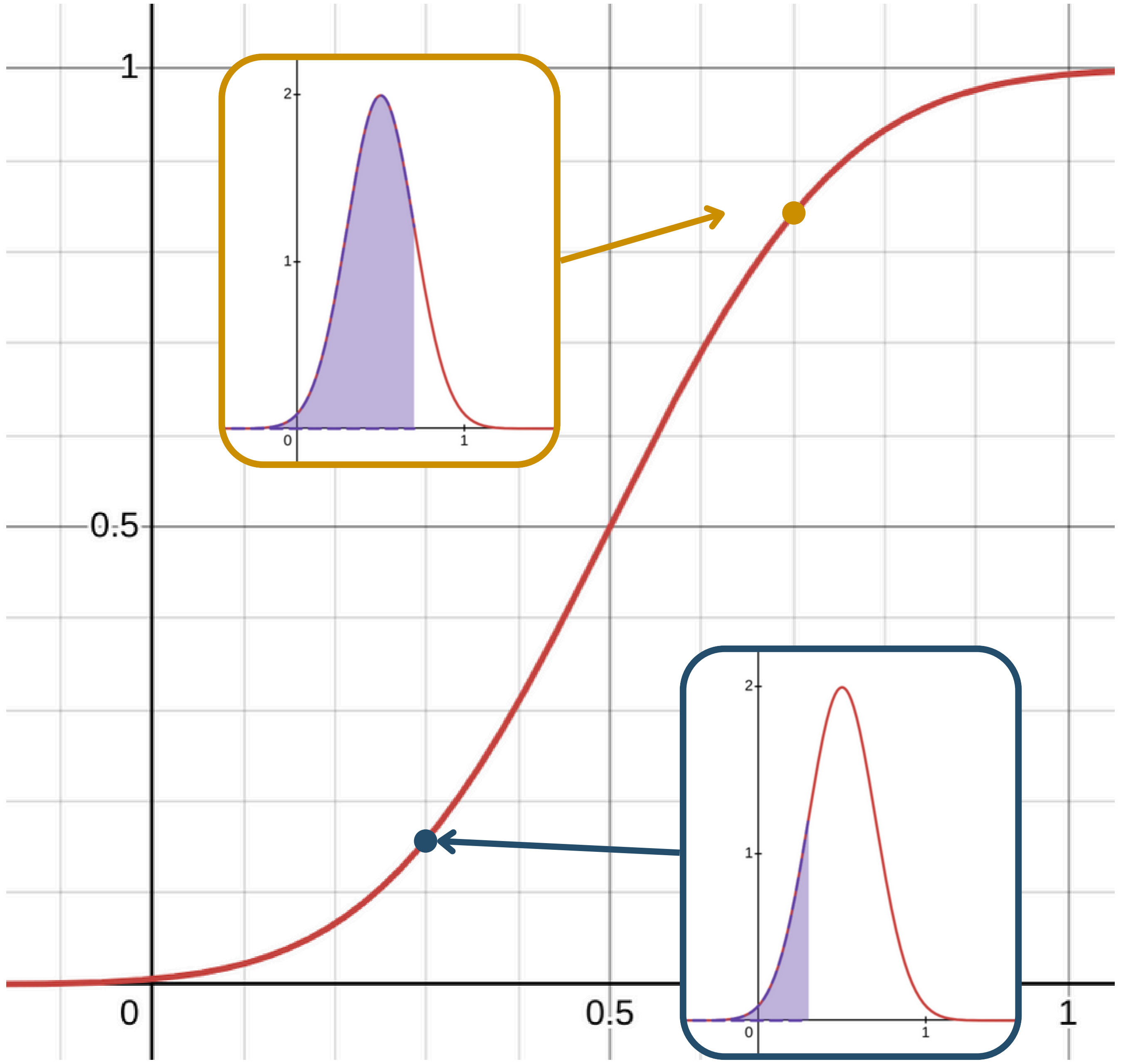
where $N[\theta \leq M(t)]$ is the number of agents with threshold less than $M(t)$. We divide both size by the number of agents n

$$x(t+1)=N[\theta \leq M(t)]/n \approx P[\theta \leq M(t)]$$

$P[\theta \leq M(t)]$ is the cumulative probability of the thresholds. For simplifying things we normalize the thresholds $\Theta=\theta/n$ and we obtain

$$x(t+1) \approx P[\Theta \leq x(t)]$$

Cumulative **Probability**



The cumulative probability $P(x)$ is the probability to extract a number smaller than a given value x. If $p(x)$ is the probability to extract a number x then

