

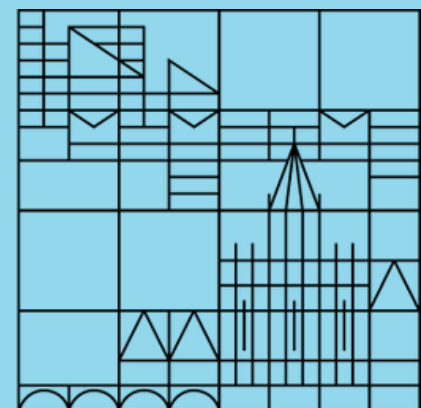
UNIVERSITÄT KONSTANZ

Opinion Dynamics

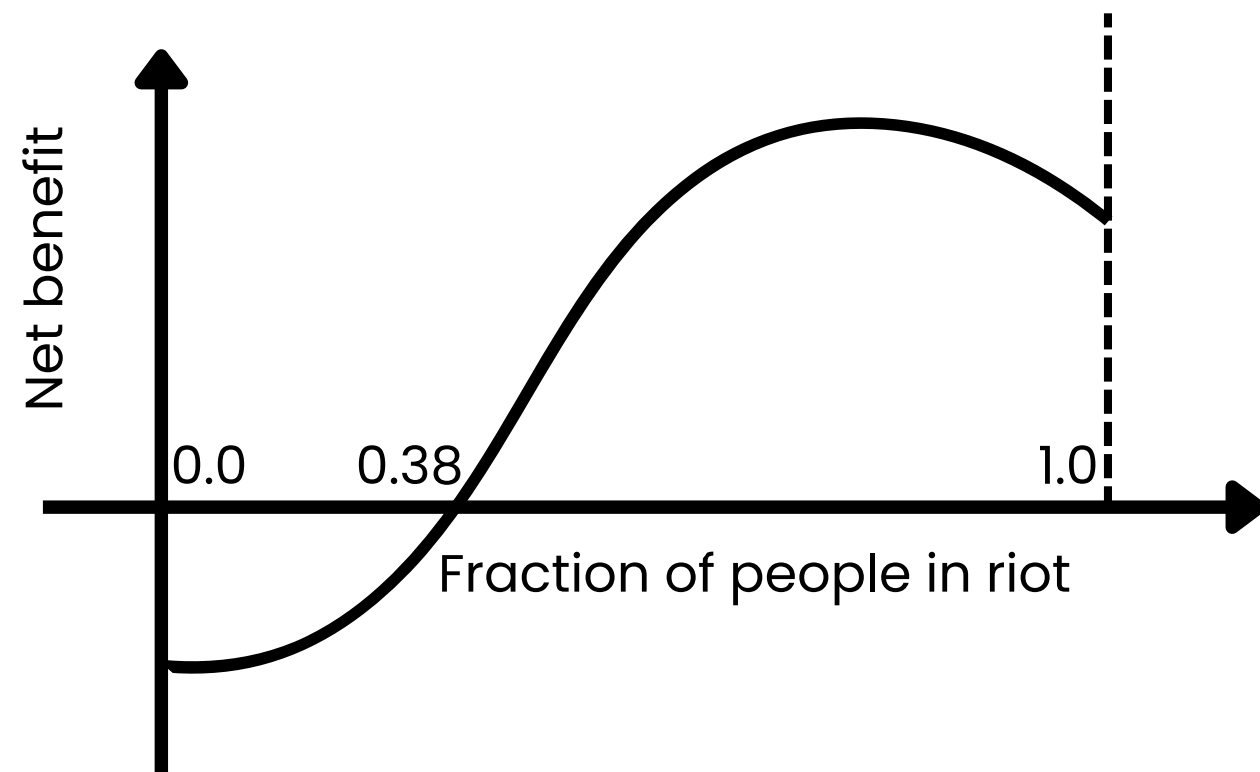
Computational Modelling of
Social Systems

Giordano De Marzo
Max Pellert

Universität
Konstanz



Recap



Diversity

What is the role of diversity in the emergence of collective behaviors?

Granovetter's Model

Simple model to describe spreading and tipping points. Based on diversity.

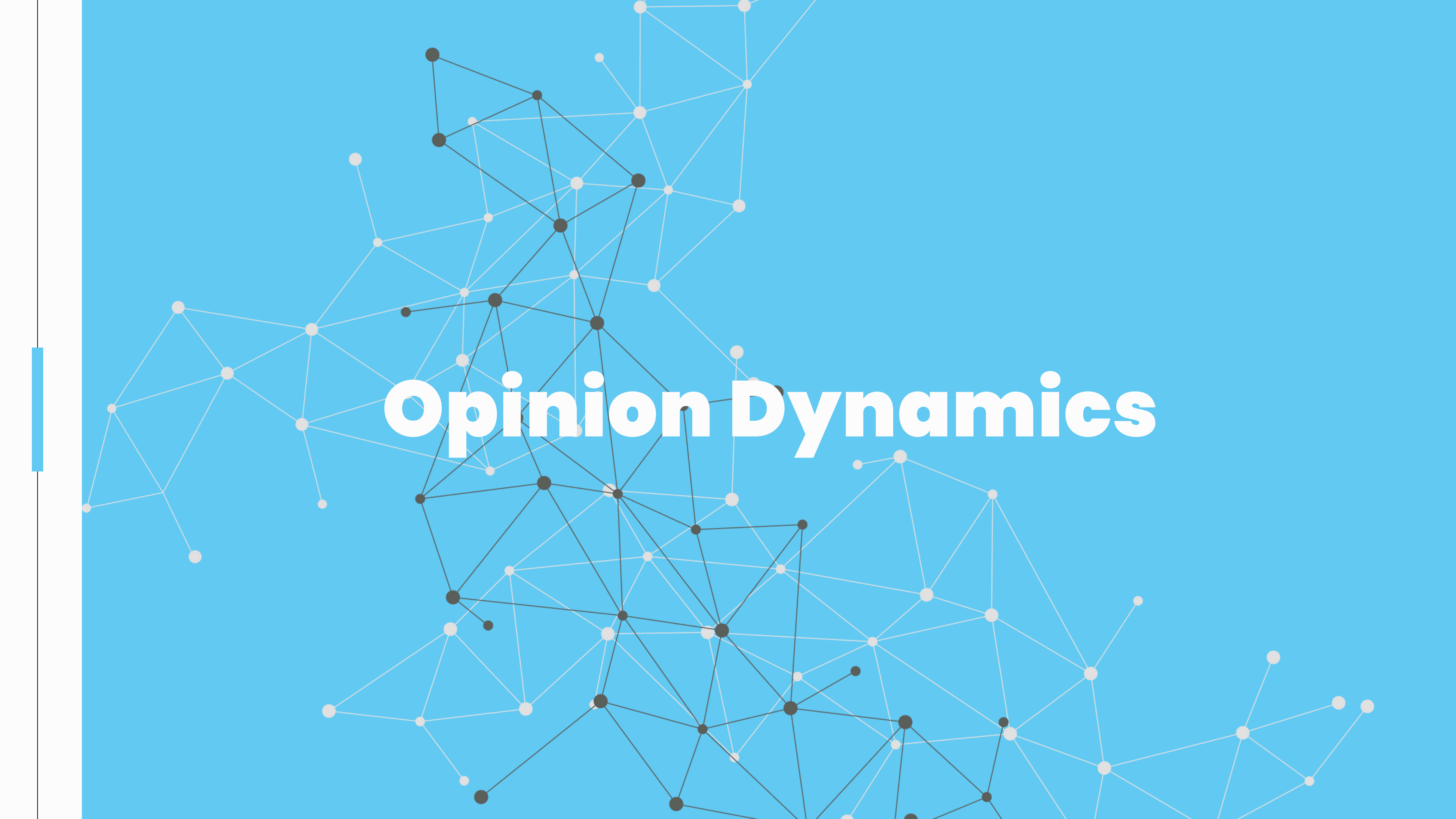
Stubborn Minorities

A stubborn minority can produce a societal change. This leads to the critical mass theory.

Outline

1. Opinion Dynamics
2. Voter Model
3. Bounded Confidence Model
4. Recommendation Algorithms and Opinion Dynamics



A network graph with nodes and edges, overlaid on a blue background. The nodes are represented by small circles, some of which are black and others are light gray. The edges are thin lines connecting the nodes. The graph is dense and interconnected, with a central cluster of nodes and several smaller clusters branching out. The overall structure is complex and non-linear.

Opinion Dynamics

What is an **Opinion**?

An opinion is a view or judgment formed about something, not necessarily based on fact or knowledge. It represents an individual's feelings or thoughts about a particular topic.

We tend to have opinions on more or less everything, examples are:

- politics
- football
- musics
- our friends' behavior



Opinion Dynamics

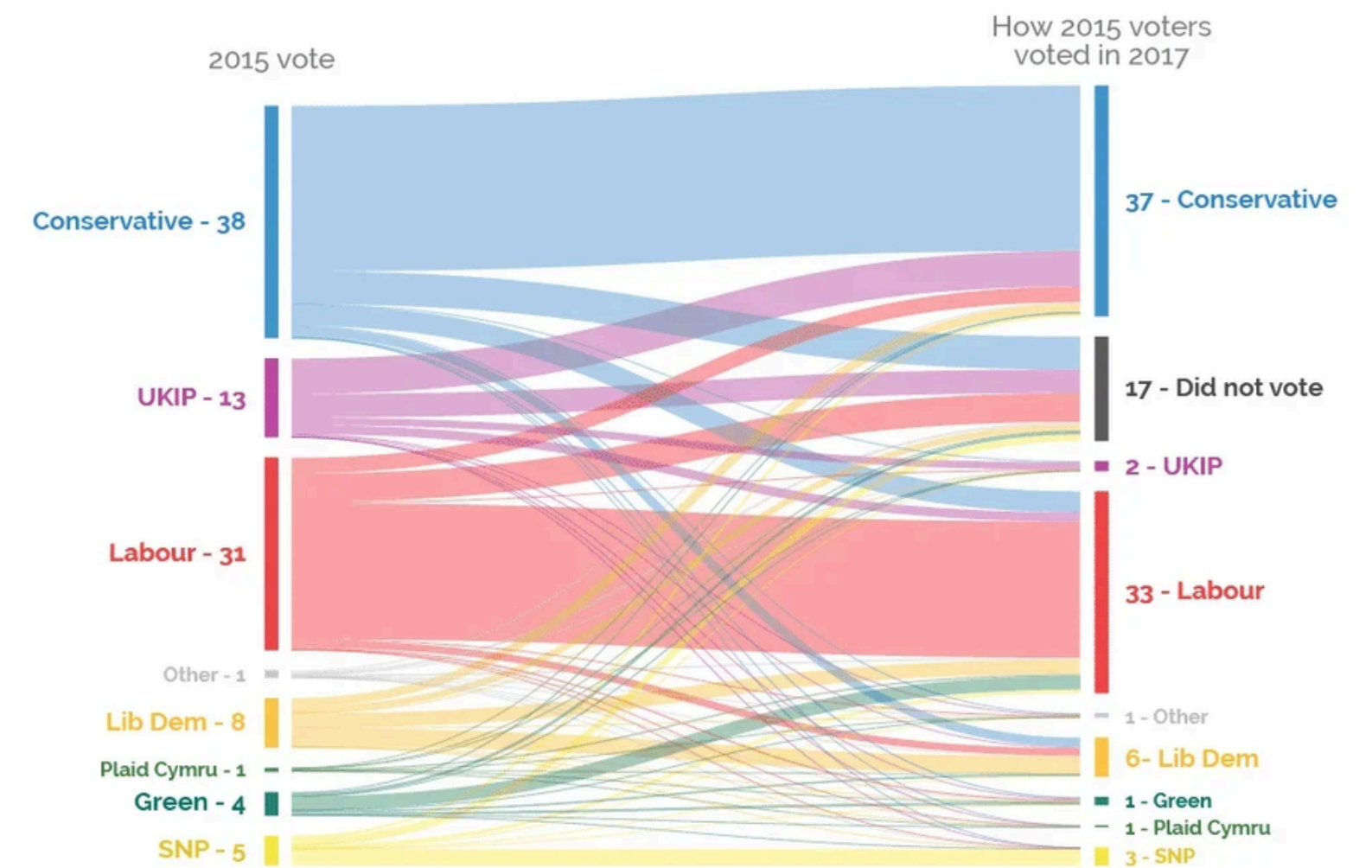
Opinions are not static, they are in continuous evolution due to the interaction with other people, the effect of mass media and social networks. We may change our opinion about

- political parties
- friends
- social norms

Opinion Dynamics studies how opinions are shared and diffused among individuals, with the aim of understanding the global opinion patterns that may emerge.

How did 2015 voters vote at the 2017 general election?

Based on a survey of 36,147 GB adults who had voted in the 2015 general election about their vote in the 2017 general election



Some Nomenclature

Consensus

All agents in the system share the same opinion.

Disorder

Agents' opinions vary randomly over time in a random way. Agents don't have a preferred opinion.

Fragmentation

Each agent has a favored opinion that hold more frequently than any other opinion.