- for discrete variables

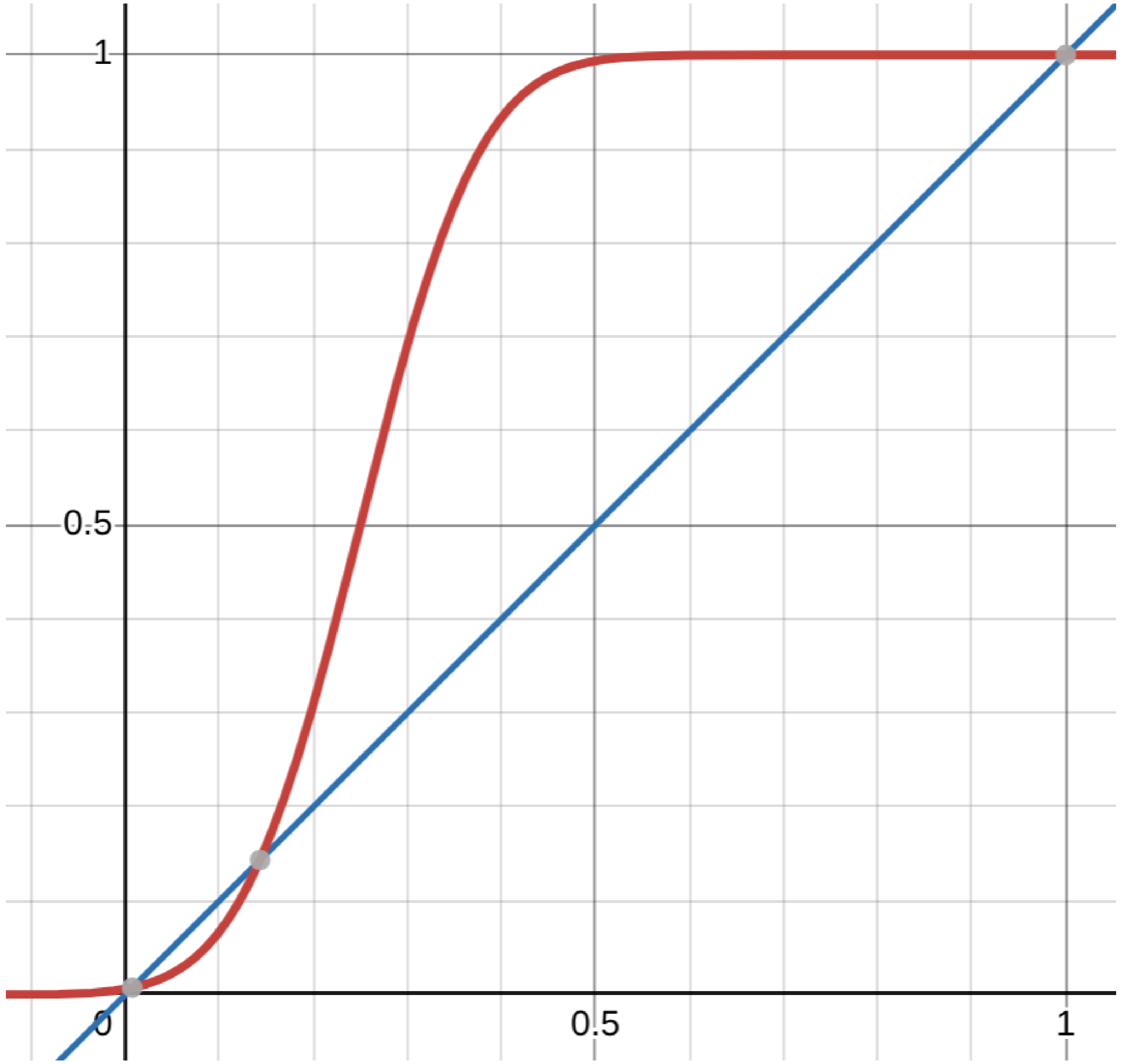
$$P(x) = \sum_{y=0}^{x} p(y)$$

- for continuous variables

$$P(x) = \int_0^x p(y)dy$$

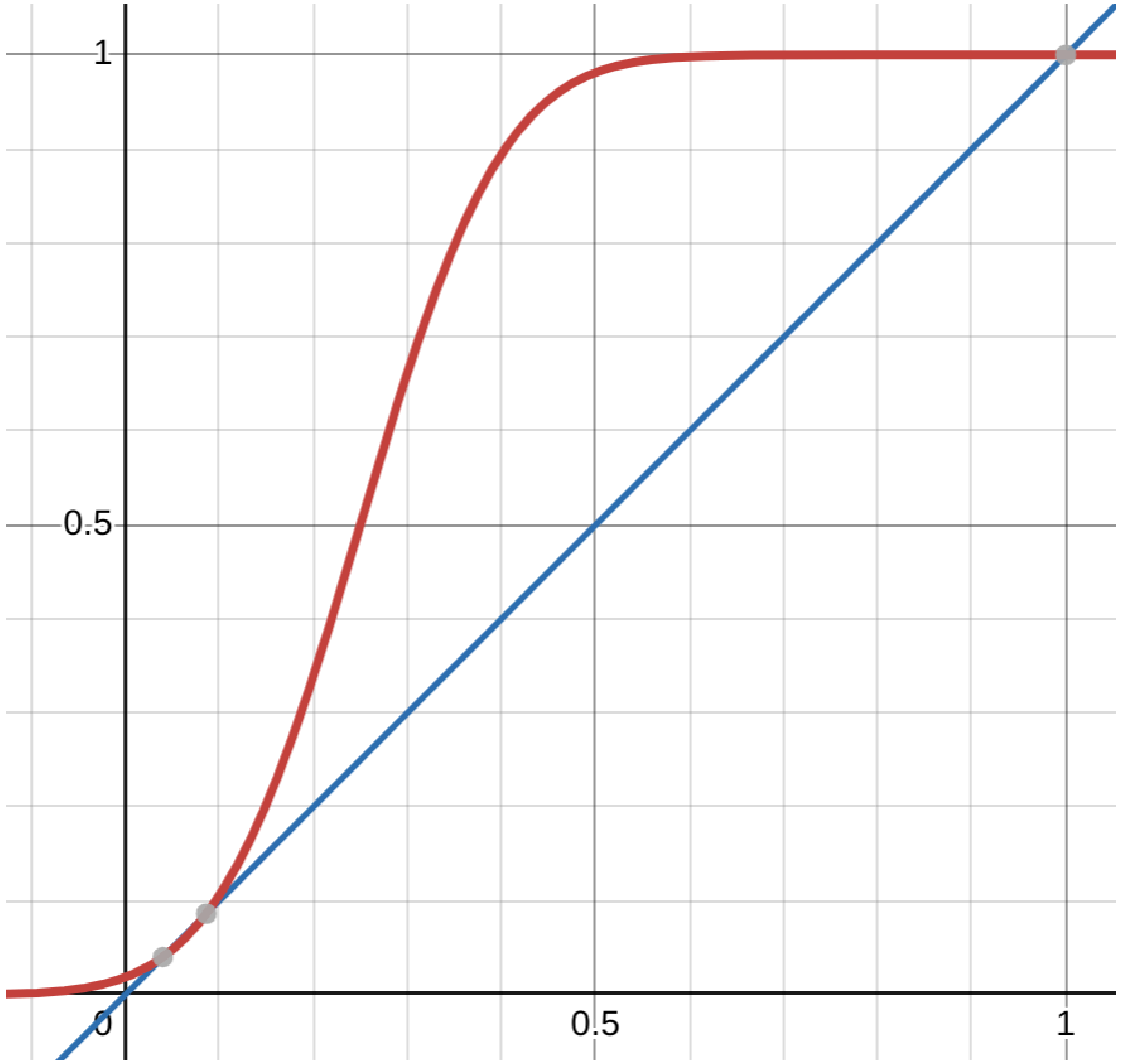
The figure shows the cumulative of a gaussian

Equilibrium with μ=0.25 σ=0.10



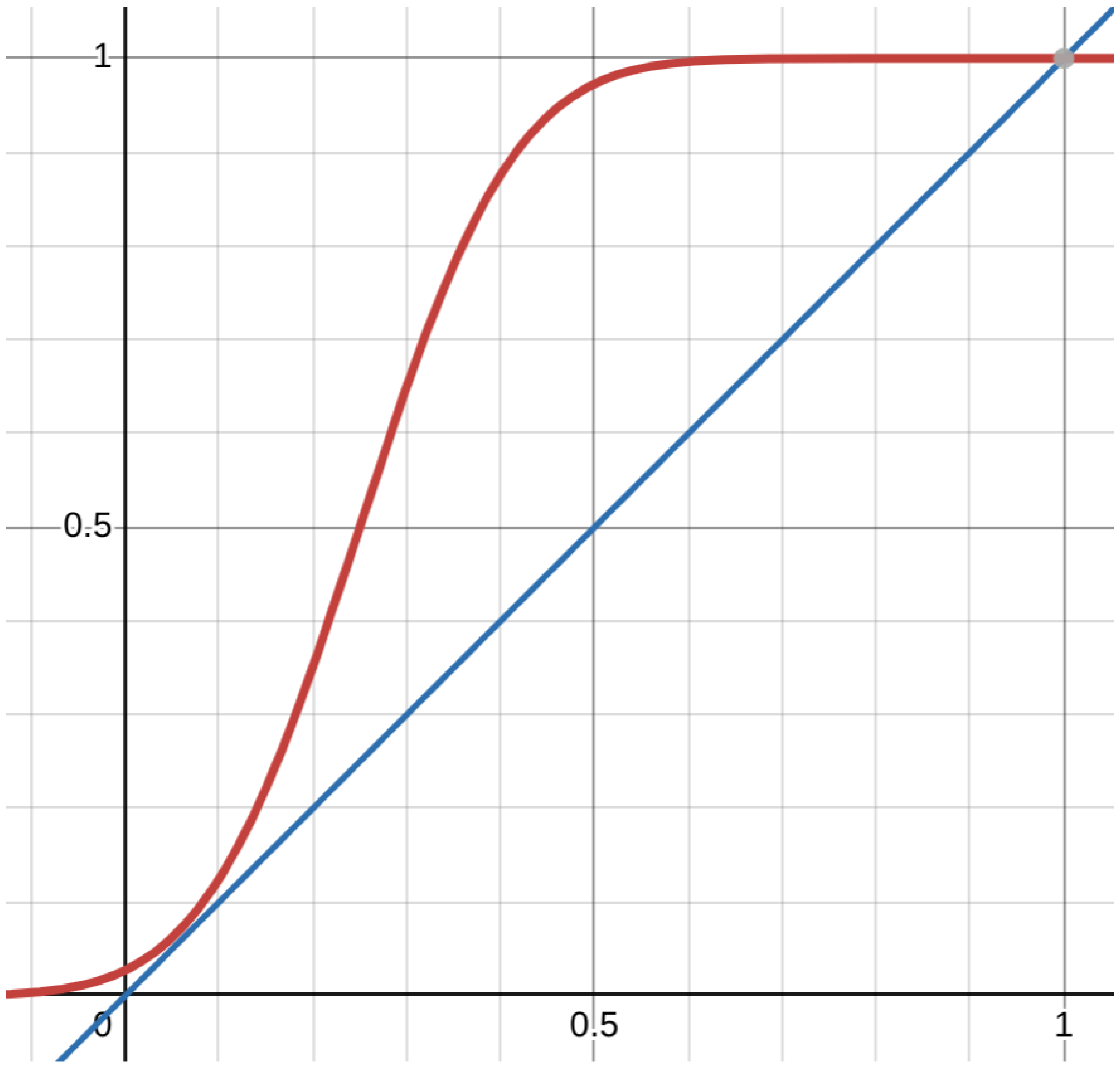
- the map has 3 equilibrium points
  - *x~0* stable
  - *x~0.14* unstable
  - *x=1* stable
- since *x(0)=0* the stable point *x=1* is never reached
- around 0% of the agents involved in the riot

Equilibrium with μ=0.25 σ=0.12



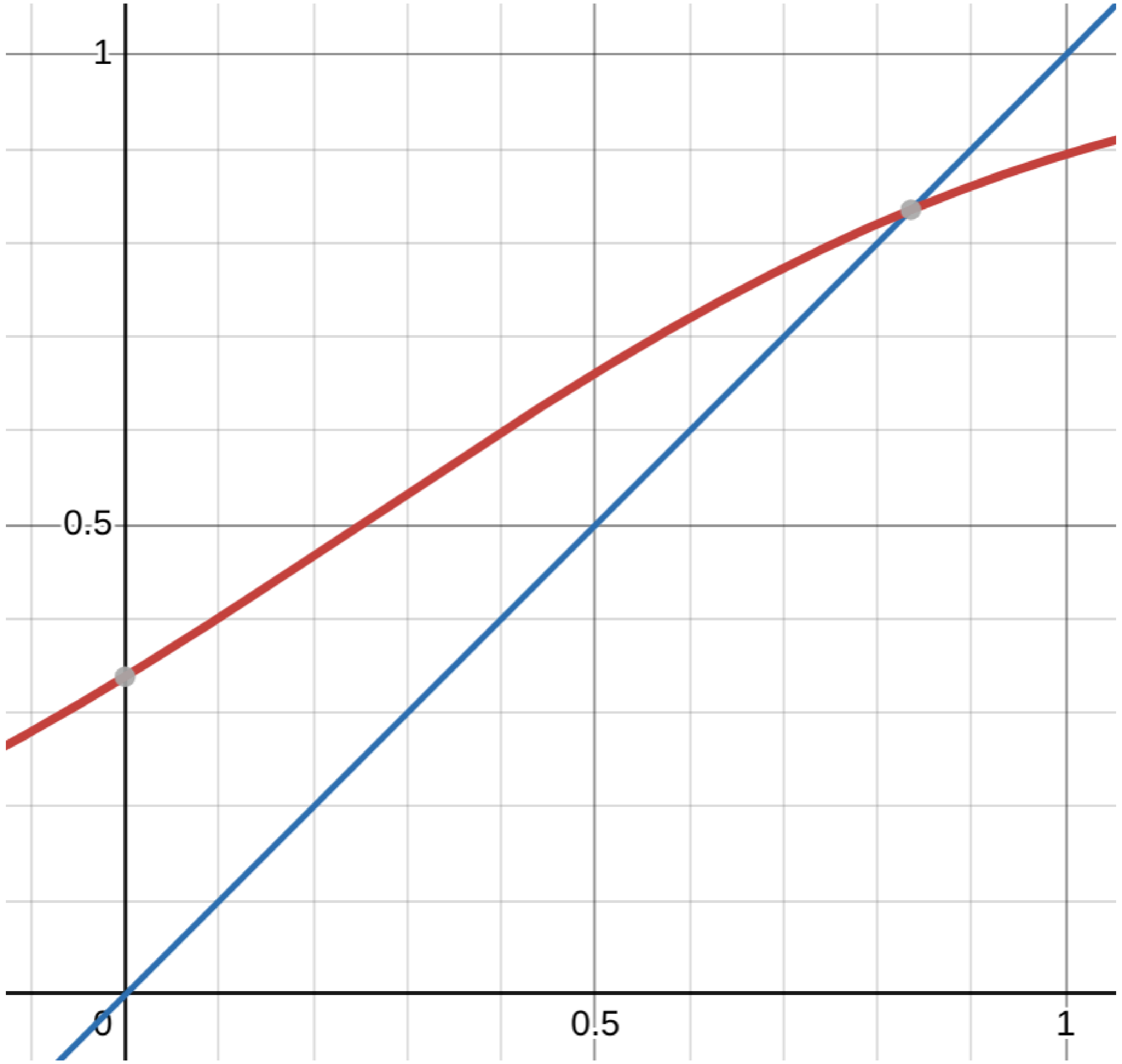
- the map has 3 equilibrium points
  - *x~0.04* stable
  - *x~0.14* unstable
  - *x=1* stable
- since *x(0)=0* the stable point *x=1* is never reached
- around 4% of the agents involved in the riot

Equilibrium with μ=0.25 σ=0.13



- the map now has only 1 equilibrium point
  - *x=1* stable
- starting from x(0)=0 the system reaches the only stable equilibrium point
- 100% of the agents involved in the riot

Equilibrium with μ=0.25 σ=0.60



- the map now has only 1 equilibrium point
  - *x=0.83* stable
- starting from x(0)=0 the system reaches the only stable equilibrium point
- 83% of the agents involved in the riot
- there is a decrease for large variance

Take Home Messages

Modelling action as rational choice

Thresholds as points where benefits outweigh costs or risks

Diversity matters

Two populations with the same average threshold have very different behaviors even if mean thresholds are the same

Tipping point or phase transition

Behavior changes dramatically at a narrow range of standard deviation of thresholds

Size effects

Small changes in threshold sequences can be important. When the population is small, you have a probability of very different outcomes. Inferring the preferences from the outcome is very hard and/or misleading

Can a Minority Win?

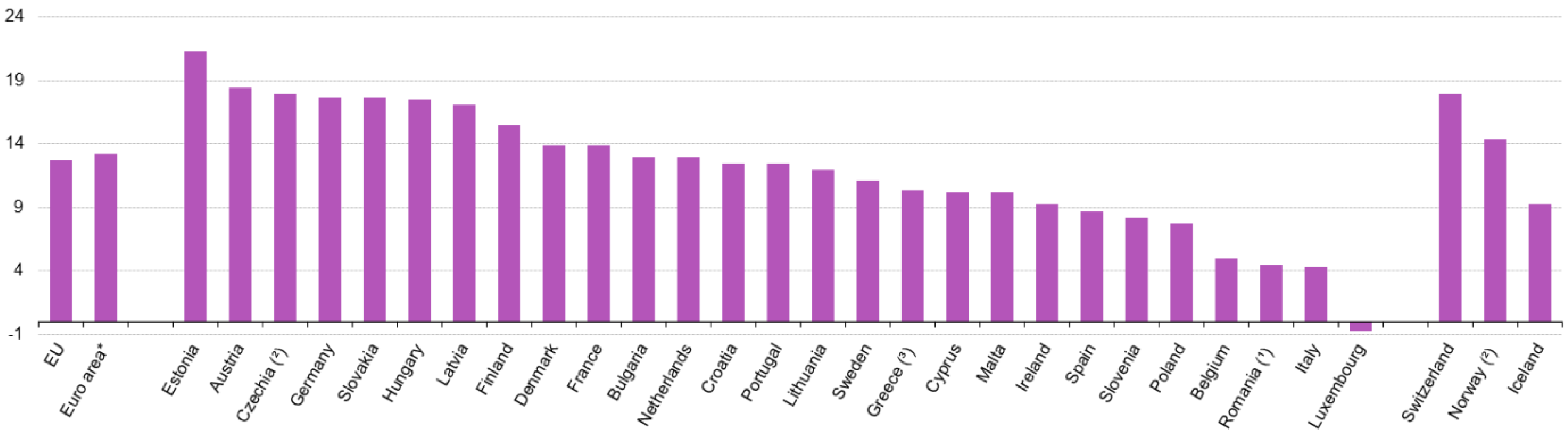
What is a Minority?

Minority groups are categories of people differentiated from the social majority

- Often based on
  - ethnicity
  - religion
  - sexual orientation
  - gender
- Not necessarily numerical
- They face inequalities and discrimination
- Play a critical role in social movements and in initiating changes in societal norms

Salary Gap

The unadjusted gender pay gap, 2022
(difference between average gross hourly earnings of male and female employees as % of male gross earnings)



Note: For all the countries except Czechia and Iceland: data for enterprises employing 10 or more employees, NACE Rev. 2 B to S (-O); Czechia: data for enterprises employing 1 or more employees, NACE Rev. 2 B to S; Iceland: NACE Rev. 2 sections C to H, J, K, P, Q. Gender pay gap data for 2022 are provisional until benchmark figures, taken from the Structure of Earnings survey, become available in December 2024.