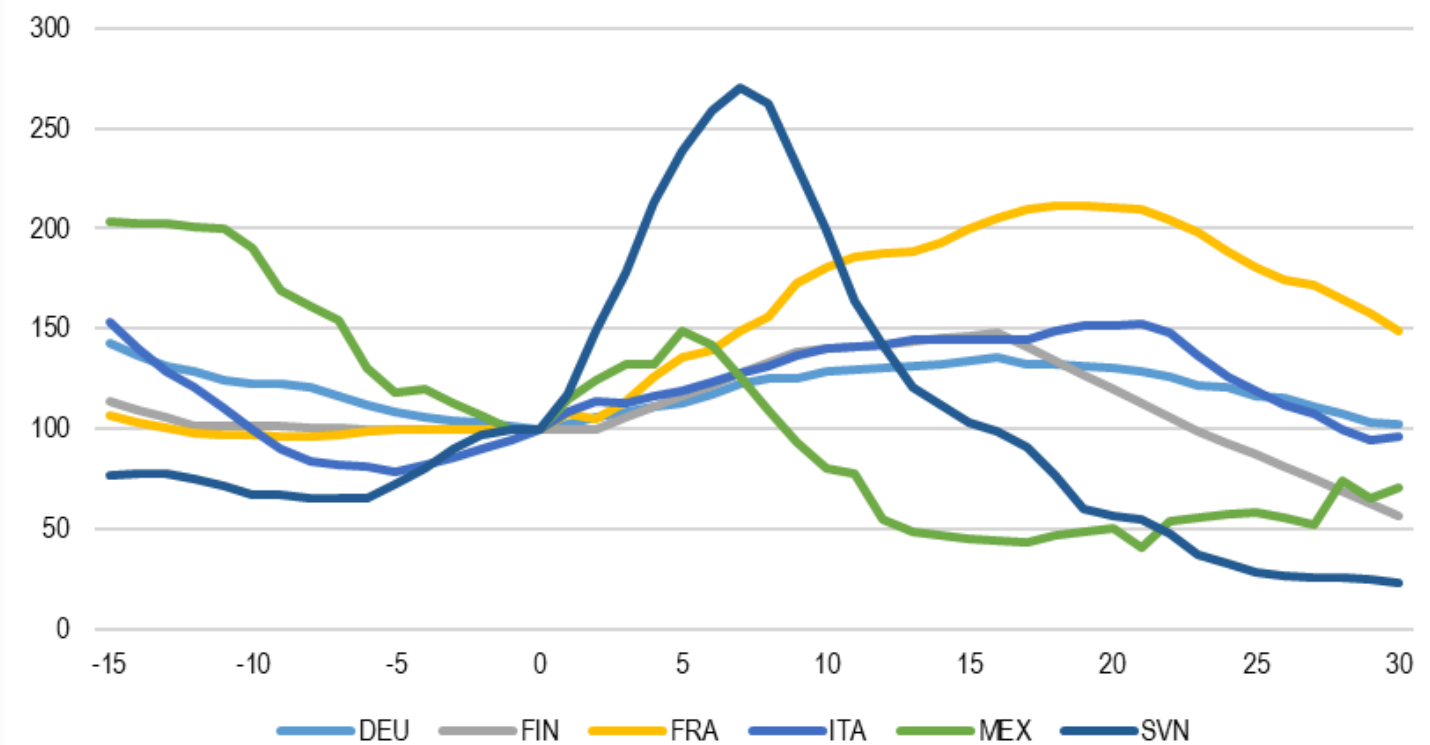
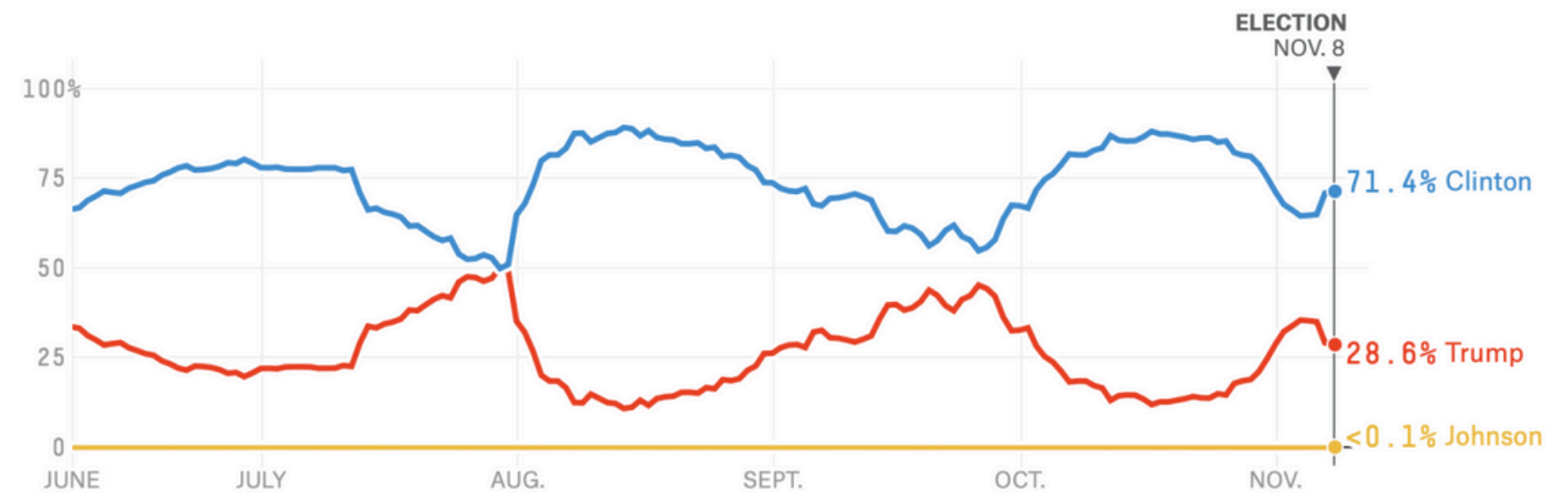
Polarization

Each agent has a favored opinion and is connected to other agents with the same favored opinion.

Examples of Opinion Dynamics

There are many examples of opinion dynamics in several areas

- **Political Elections.** During political campaigns, opinions about candidates and issues can shift rapidly due to debates, advertisements, and news coverage.
- **Public Health.** Opinions on vaccines can fluctuate widely due to misinformation, scientific reports, and celebrity endorsements.



Key Questions in Opinion Dynamics

We want to understand the main factors driving and influencing opinion dynamics

- Under which circumstances does a group of people reaches consensus on a given opinion?
- Is a central authority needed for reaching consensus?
- What are the drivers of opinion polarization and fragmentation?
- What is the role of social networks and mass media on opinion dynamics?
- How can we forecast how the opinion of a large group will evolve over time?

A network graph visualization on a blue background. The graph consists of numerous nodes, represented by small circles, connected by thin lines (edges). The nodes are arranged in a complex, interconnected pattern, with some nodes being larger and darker (black) than others. The overall structure suggests a social network or a complex system of relationships. The text 'Voter Model' is overlaid in the center of the graph.

Voter Model

Binary Opinions

In many contests we face binary opinions or options

- vote in a referendum
- science or conspiracy
- vax or no-vax

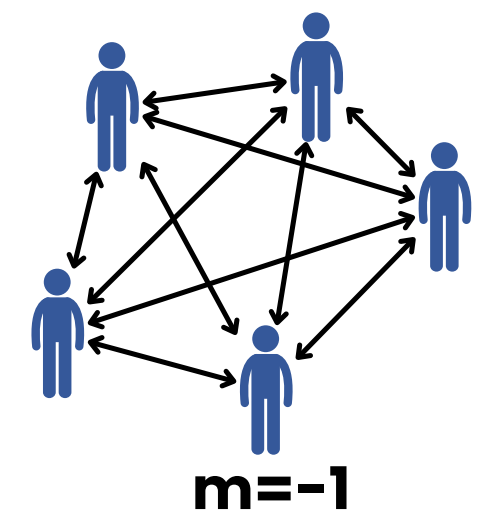
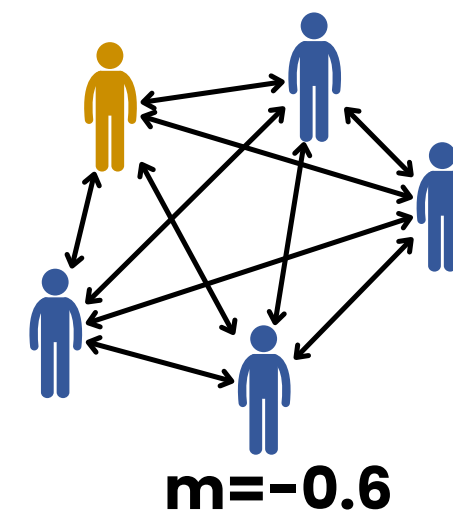
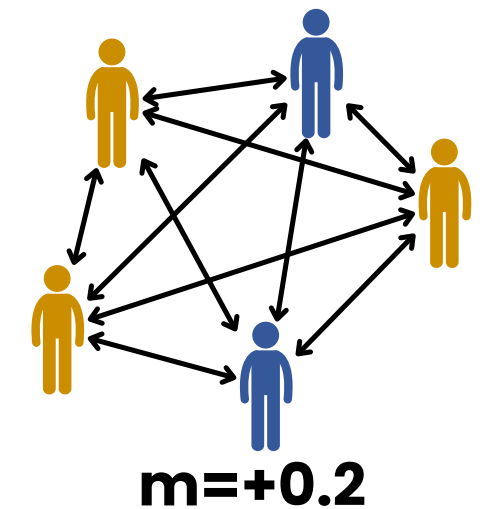
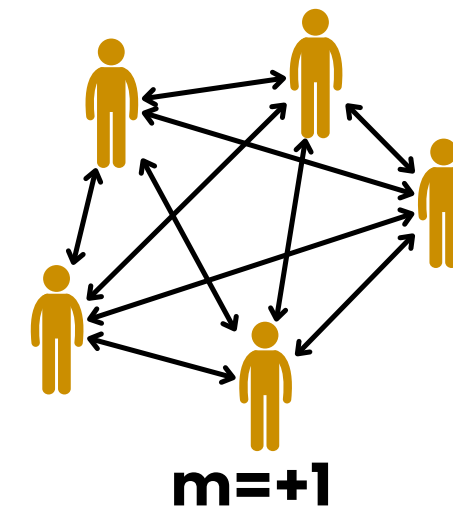
The two opinions are generally described as +1 and -1 states. Each agent is then assigned a number, either +1 or -1, depending on its opinion.

The state of the system is described by the magnetization m
$$m = (2N_+ - N) / N$$