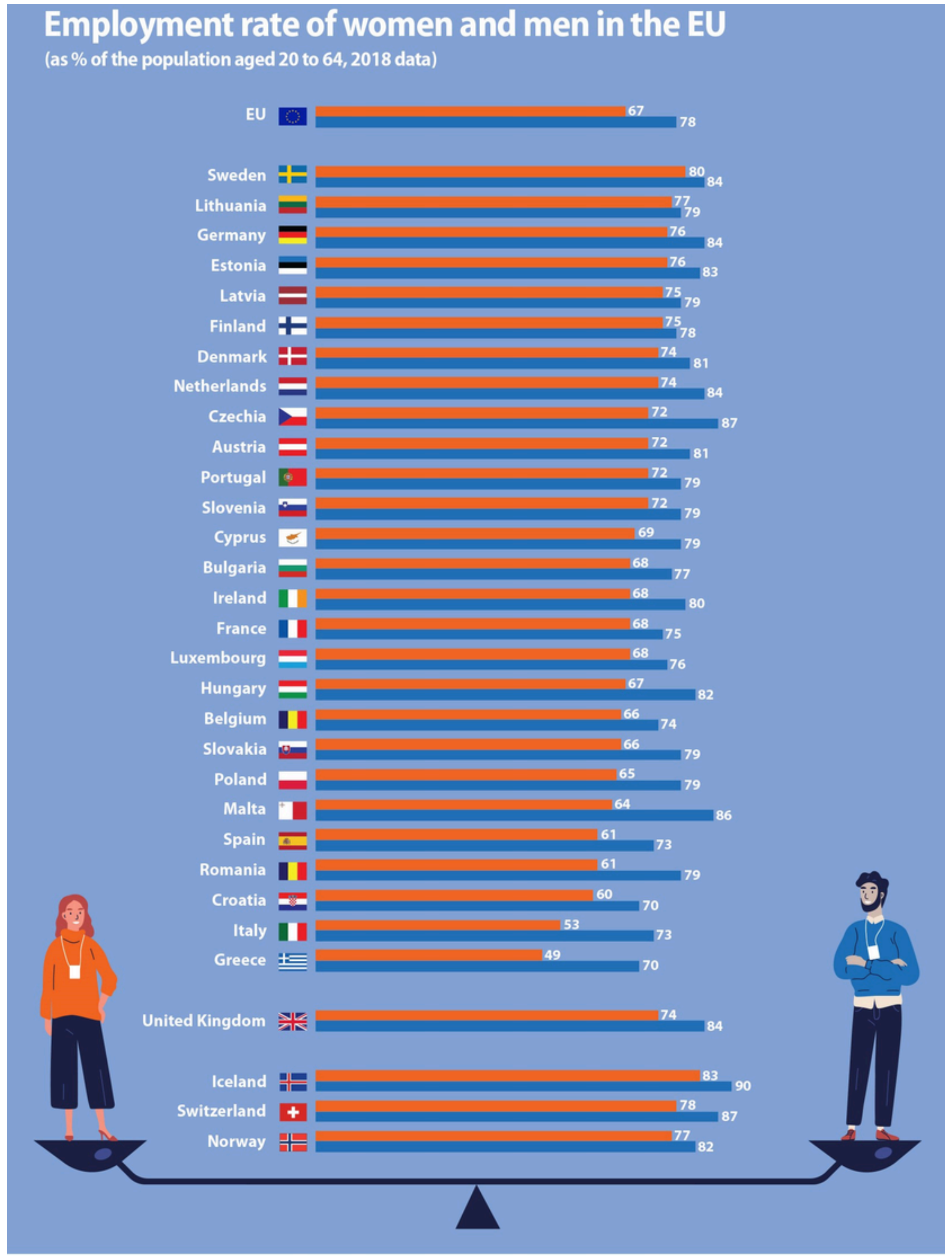
* Euro area (2015-2022)

(¹) Estimated data.
(²) Definition differs (see metadata).
(³) 2018 data.

Source: Eurostat (online data code: sdg_05_20)

eurostat

Never Stop at the First Graph!

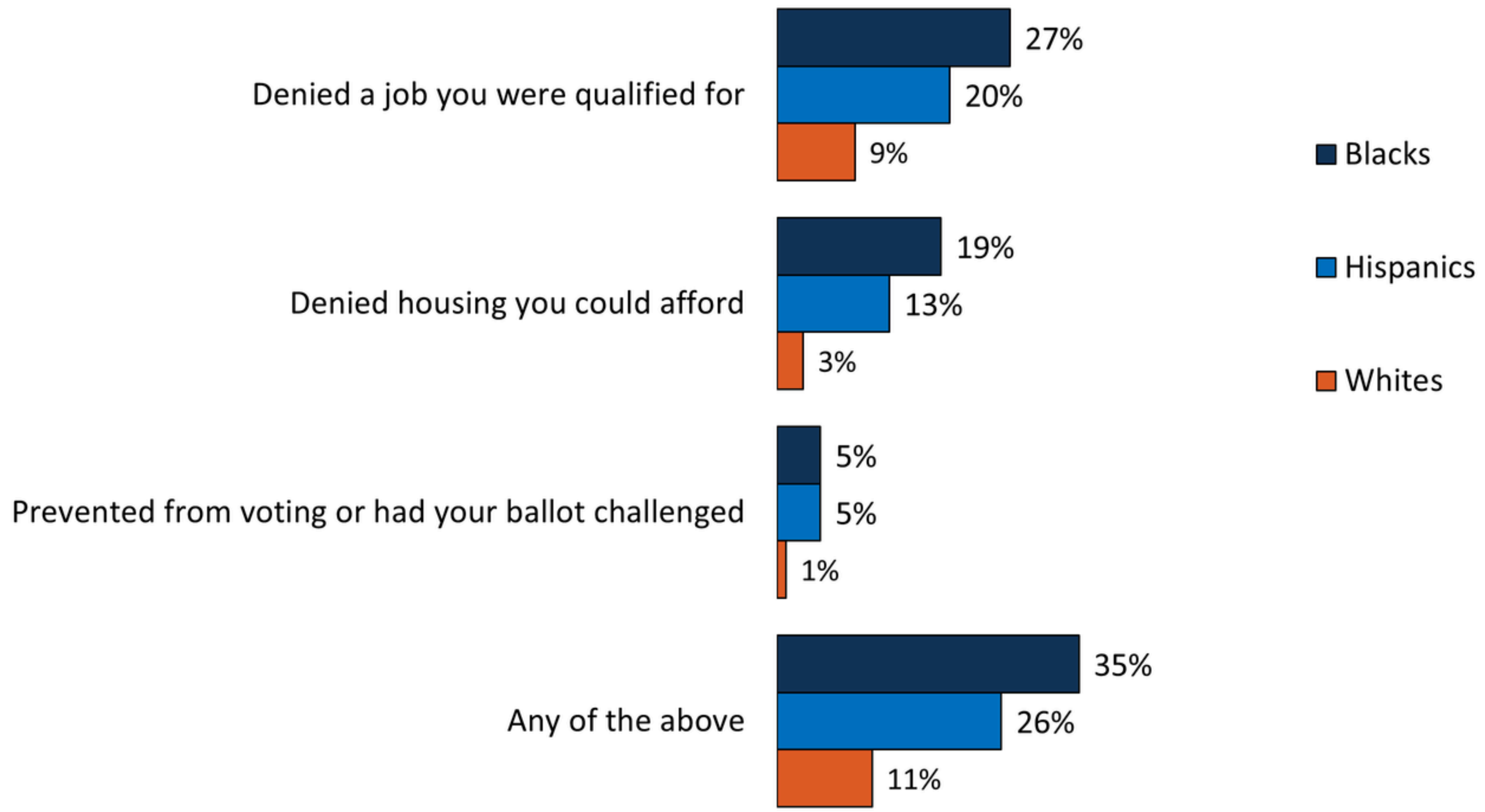


ec.europa.eu/eurostat

Ethnic Discrimination

About A Third Of Blacks And A Quarter Of Hispanics Say They Have Experienced Some Types Of Racial Discrimination

Percent who say they have ever experienced each of the following because of their racial or ethnic background:

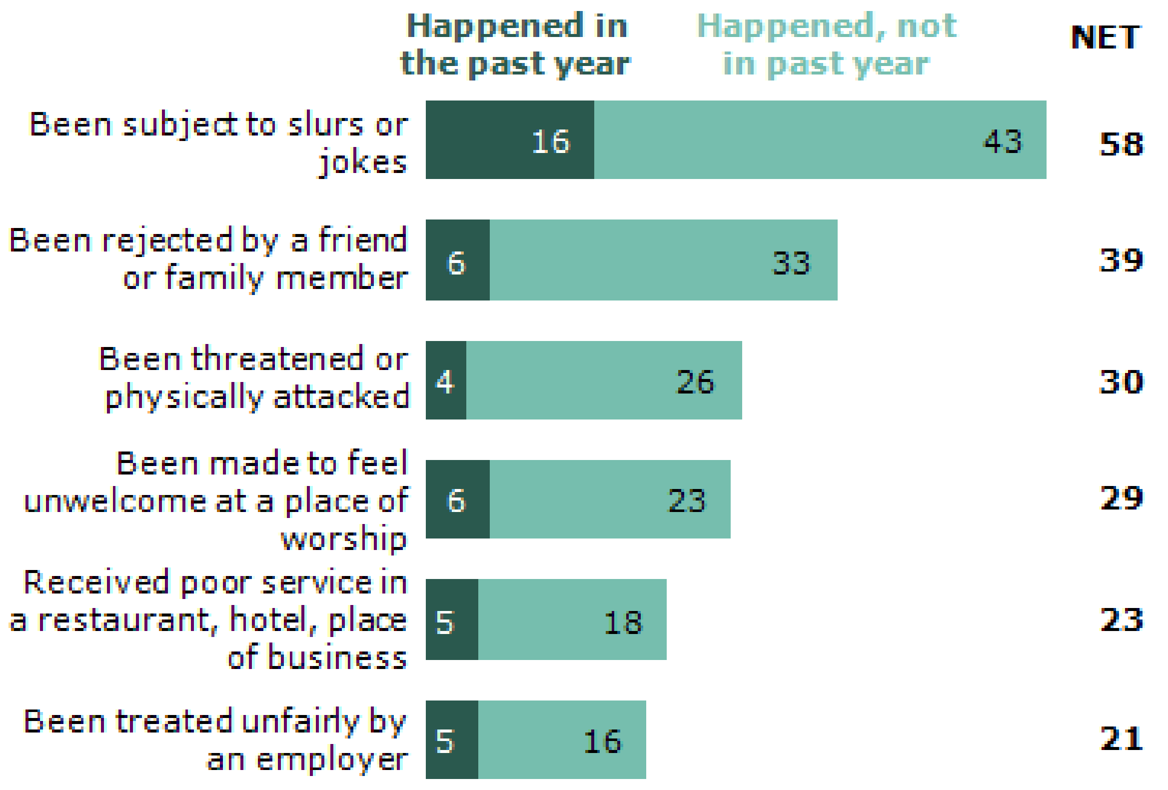


NOTE: Items asked of a half sample of Whites.
SOURCE: CNN/Kaiser Family Foundation Survey of Americans on Race (conducted August 25-October 3, 2015)