Positive Opinion



Negative Opinion



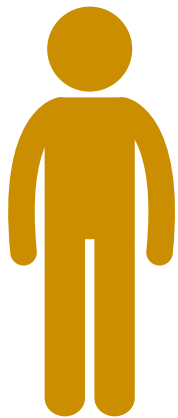
Following Majority: Glauber Dynamics

In Glauber Dynamics agents experience a strong social pressure

Agent

Agents are described by their opinion, either positive or negative

Positive Opinion

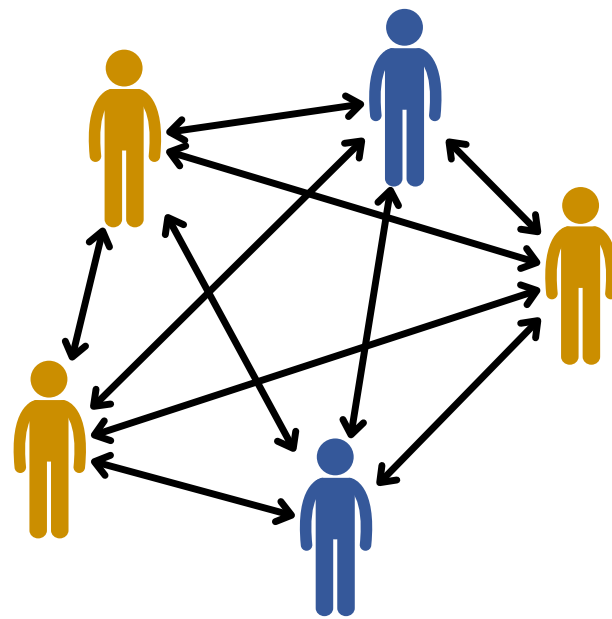


Negative Opinion



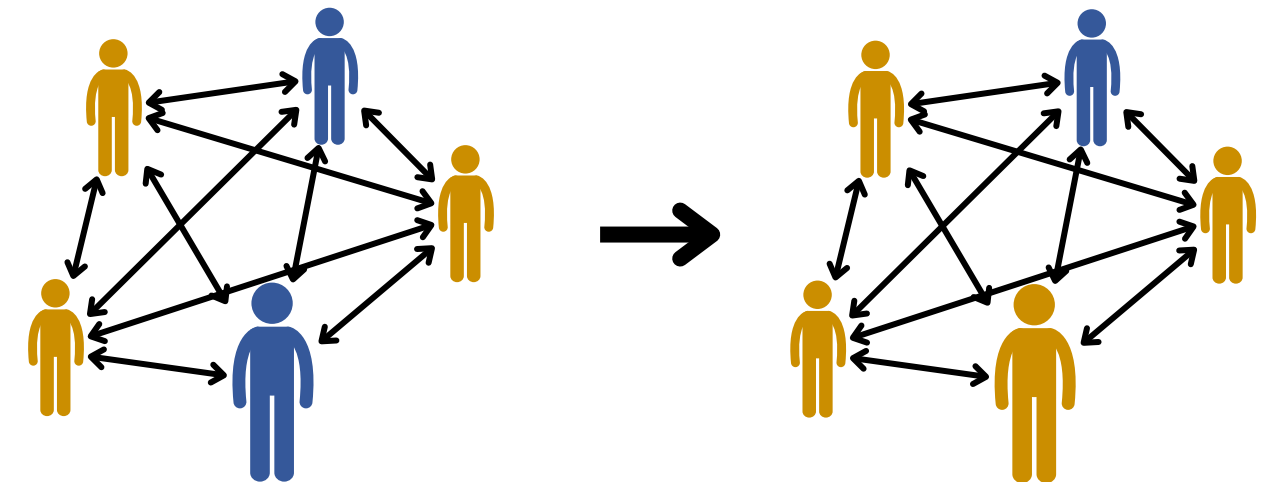
Space

Agents interact on a network or on a lattice



Dynamics

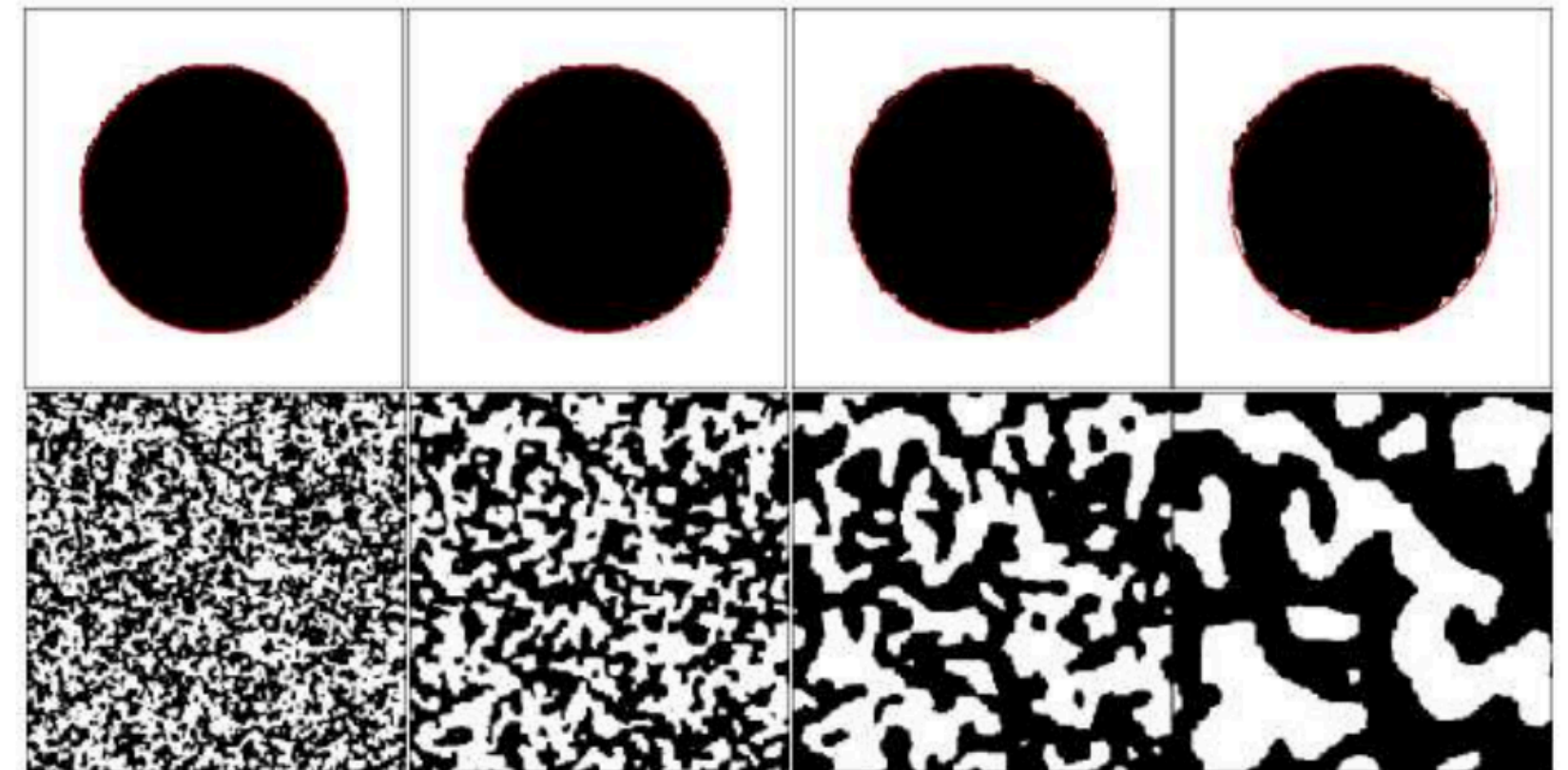
At each time step an agent is selected and its opinion becomes equal to that of the majority of its neighbors



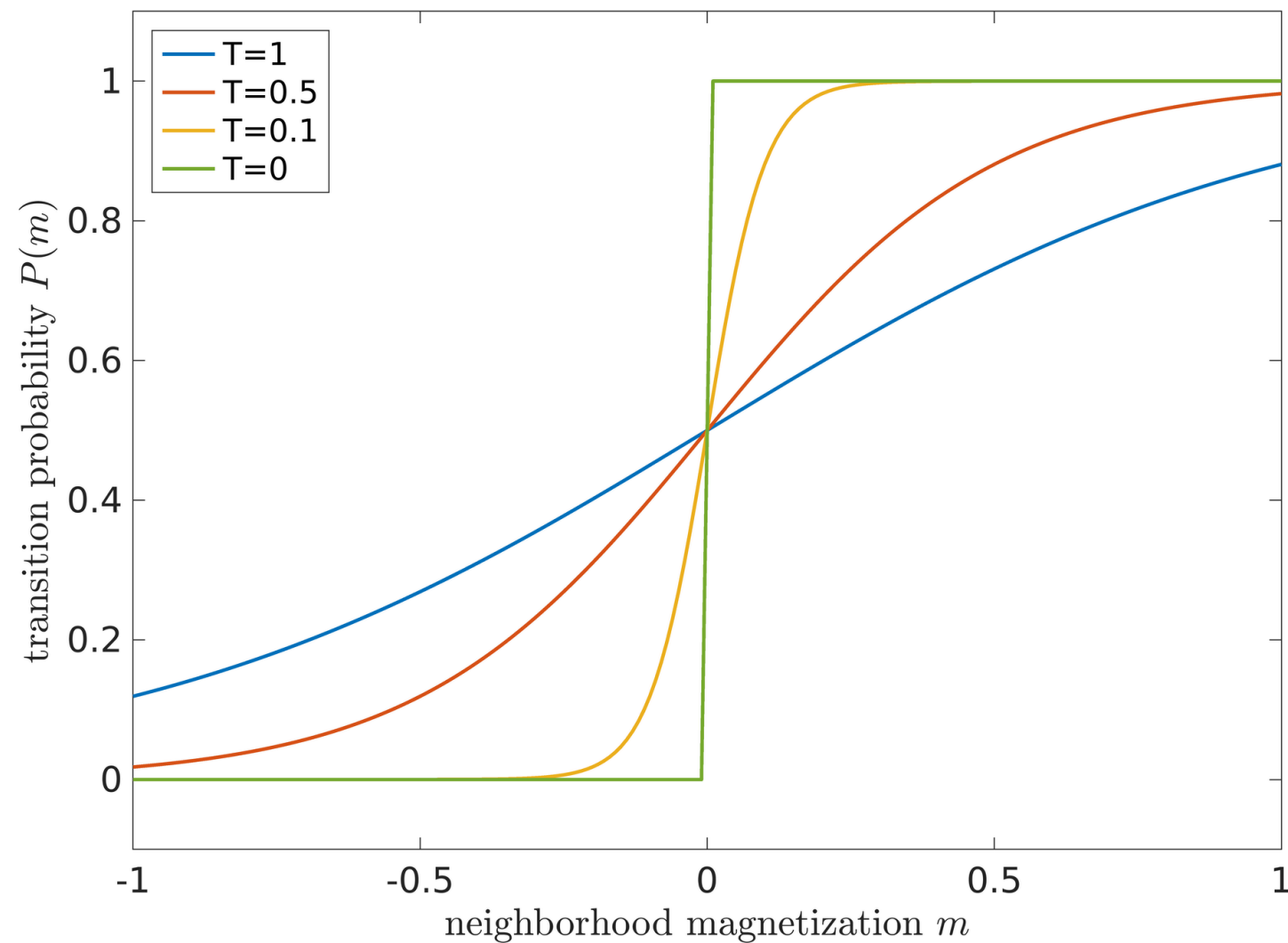
Simulation on 2D Lattice

The system has tendency to evolve toward consensus, that is reached through a coarsening process driven by surface tension

- Initially, small coherent islands are formed
- then these islands grow in size till covering the whole system
- while in the mean field case consensus is always reached, on lattices or networks, the process can take very long (infinite) time
- metastable states can form



Introducing Randomness



In Glauber Dynamics agents always follow the majority in a deterministic way

- To make the model more realistic some randomness (or temperature T) can be included.
- This modifies the transition probability (probability to choose the first opinion)
- Depending on T , the consensus may disappear

As T increases, the transition probability becomes more and more smooth.

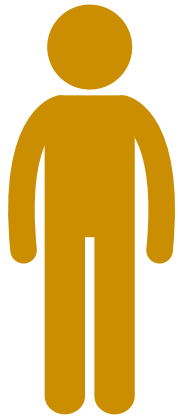
The **Voter** Model

In the Voter Model agents copy their neighbors

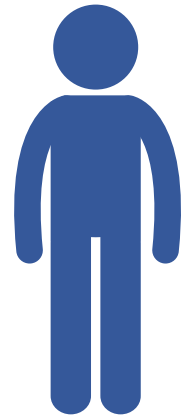
Agent

Agents are described by their opinion, either positive or negative

Positive Opinion

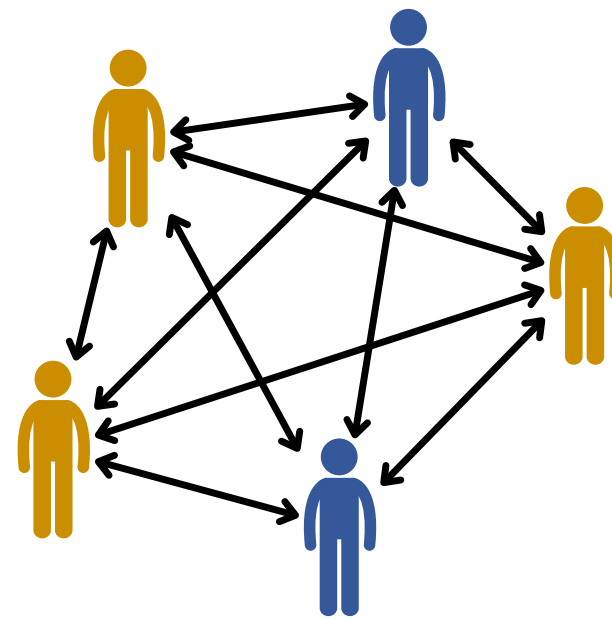


Negative Opinion



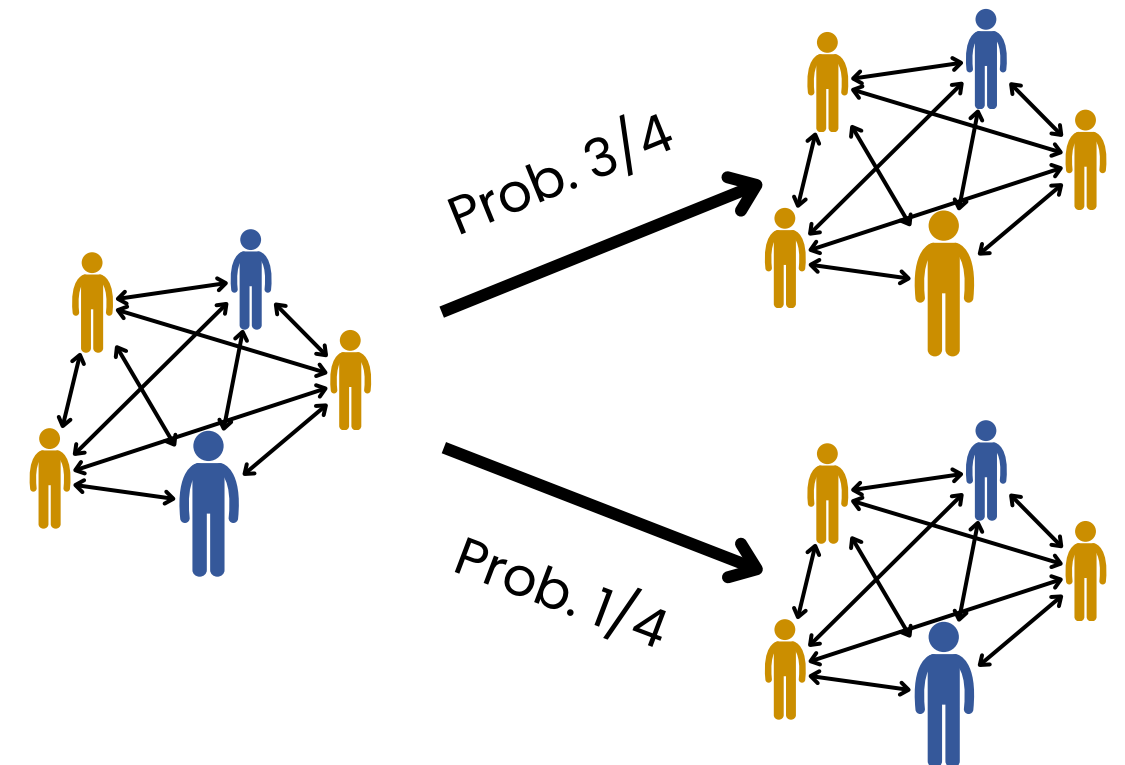
Space

Agents interact on a network or on a lattice



Dynamics

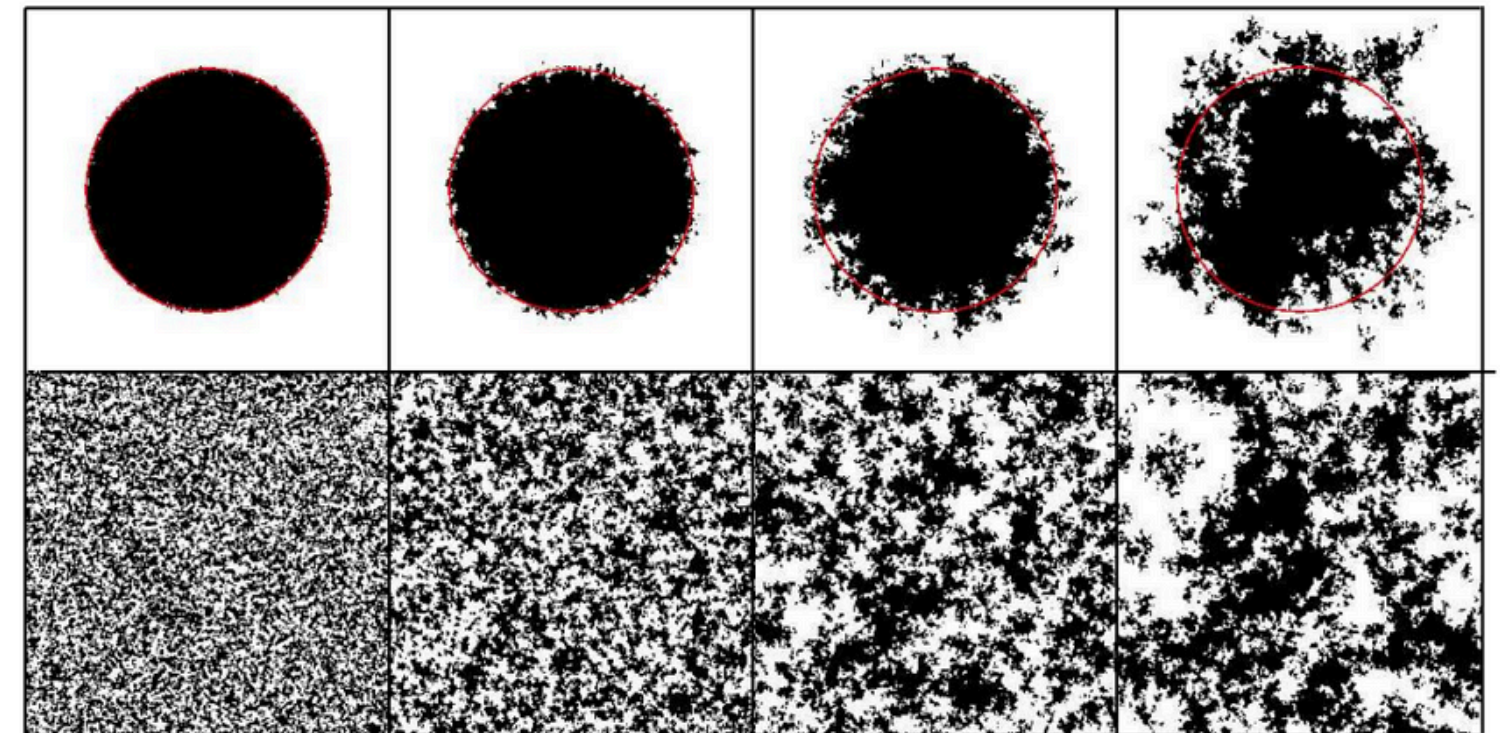
At each time step an agent is selected and it copies a random neighbor's opinion



Simulation on 2D Lattice

The system has a tendency to evolve toward consensus, but this is weaker than in the Glauber dynamics

- the process is diffusion driven (there is no drift)
- the magnetization is conserved
- there is no surface tension
- due to fluctuations there are no metastable states



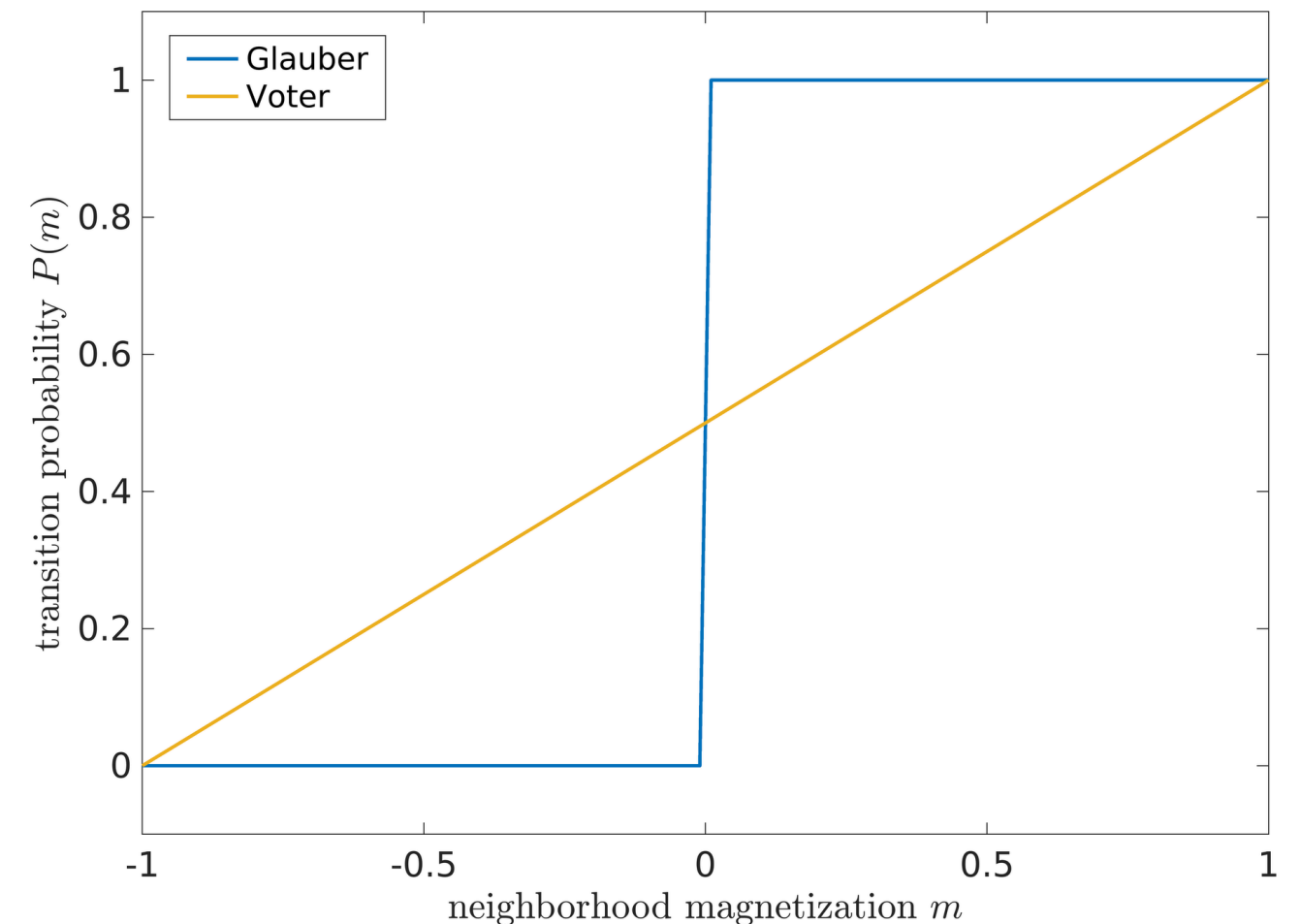
Properties of the Voter Model

The Voter Model behavior is strongly influenced by the topology (space):

- for lattices $D < 3$ consensus is reached also in infinite systems thanks to coarsening
- for lattices with $D > 2$ consensus is reached only in finite systems and thanks to fluctuations

An important quantity is the consensus time T

- number of updates of the whole system needed for reaching consensus
- $T \sim N \log(N)$ for $D=2$
- $T \sim N$ for $D > 2$



Including Memory

In both Glauber dynamics and the voter model agents change opinion independently of their past history

- people don't change opinion easily
- the more you hold an opinion, the less likely you are to change it

In order to account for this we introduce a memory effect in the voter model

- in the standard model the transition probability of agent i is determined by the fraction of its positive neighbors f_i

$$P_i = f_i$$

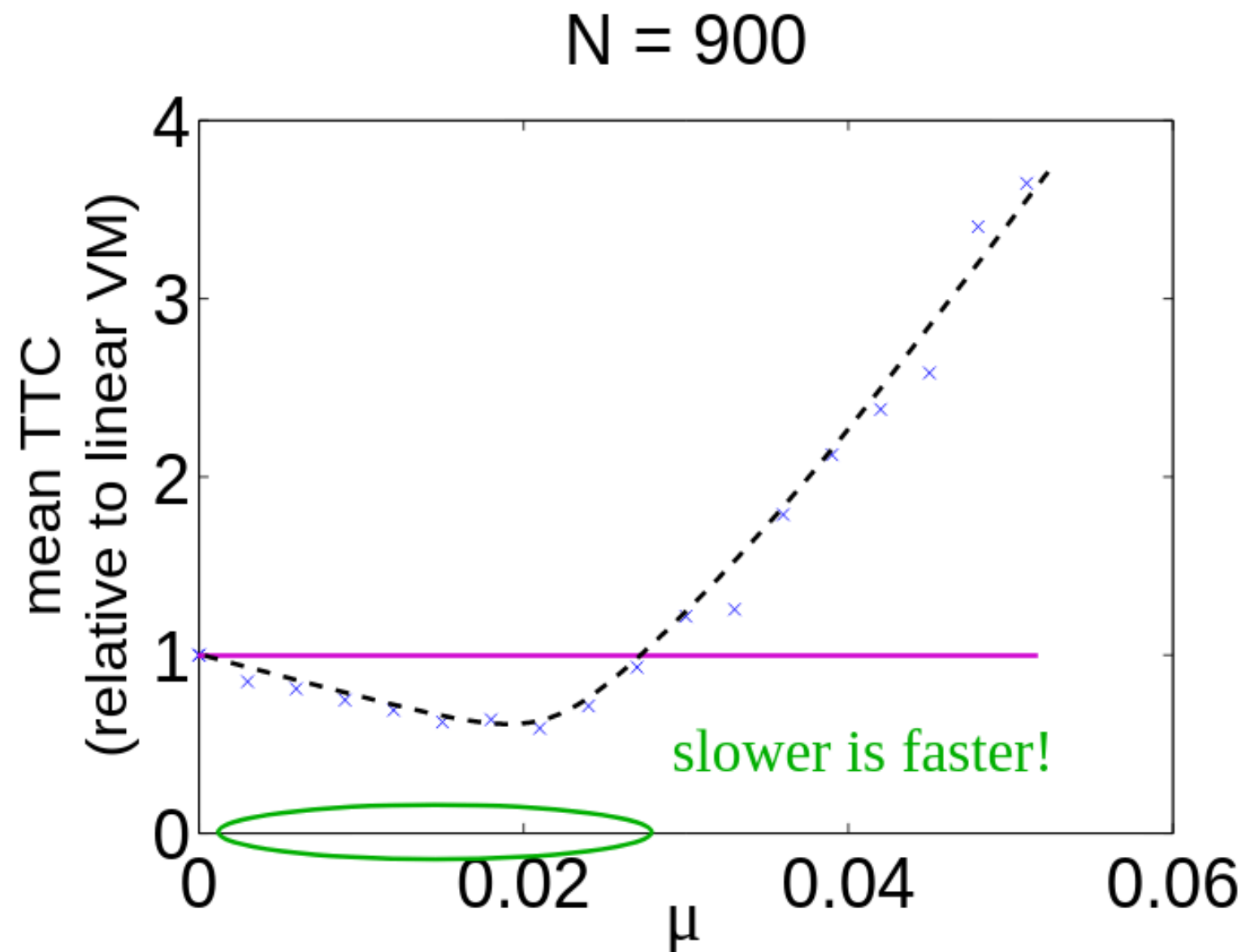
- we modify it to include a memory term

$$P_i = [1 - v_i(\tau_i)] f_i$$

- τ_i is the amount of time since the last change of opinion of agent i
- the evolution of $v_i(\tau_i)$ is linear up to a saturation v_s and is governed by the inertia μ

$$v_i(\tau_i) = \min[\mu\tau_i, v_s]$$

Slower is Faster!



Stark, Hans-Ulrich, Claudio J. Tessone, and Frank Schweitzer.
"Decelerating microdynamics can accelerate macrodynamics
in the voter model." *Physical review letters* 101.1 (2008): 018701.

For $\mu > 0$ the micro-dynamics is slower

- agents change opinion more reluctantly
- one would expect consensus time to increase

Numerical simulations show that consensus time is not monotonic in the inertia μ

- there is an optimal value of $\mu > 0$ for which consensus time is minimum
- slowing down the micro-dynamics makes the macro-dynamics faster
- micro-macro gap once again

More than two Opinions

Binary opinions are a nice schematization, but life is more complex

- many political parties
- many football teams
- many possible favorite artists

We can consider M opinions, each is assigned a different color. We denote by N_k the number of agents sharing opinion k . In this way we can define opinion k magnetization as $m_k = (2N_k - N)/N$