Sexual Orientation

Perceptions of Discrimination

% saying this ... because of their sexual orientation or gender identity

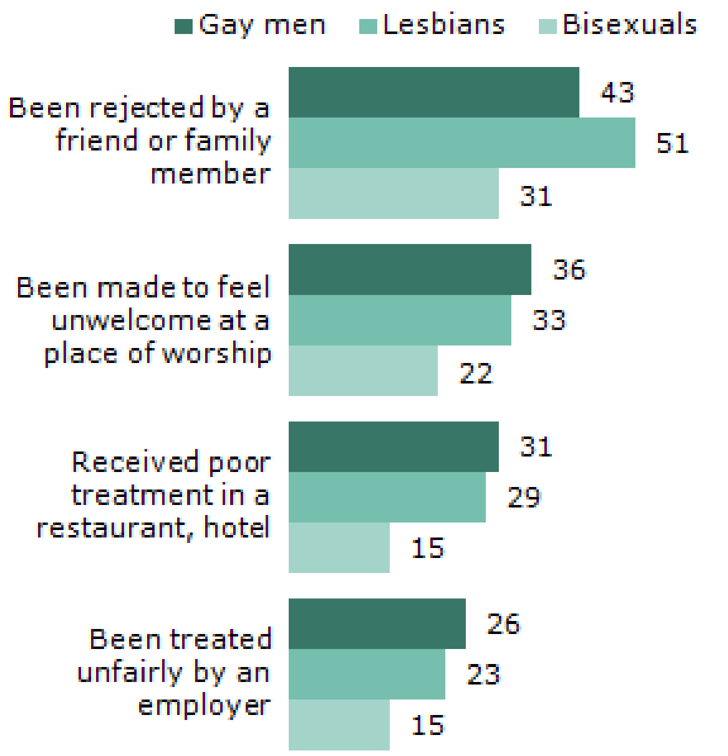


Notes: Based on all LGBT (N=1,197). "Net" was computed prior to rounding.

PEW RESEARCH CENTER LGBT/82a-f

Bisexuals Report Less Discrimination

% saying they have ever ... because they are or were perceived to be gay/lesbian/bisexual



Note: Based on gay men (n=398), lesbians (n=277) and bisexuals (n=479).

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Stubborn Minorities

A **stubborn minority** is a minority that will never change its habits, no matter what:

- Jewish and Kosher food
- Sexual habits
- Religion

When the majority has no interest in the specific matter, a change in the social norms may occur.

The idea is that it is easier to have all Kosher beverages than having to produce and distribute two different products.

The Most Intolerant Wins: The Dictatorship of the Small Minority

Nassim Nicholas Taleb

Critical Mass Theory

Apparently stable societal norms can be effectively overturned by the efforts of small but committed minorities. This leads to the Critical Mass Theory

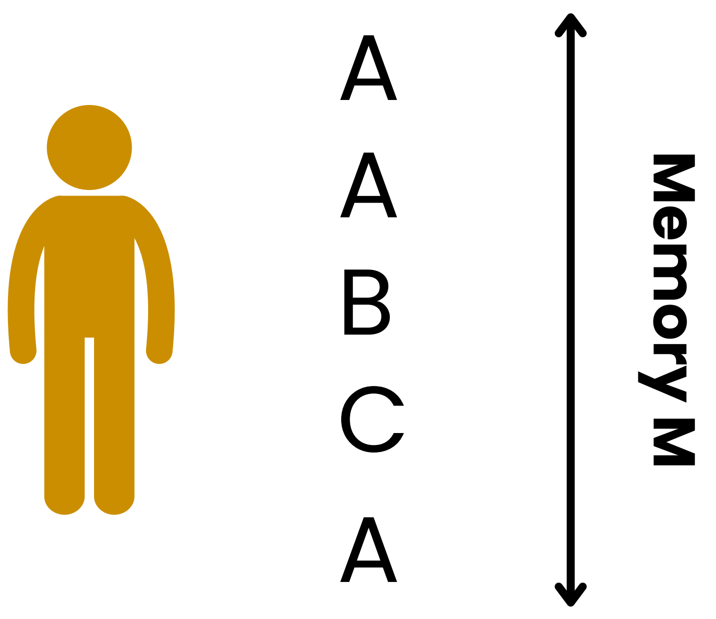
- when a committed minority reaches a critical group size the social system crosses a tipping point
- Once the tipping point is reached, the actions of a minority group trigger a cascade of behavior change

Modeling the Tipping Point

We consider a model similar to the Naming Game

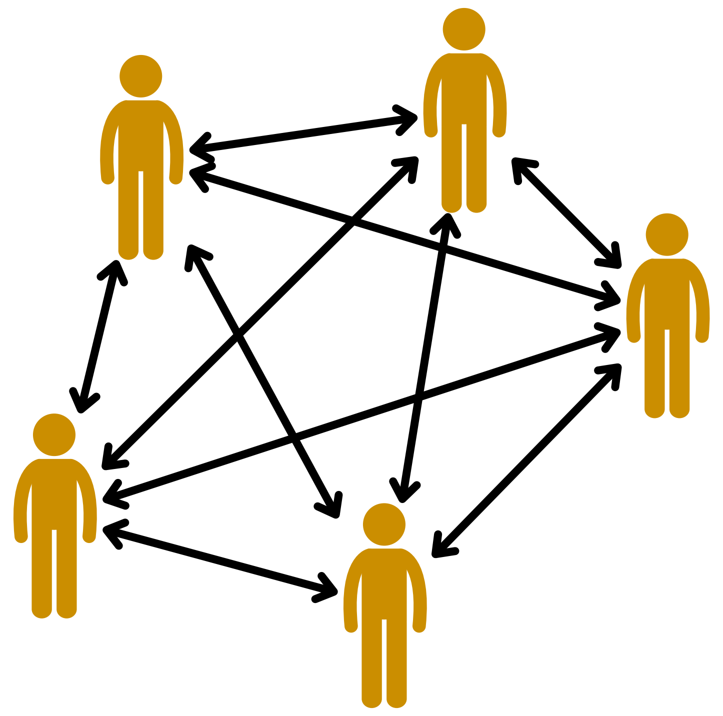
Agent

Agents store the last M names (or strategies) they heard



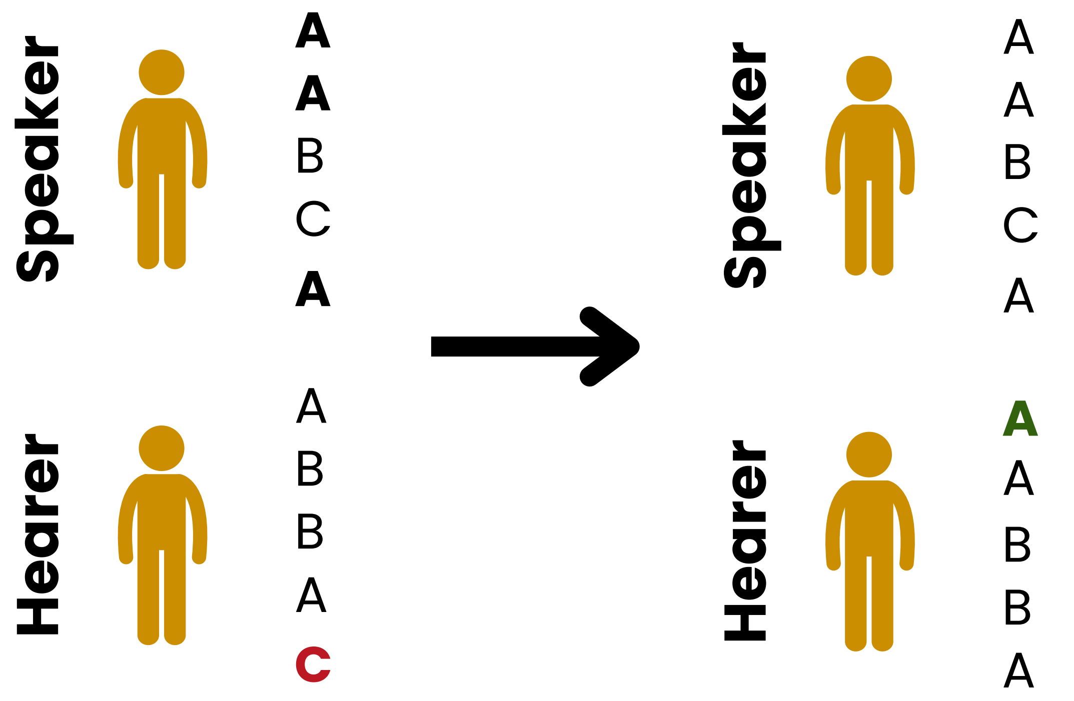
Space

Agents interact on a fully connected network (mean field)



Dynamics

The speaker communicates the most common word in its memory. The speaker records it.