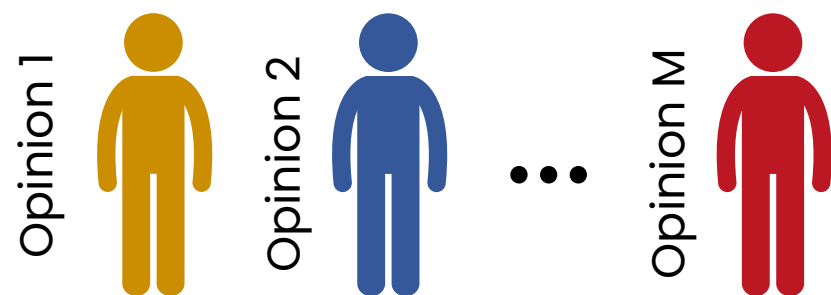


The Multistate Voter Model

The Multistate Voter Model generalizes the Voter Model to multiple opinions

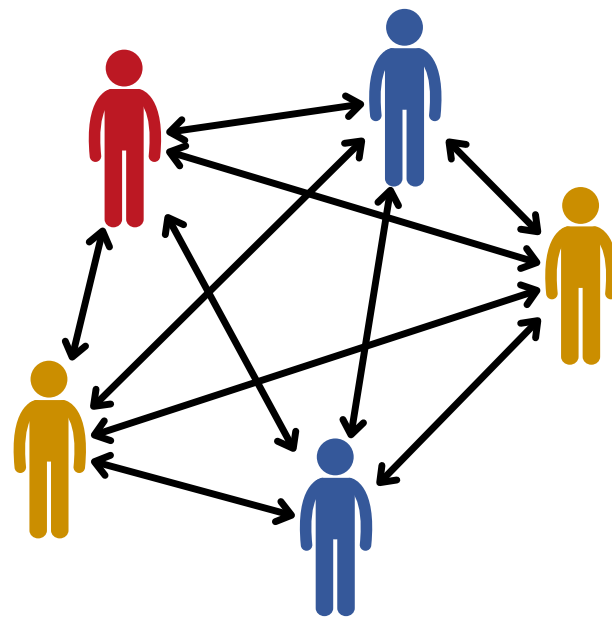
Agent

Agents are described by their opinion, which can be one out of M



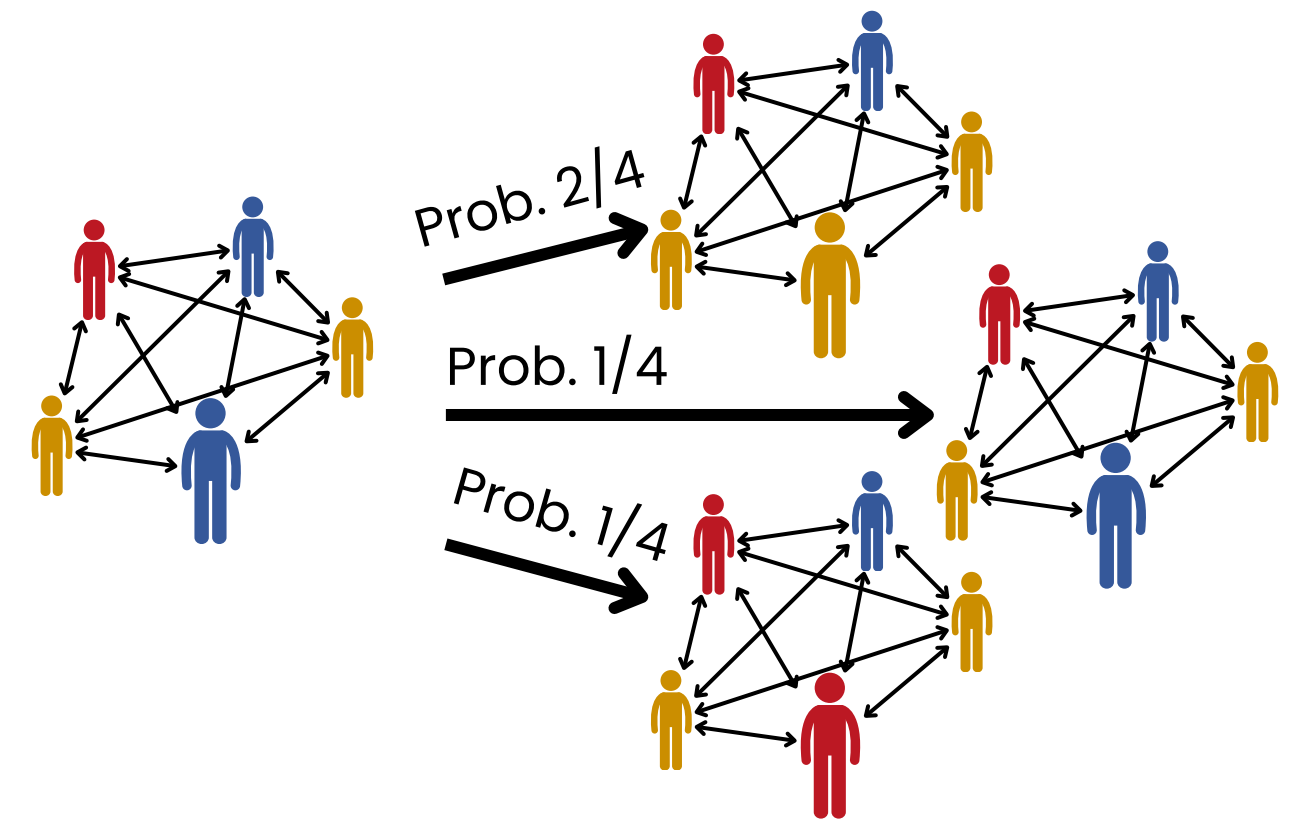
Space

Agents interact on a network or on a lattice

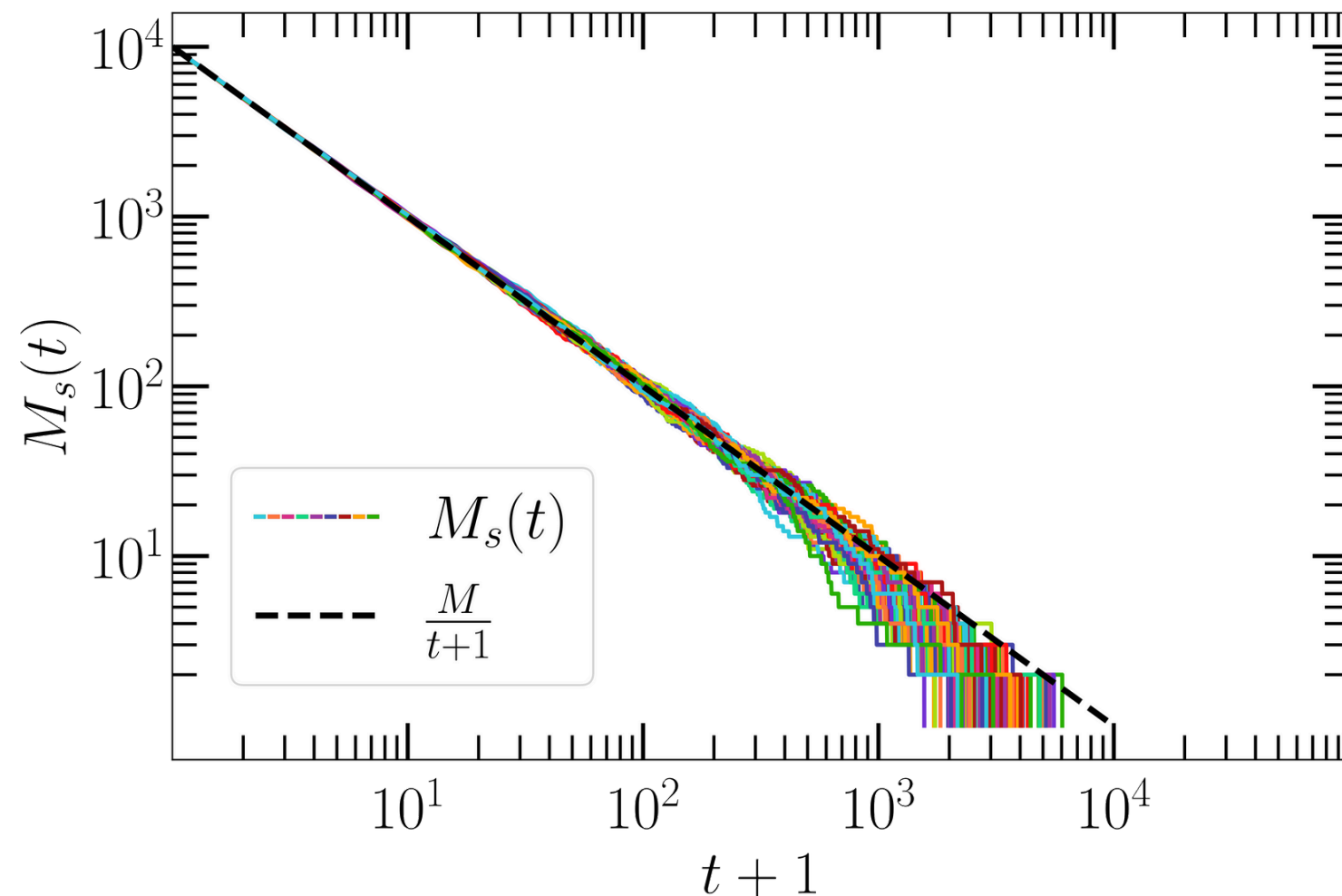


Dynamics

At each time step an agent is selected and it copies a random neighbor's opinion



Convergence to Consensus



Starnini, Michele, Andrea Baronchelli, and Romualdo Pastor-Satorras.
"Ordering dynamics of the multi-state voter model." *Journal of Statistical Mechanics: Theory and Experiment* 2012.10 (2012): P10027.

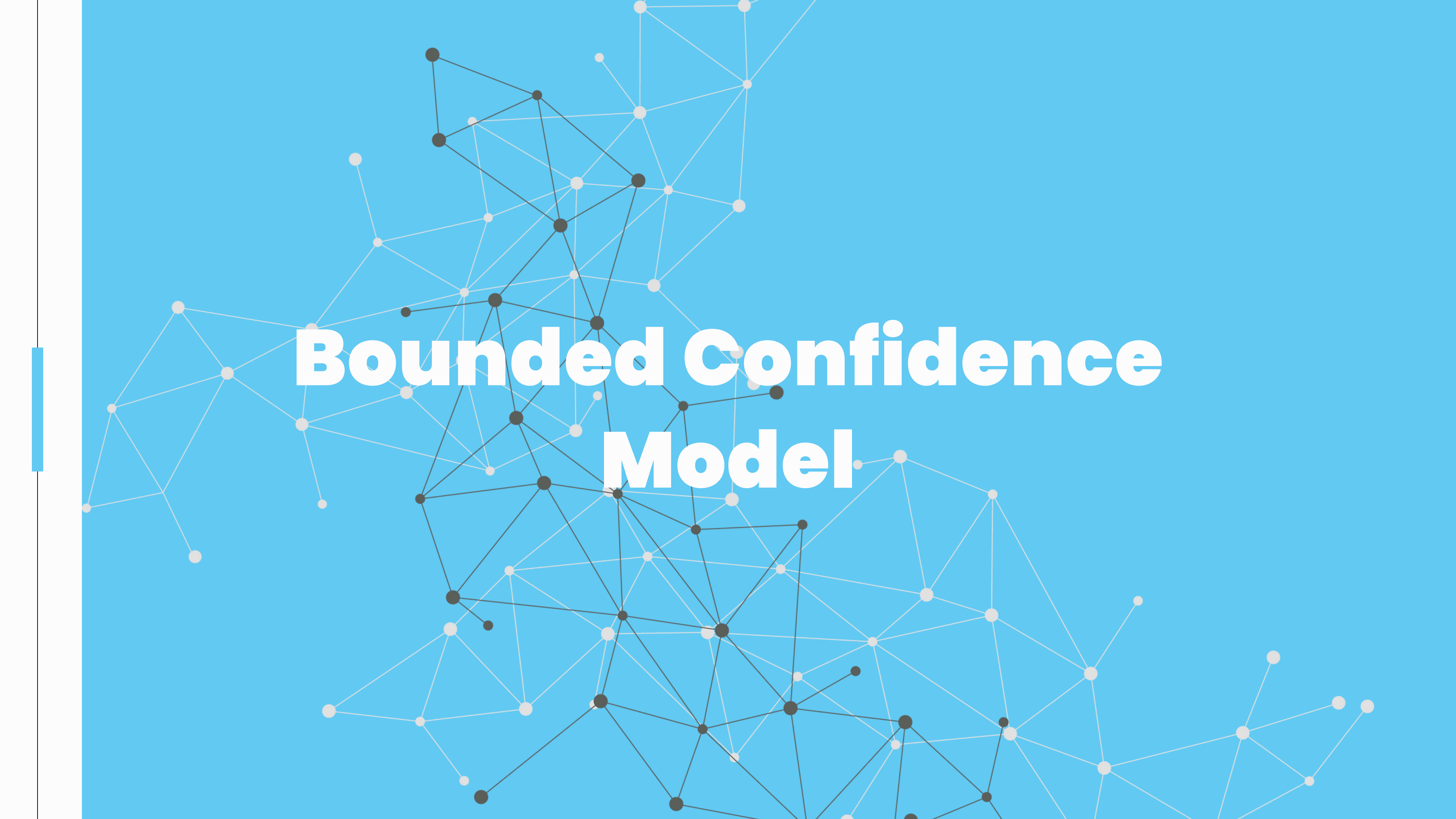
The phenomenology is similar to the binary opinion case

- there is no drift
- consensus is always reached in finite systems

An interesting quantity to study is the number of surviving opinions M_s over time

- the evolution of M_s describes how the system reaches consensus
- on a complete graph M_s decays slowly as a power law

$$M_s \sim M/t$$

A network graph with nodes and edges, overlaid on a blue background. The nodes are represented by small circles, some of which are black and some are light gray. The edges are thin lines connecting the nodes. The graph is dense and interconnected, with a central cluster of nodes and several smaller clusters branching out. The text "Bounded Confidence Model" is centered over the graph in a large, white, sans-serif font.

Bounded Confidence Model

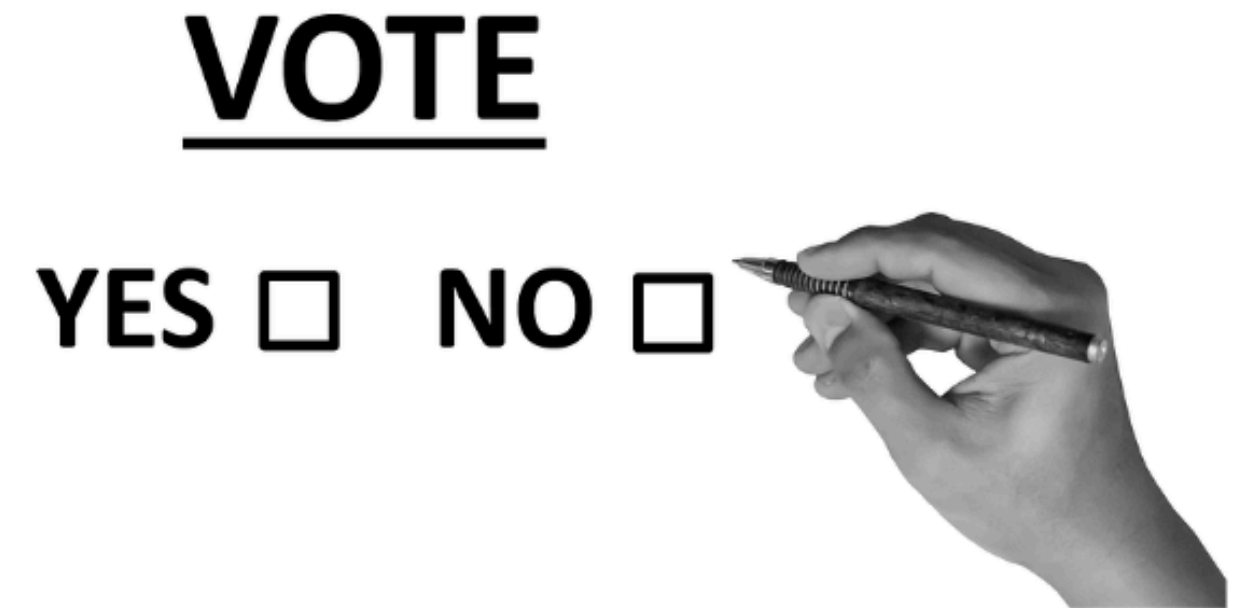
Discrete vs Continuous Opinions

Life is not black and white, there are many possible shades:

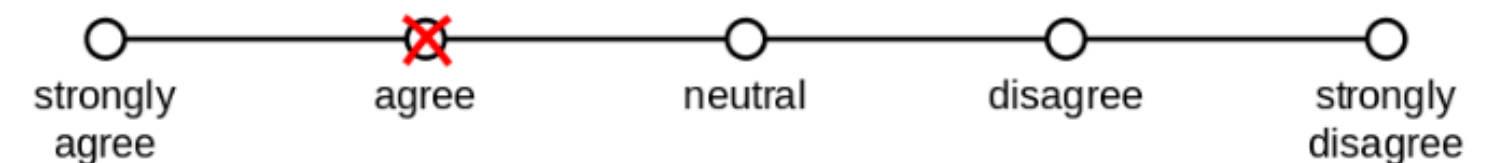
- discrete opinions are not enough to describe the full spectrum of human opinions
- for instance political parties are discrete, but political ideology is not

In order to overcome this limitation we introduce continuous opinions

- there are two extremes +1 and -1
- all values between +1 and -1 are possible
- the value 0 corresponds to a centrist perspective



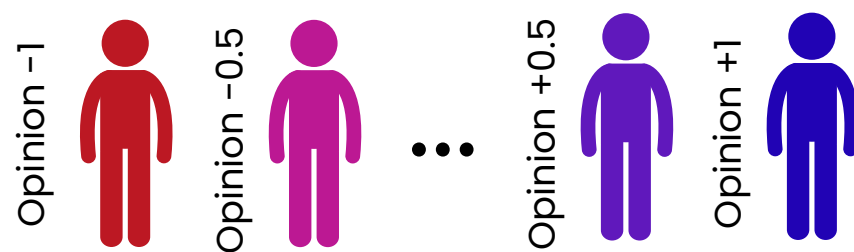
1. The website has a user friendly interface.



Bounded Confidence Model

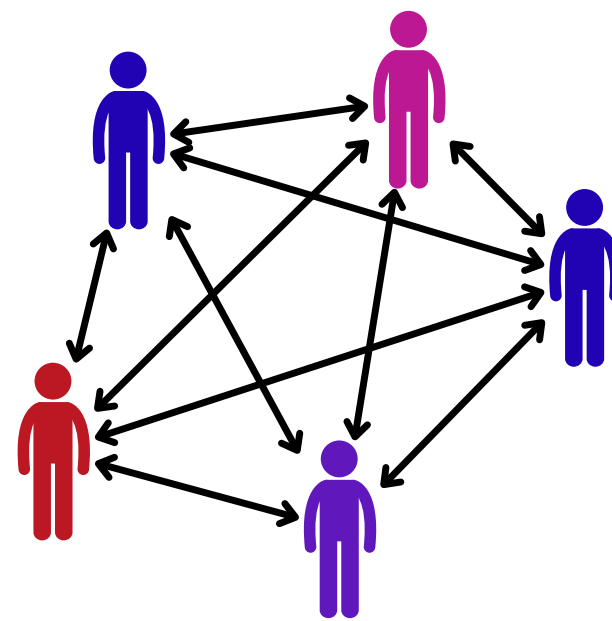
Agent

Each agent i is described by its opinion x_i , which ranges from -1 to 1



Space

Agents interact on a network



Dynamics

At each time step two agents i, j are randomly selected. If $|x_i - x_j| < \epsilon$ agents interact and their opinions get closer

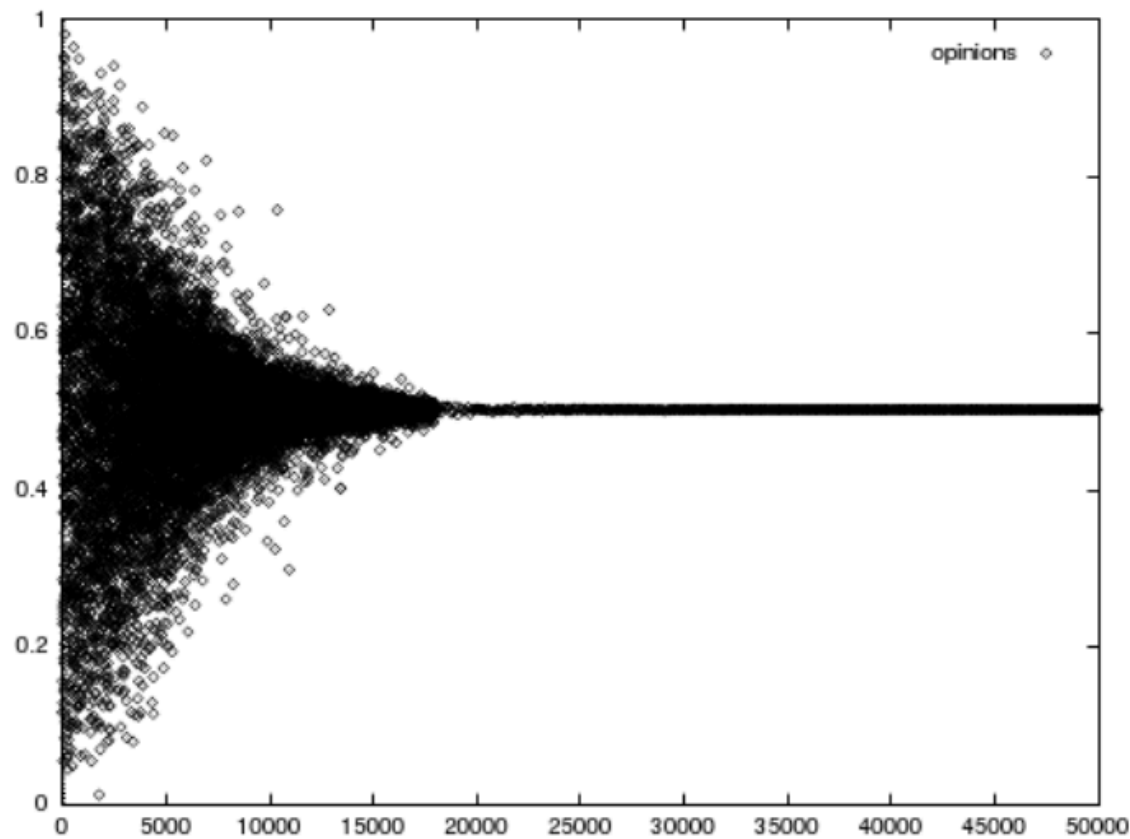
$$x_i(t+1) = x_i(t) + \zeta [x_j(t) - x_i(t)]$$

$$x_j(t+1) = x_j(t) + \zeta [x_i(t) - x_j(t)]$$

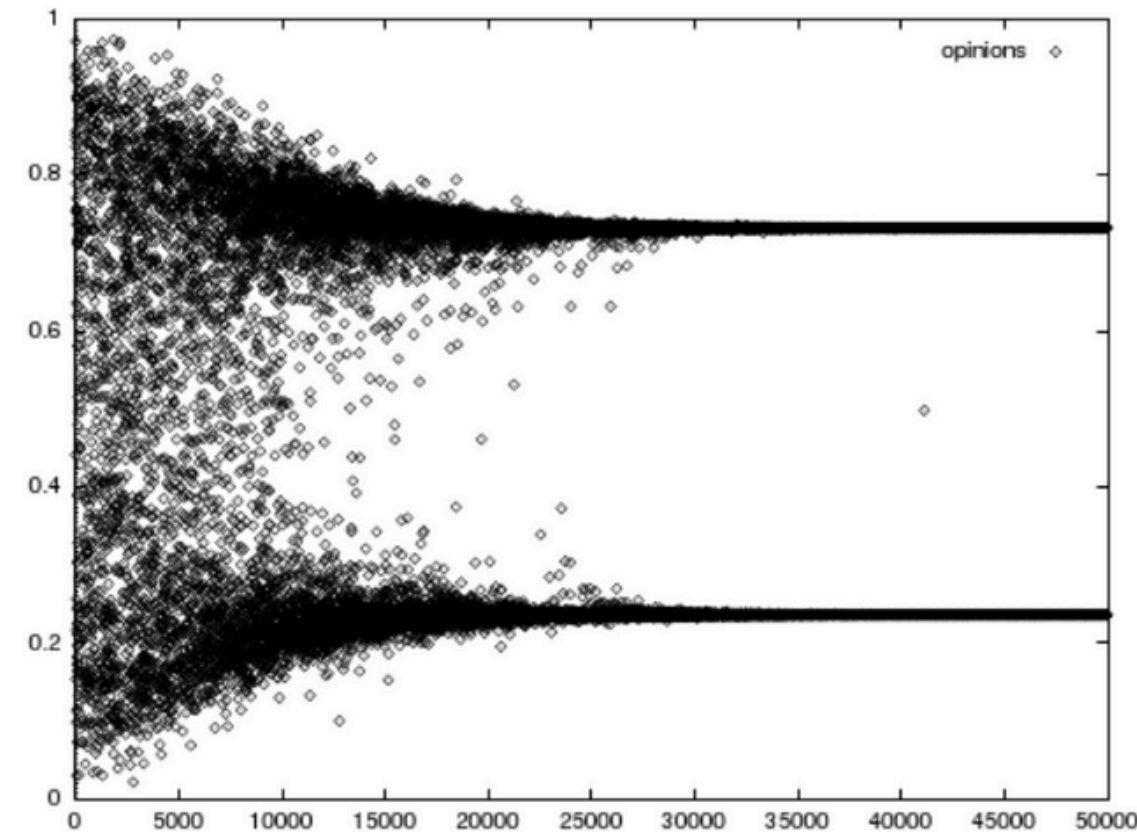
ϵ is the threshold for interaction (opinion difference tolerance)
 ζ sets the convergence time

Simulation Examples

Starting from an initial uniform distribution of opinions, we observe agents to get closer and closer in opinion. Asymptotically, all agents have a given (or few) opinion value. However also a fragmented configuration can emerge.

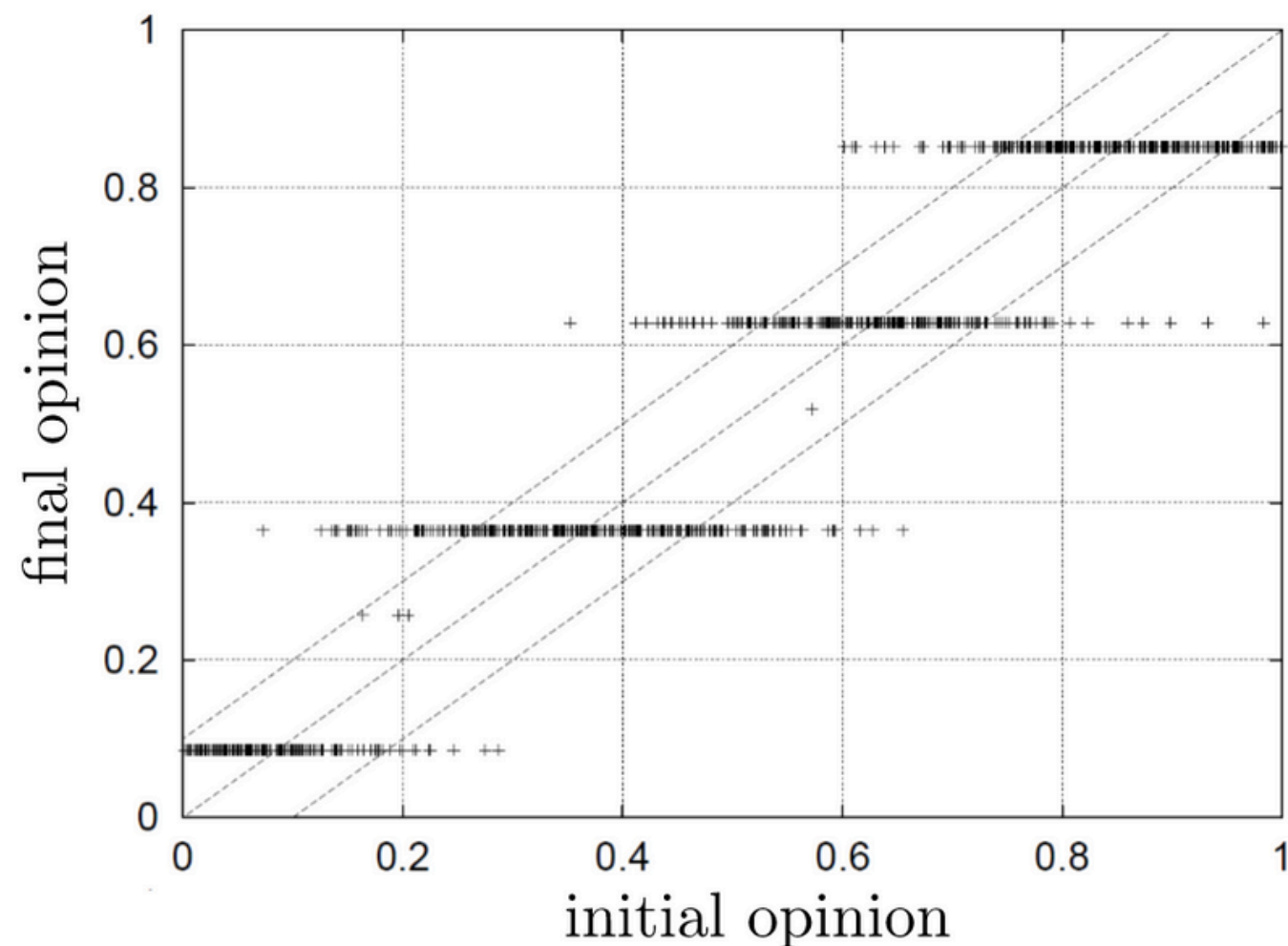


$$\epsilon = 0.5, \zeta = 0.5, N = 2000$$



$$\epsilon = 0.2, \zeta = 0.5, N = 1000$$

Final Opinion Distribution



Deffuant, Guillaume, et al. "Mixing beliefs among interacting agents." *Advances in Complex Systems* 3.01n04 (2000): 87-98.

We want to understand how a given initial opinion distribution evolve over time

- a relevant parameter is the number of peaks in the final distribution of opinions
- qualitative dynamics mostly depend on the threshold ε
- the number of peaks is $1/(2\varepsilon)$
- ζ and N only influence convergence time and width of the distribution of final opinions

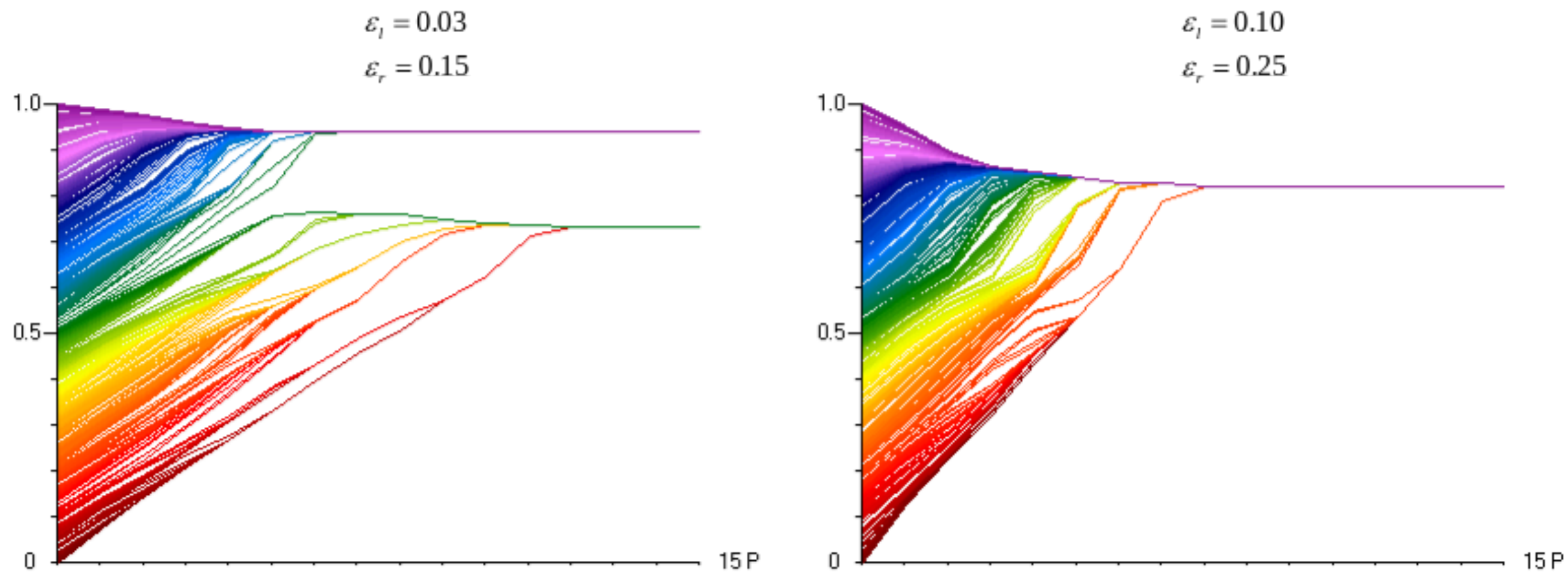
For small threshold (tolerance) we then expect to observe a very fragmented configuration.