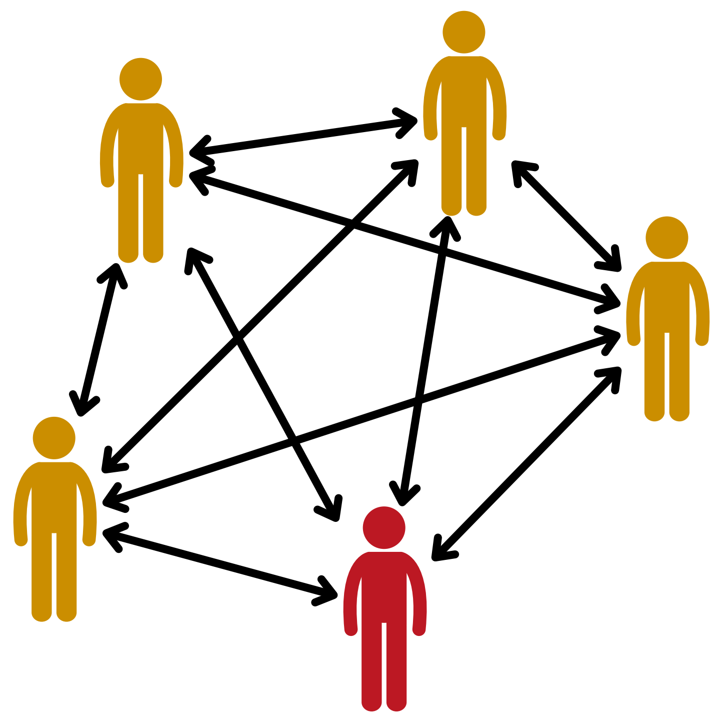


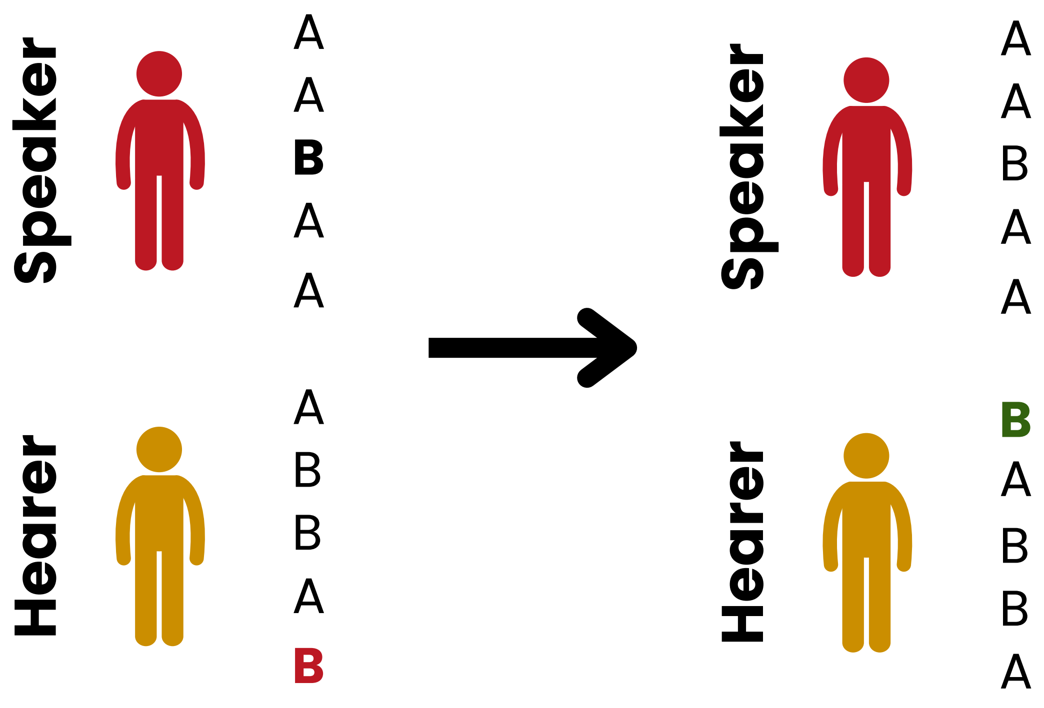
Committed Agents

We want to study if a committed minority can change social norms

- we set the initial state with all memories full of the name A (consensus)
- we introduce stubborn or committed agents
- committed agents always communicate name B, independently how their memory