Asymmetric Bounded Confidence

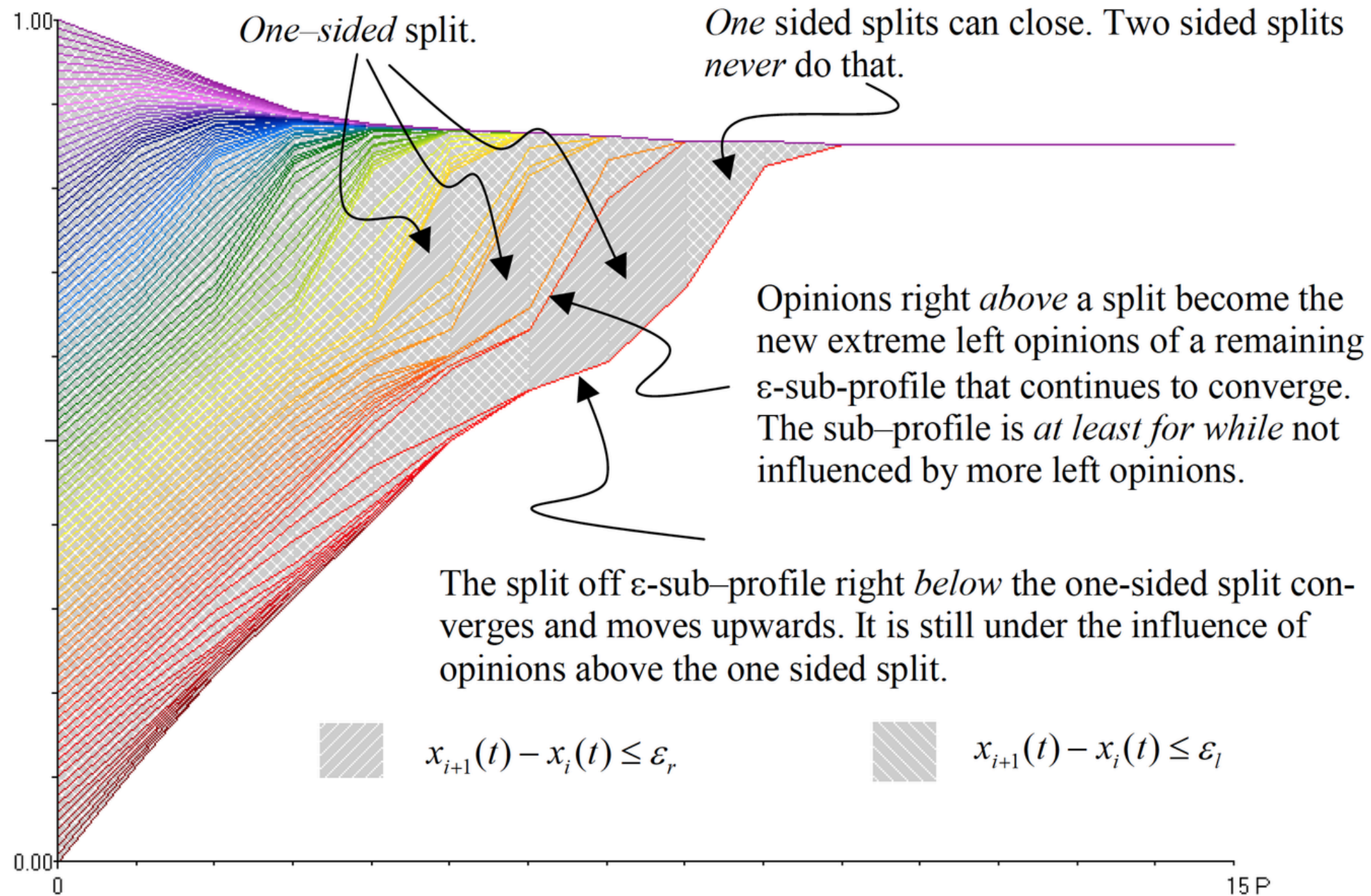
A simple modification of the Bounded Confidence model consists in using different threshold for the left and right opinion. Now interactions take place if


$$-\varepsilon_l < X_i - X_j < \varepsilon_r$$

This makes the collective opinion drift in the direction favored by the asymmetry.



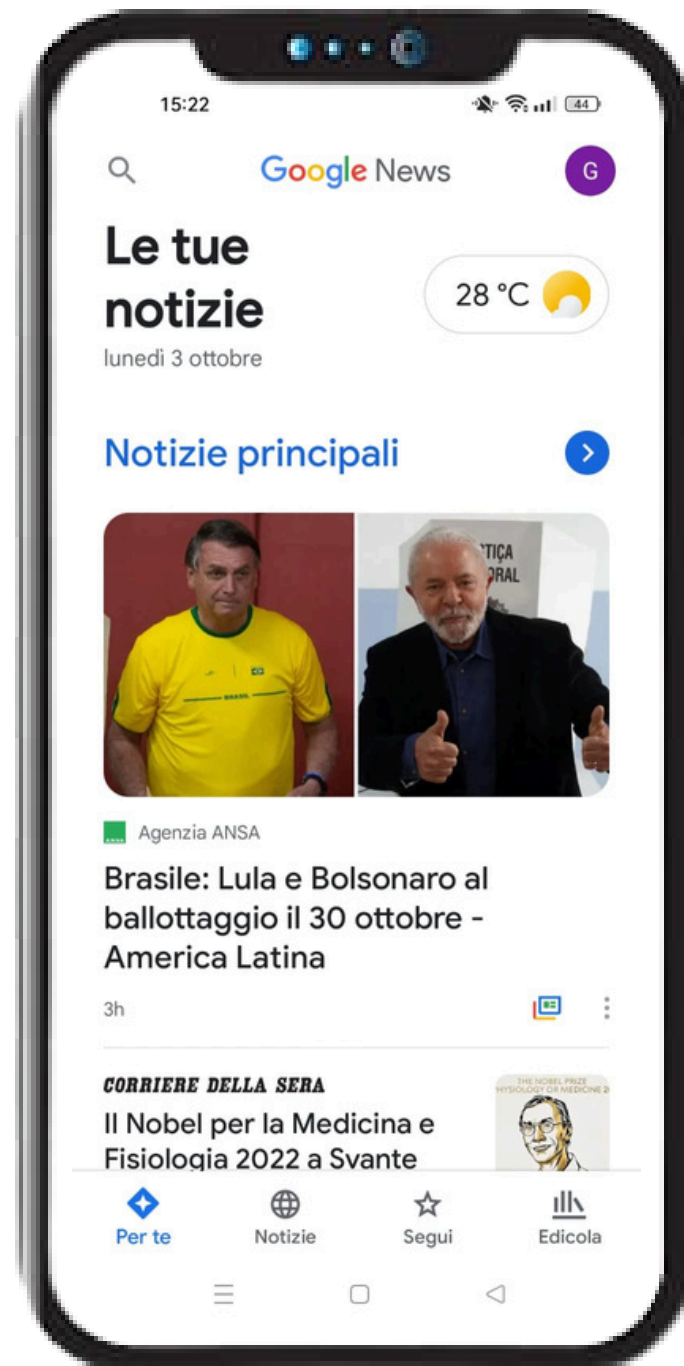
One-Sided Splits



A network graph with nodes and edges, some nodes are highlighted in black, set against a blue background. The graph is composed of numerous nodes connected by thin lines, with a few nodes in the center and top-left being larger and black, while others are smaller and light grey.

Recommendation Algorithms and Opinion Dynamics

The New Information Age



Sources of information are central in Opinion Dynamics. We live in a digital society

- social networks
- streaming platforms
- e-commerce
- online information

Previously information was mainly diffused from mass media, now it mainly travels on online platforms.

Online platforms influence the information we have access to!

Recommendation Algorithms

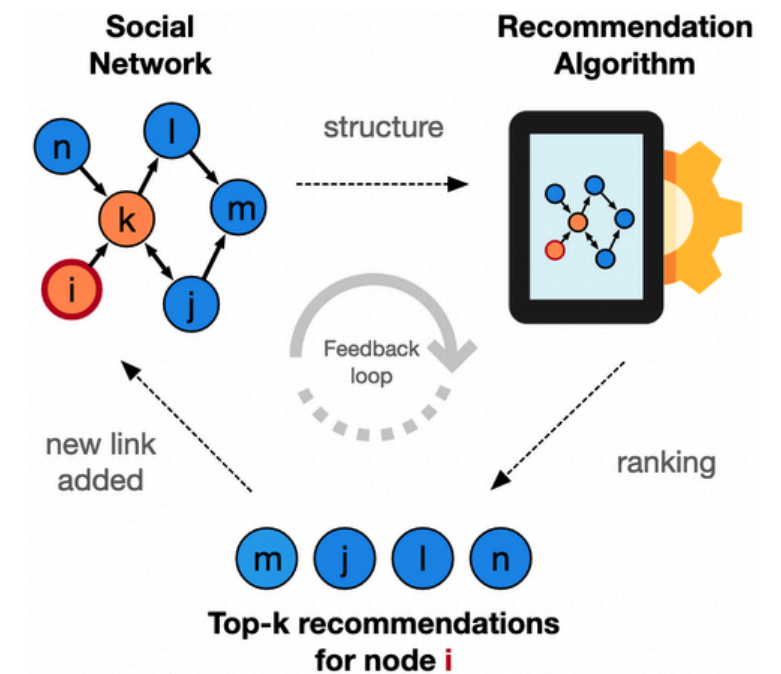
Most online platforms use recommendation algorithms. There are two main types:

- **link-recommendations** Recommend people/influencers we may be interested in connecting with/following
- **content-recommendations** Recommend content (posts, images, music, items) that is in line with our taste

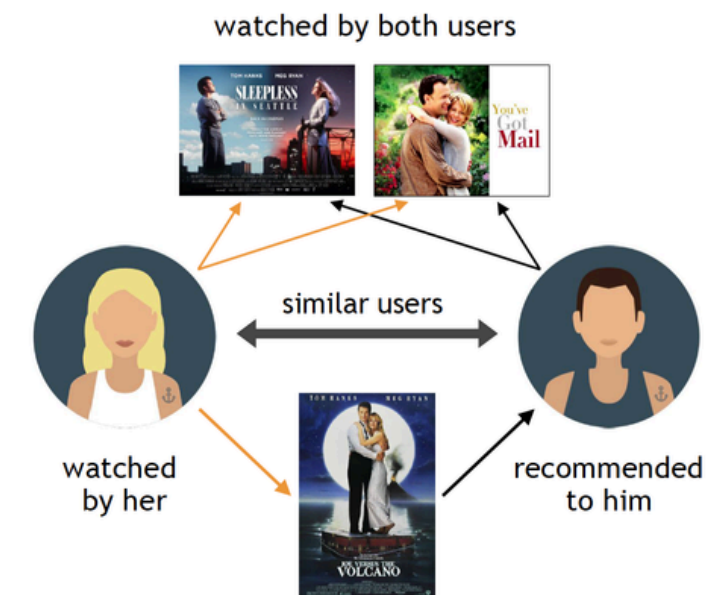
Recommendation algorithms tend to alter the information we have access to, producing biases

How is Opinion Dynamics influenced by recommendation algorithms?

Link-Recommendations



Content-Recommendations



Echo Chambers

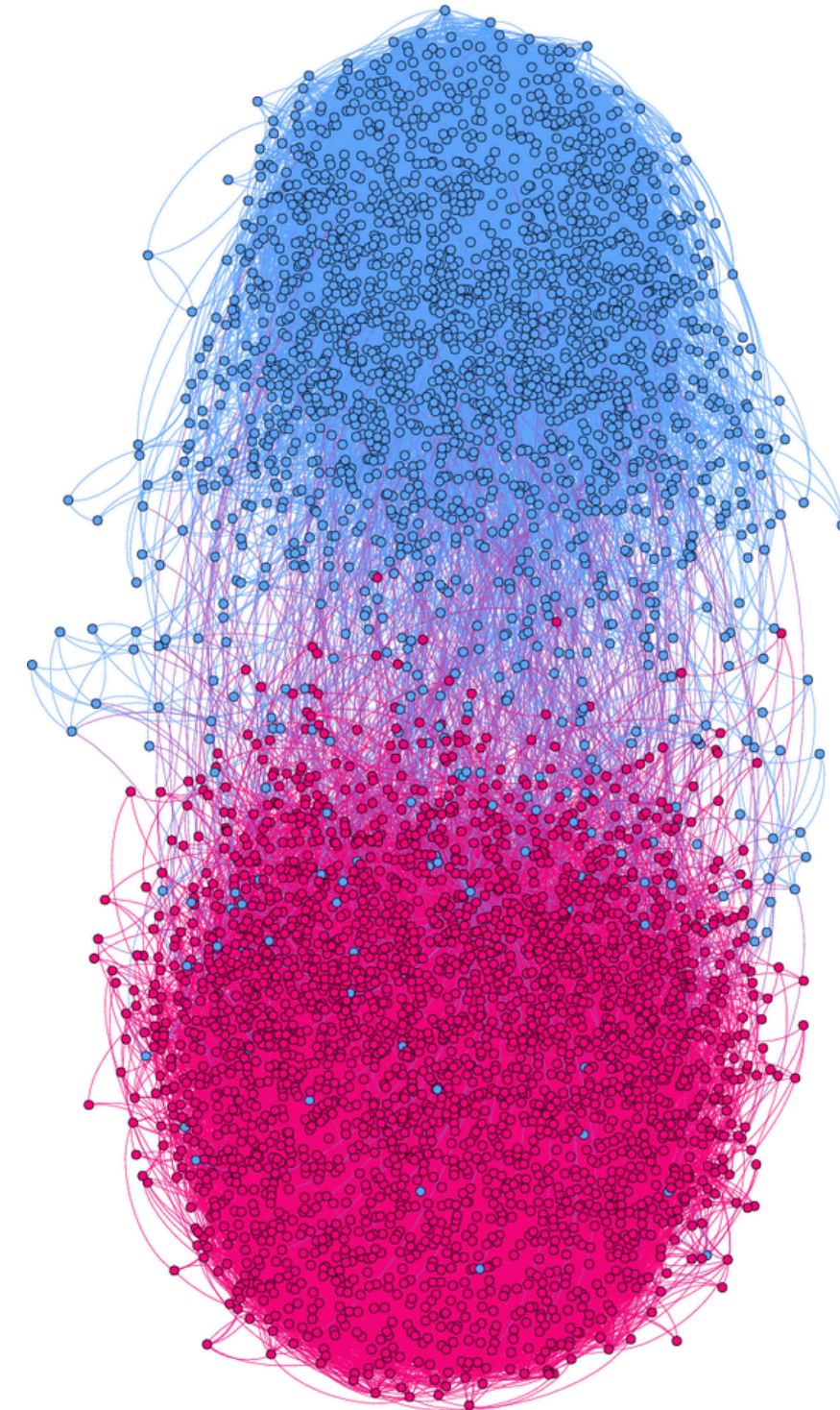
Echo Chamber Effect:

each individual is connected to other individuals sharing its same ideas and believes

Echo Chambers form spontaneously due to homophily, but are strongly favored by recommendation algorithms:

- link recommendations may suggest similar users
- content recommendations may suggest items shared by friends

When you are in an echo chamber, it looks like everybody around you has your same opinion.