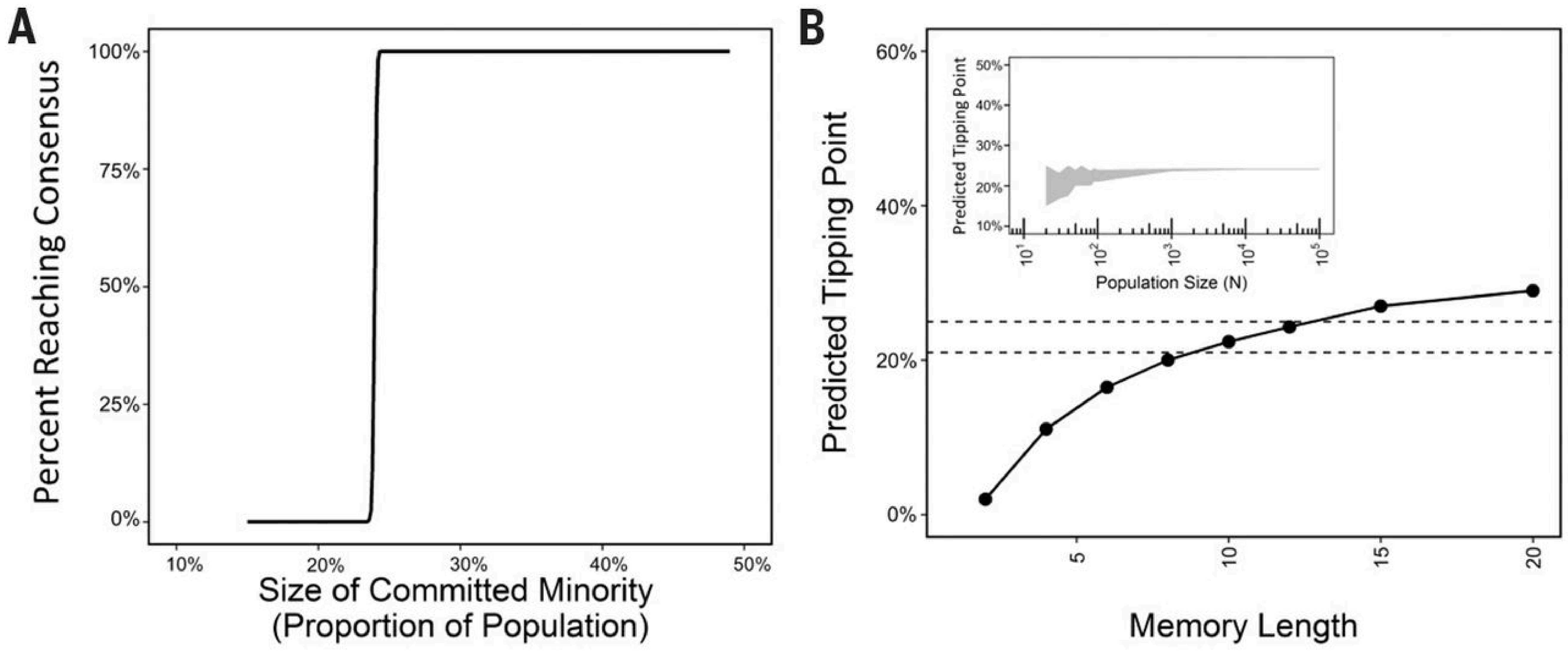
d age





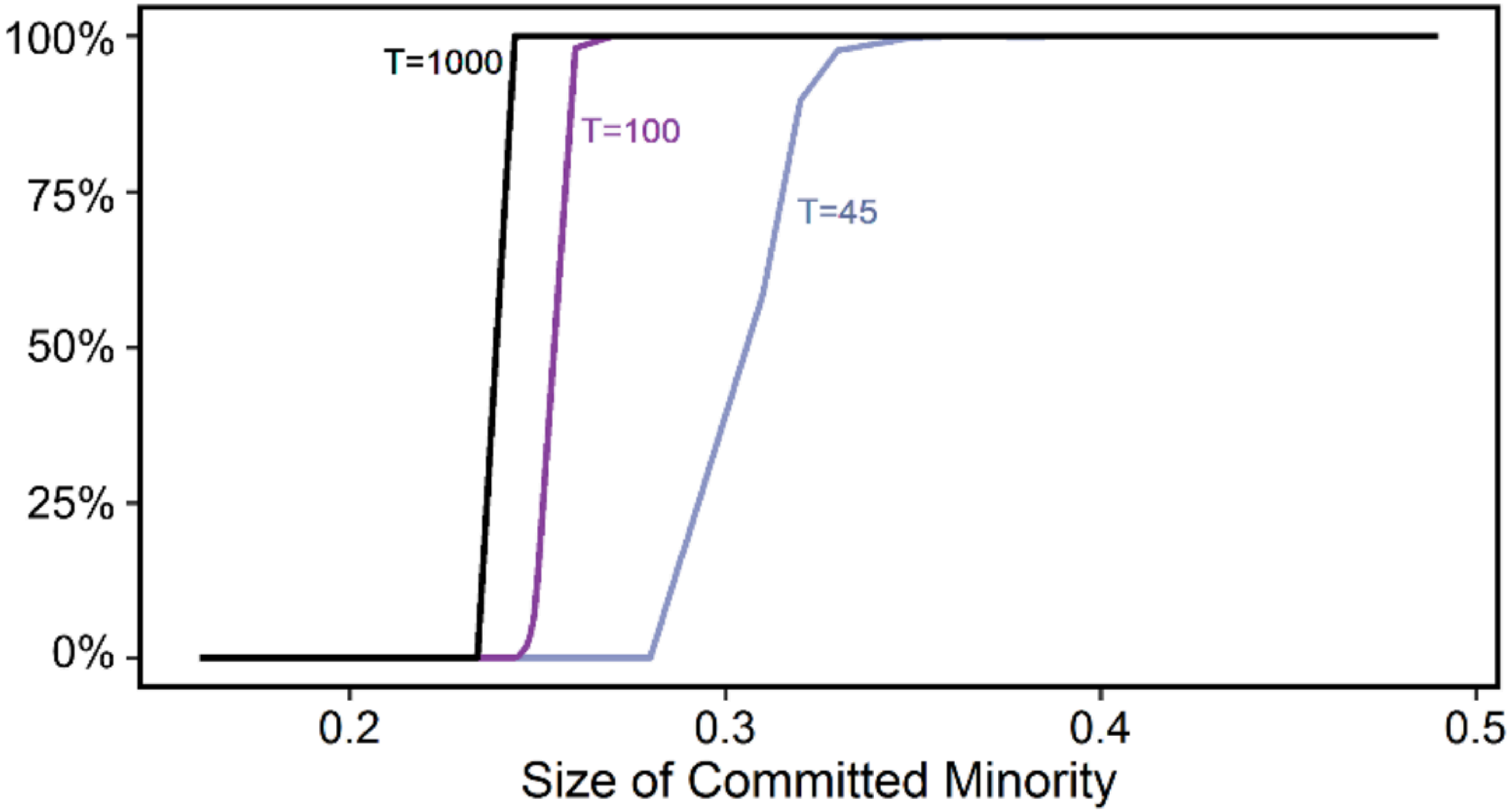
Simulation Results

The model show a tipping point for a fraction of committed agent around 20/30%. The exact tipping point depends on the memory length.

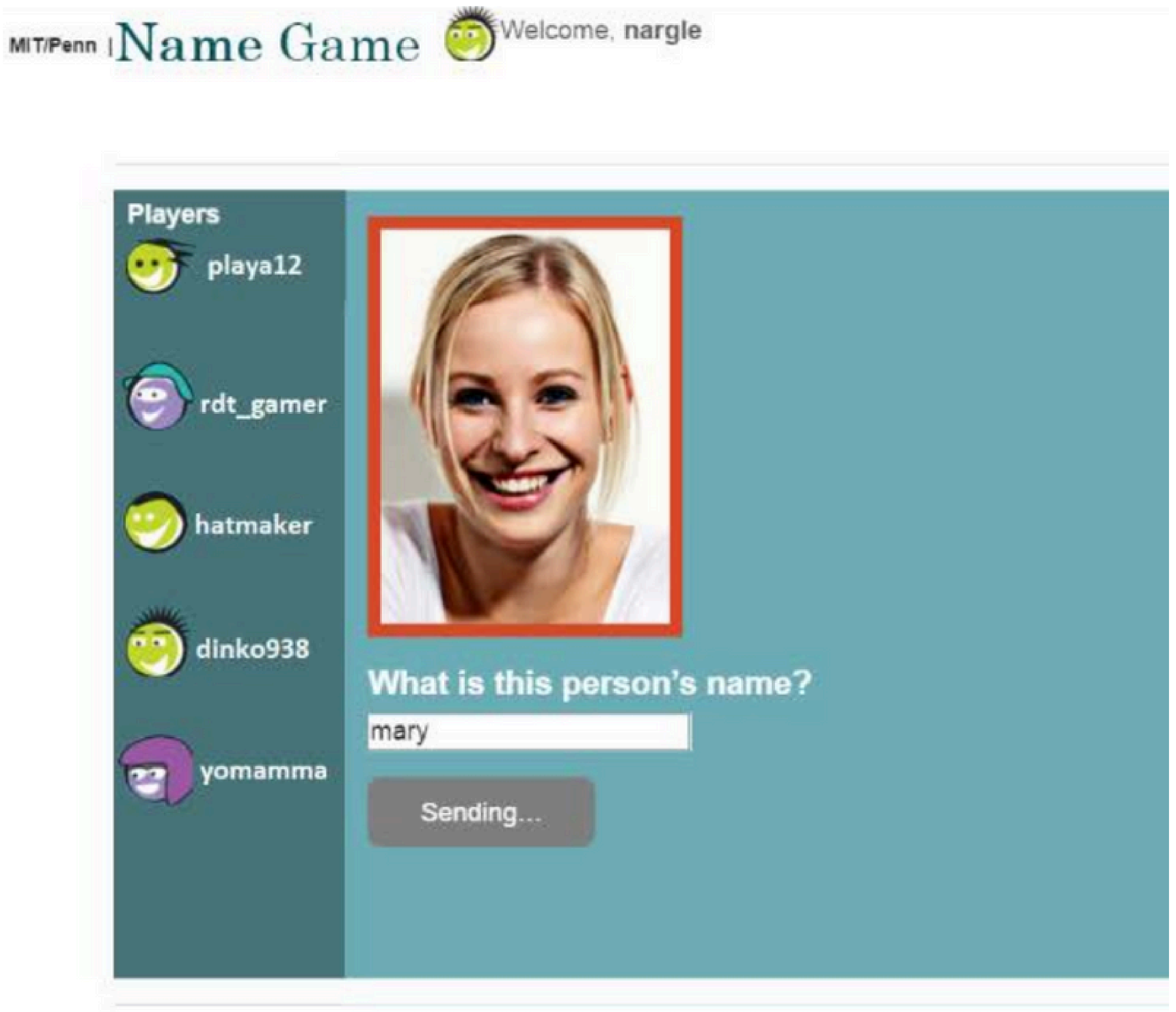


Robustness

Results are quite stable, for instance if we increase the simulation time the tipping point only slightly varies.



Testing the Model



194 Participants divided into 10 independent online groups

Procedure:

Participants randomly paired in rounds within their groups to name a pictured object

The objective was to coordinate on the same name with their partner

- Successful coordination -> financial reward
- Failure -> financial penalty.

Goal and Incentives:

The aim was not to achieve a global consensus but to coordinate successfully in each pairwise interaction.

Over time a common name emerges!

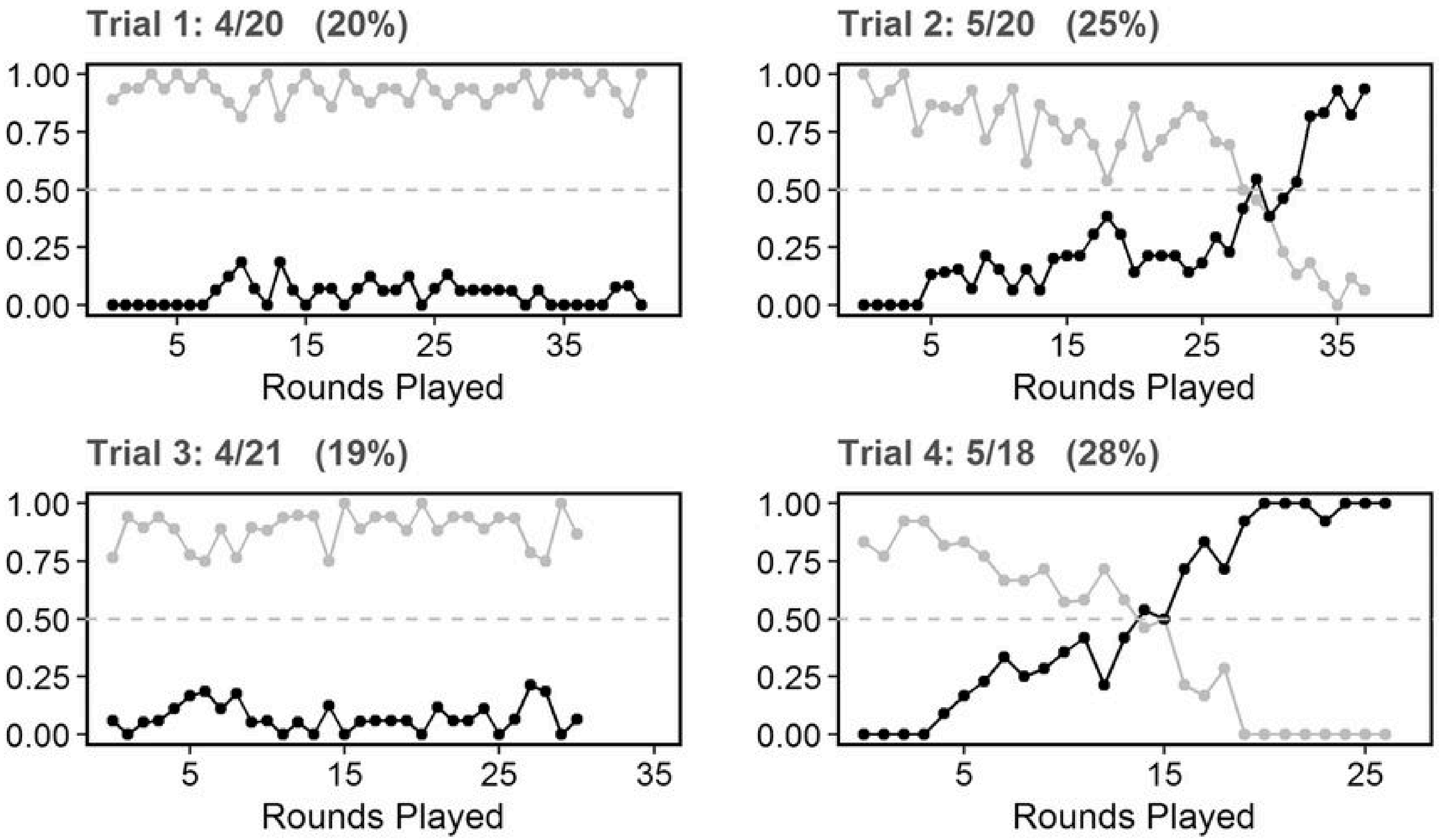
Adding a Committed Minority

After establishing a convention among all participants, a small number of confederates, termed as "committed minority," is introduced into each group:

- Their role was to challenge and attempt to change the established naming convention by consistently using a novel alternative (stubborn)
- The size of the committed minority varied across the 10 groups, designed to study the dynamics of how a critical mass can influence social norms.
- Minority sizes ranged from 15% to 35% of each group's population.

Results

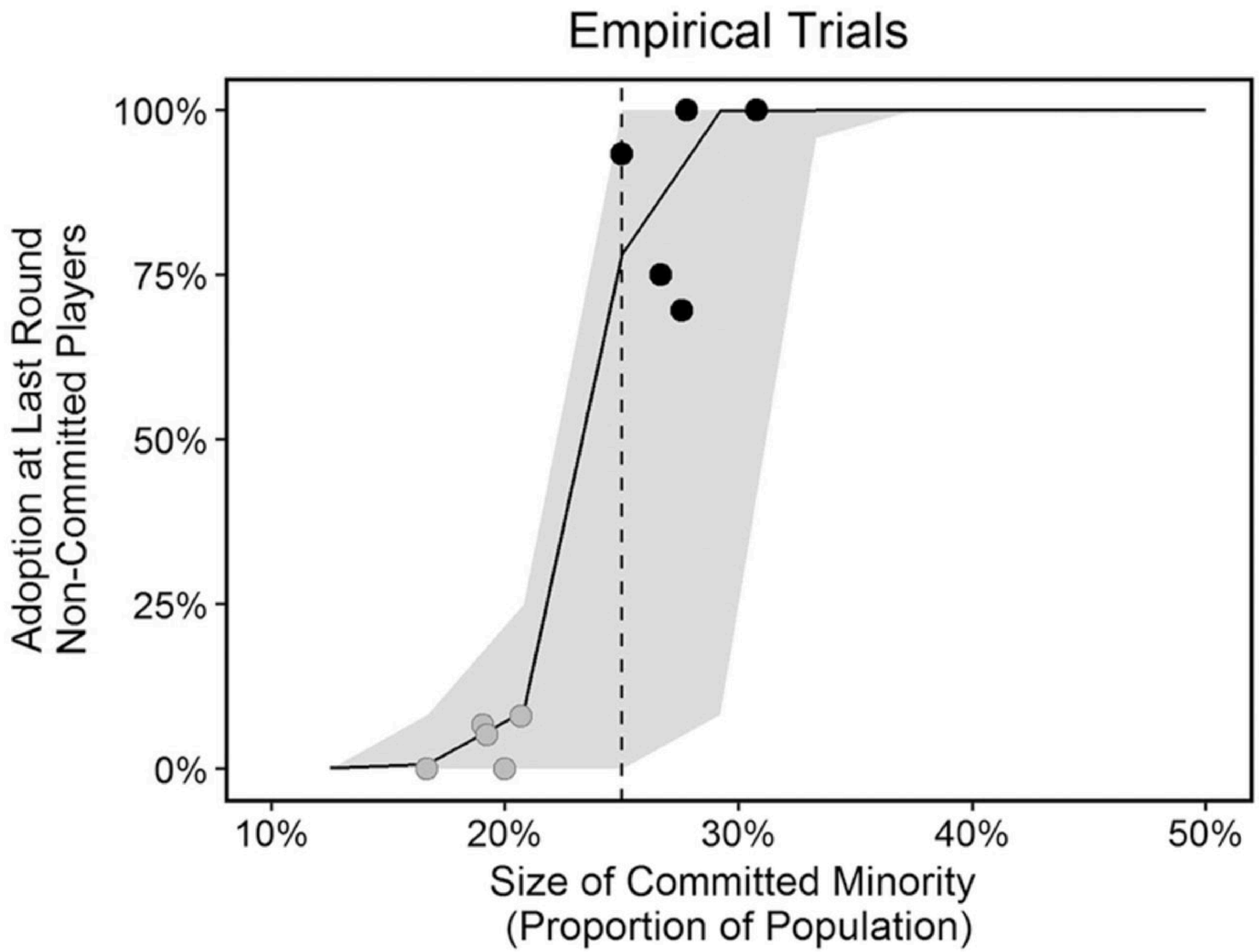
When the committed minority overcomes a threshold, there is a shift of the social norm



Tipping Point

The plot shows the aggregated results from the 10 groups:

- tipping point at 25%
- sharp (first order) transition
- a committed minority can overturn a social norm
- results similar to model



Take Home Messages

Committed Minority

A minority that will never conform to the social norm independently of the social pressure of the majority

Social Tipping Points

A committed minority can overturn a societal norm producing an abrupt transition in the system. This occur in correspondence to a critical mass.

Modeling Tipping Points

We can include stubborn or committed agents in a variation of the Naming Game. Results show a norm transition when the minority size is around 25%.

Experimental Results

The Agent Based Model is replicated using human participant on an online platform. Similar norm transitions are observed when the committed minority size is 25%.

Conclusions

Diversity Induced Collective Behavior

Complex Systems show collective behavior that originates from the presence of differences among the individuals

Granovetter's Threshold Model

A model to describe the activation of individuals based on thresholds. Individual differences have very relevant outcomes.

Analysis of Granovetter's Threshold Model

The model can be described as a map that shows a first order transition (tipping point) when the variance of the thresholds distribution is increased

Can a Minority Win?

A stubborn minority can generate an abrupt societal norm change when a critical mass is reached. Model and experiment support this theory.

Quiz

- Which of these is complex and which is complicated?
  - An airplane
  - The Internet
  - The Web
  - A deep neural network
- Do you know any example of spreading?
- For a given σ, does μ change the outcome in Granovetter's model?
- Do you think there are minorities in Konstanz? Are they discriminated?
- Which are some examples of stubborn minorities?
- Do you know any example of tipping point in society?
- Which are the limits and strengths of the tipping point model?

Play Yourself to Understand!

Logistic Map

https://www.complexity-explorables.org/flongs/logistic/