Filter Bubbles



Filter Bubble Effect:

each individual is exposed to algorithmically personalized content that confirms its believe

Filter Bubbles form due to content-recommendations and didn't exist before the advent of internet:

- content recommendations suggest items similar to those previously liked by users
- this increases engagement, but limit content diversity

When you are in a filter bubble, your feed is dominated by just few topics that you like.

Modeling Personalized Information

We can modify the Voter Model to model the effect of content recommendation algorithms.

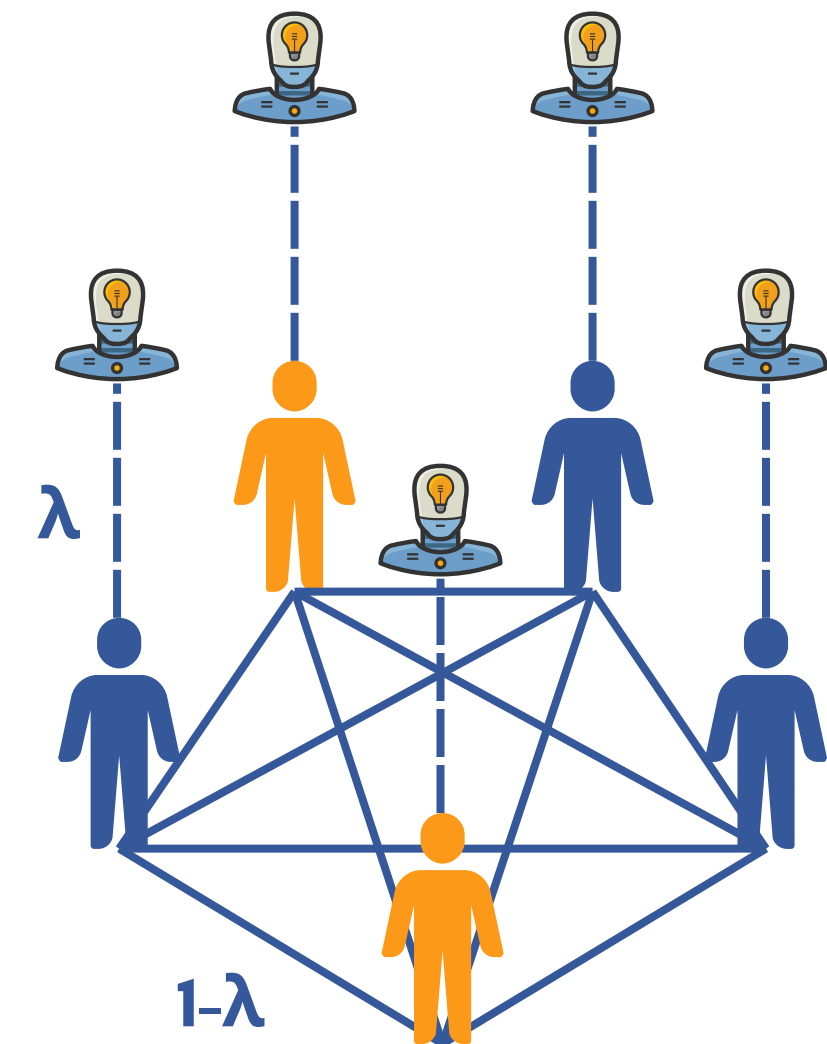
Each agent is exposed to a source of personalized information e_i :

- with prob. λ copies personalized information
- with prob. $1-\lambda$ copies random agent

Personalized information reinforces users' past choices

n_i = positive clicks – negative clicks

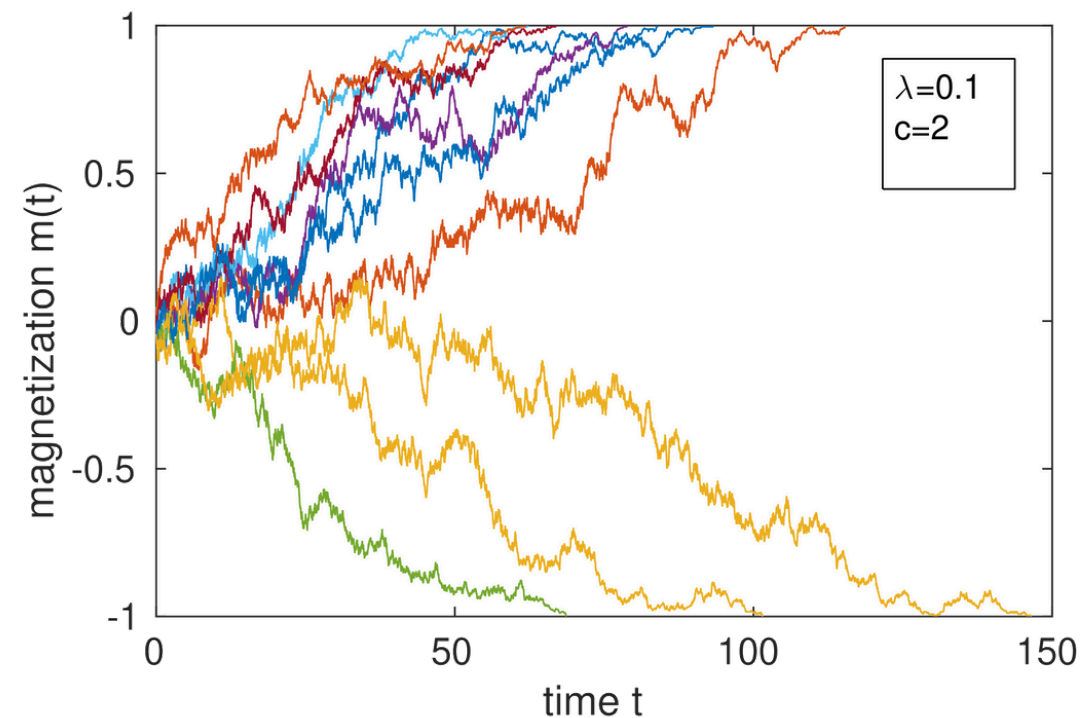
$$P[e_i(t) = 1] = P[n_i] = \frac{c^{n_i}}{1 + c^{n_i}}$$



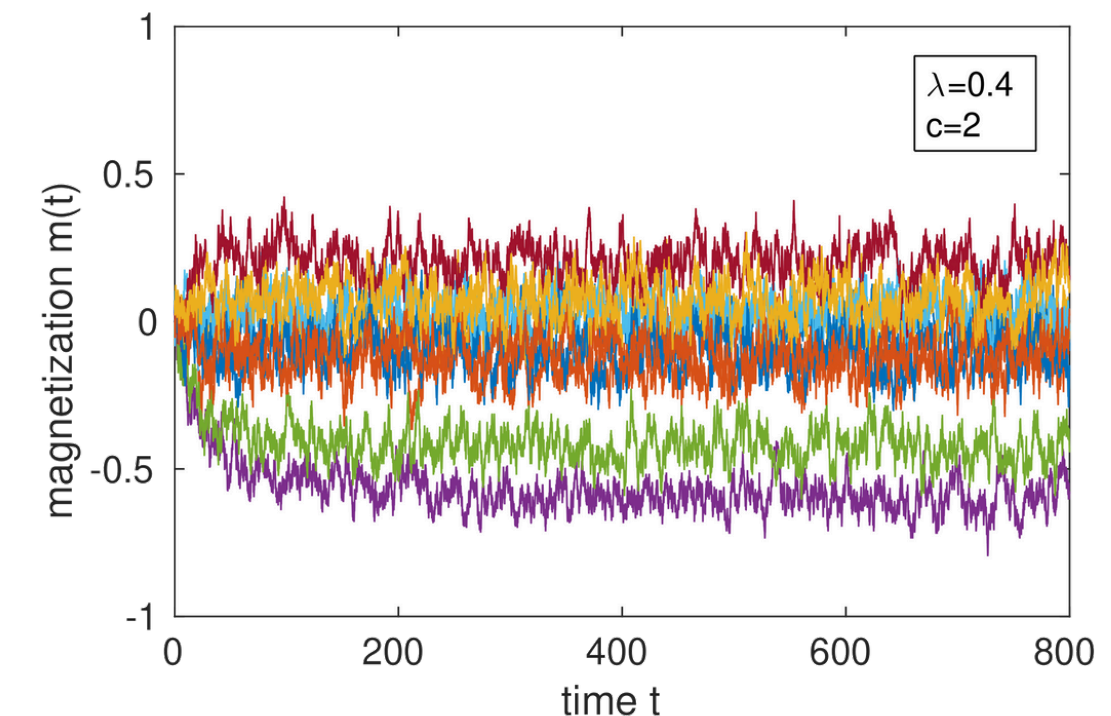
De Marzo, Giordano, Andrea Zaccaria, and Claudio Castellano. "Emergence of polarization in a voter model with personalized information." *Physical Review Research* 2.4 (2020): 043117.

Disordered States

The evolution of the system can be visualized using the magnetization parameter $m = (2N_+ - N) / N$



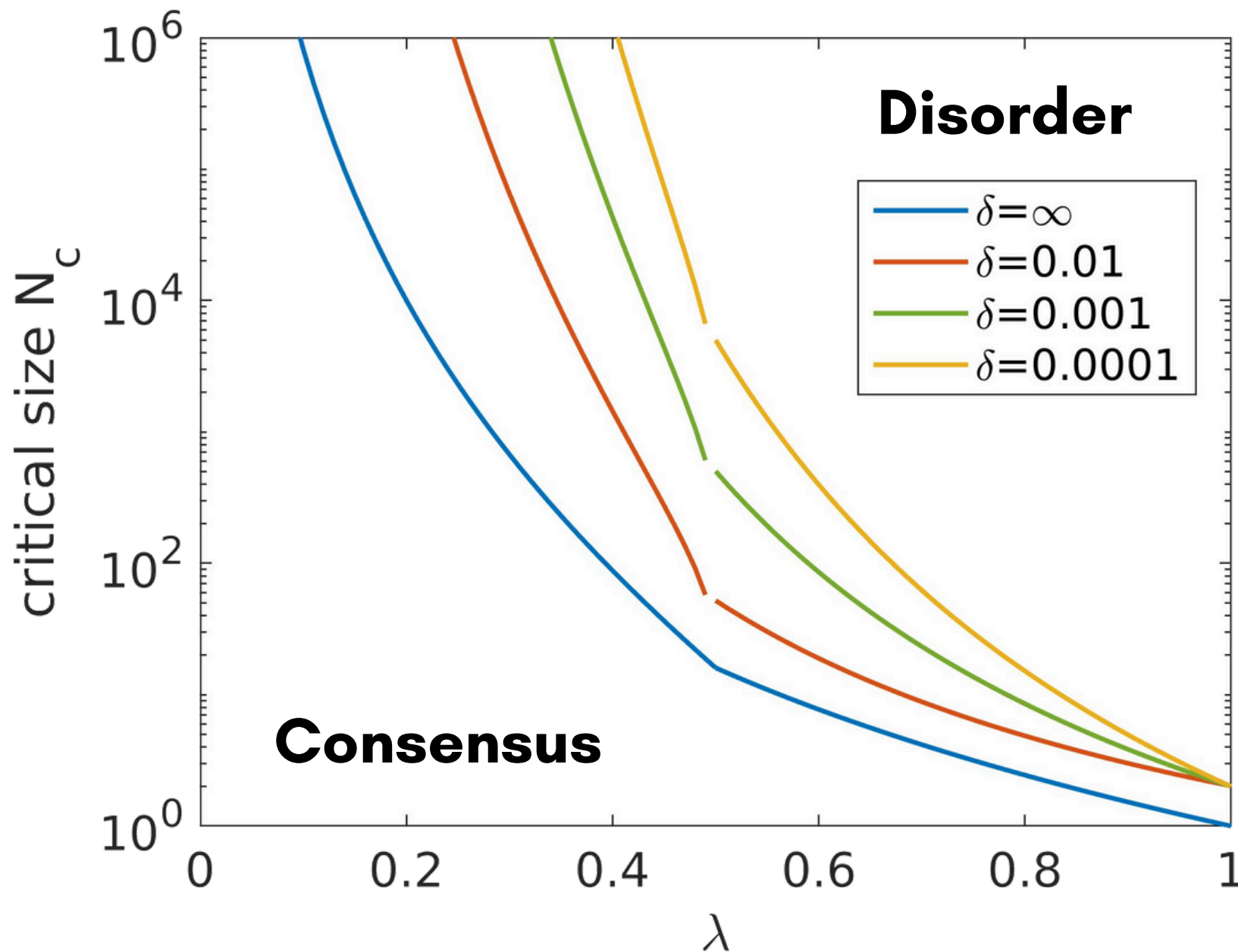
Stronger personalized
information \rightarrow



When the strength of the personalized information is increased, the system remains trapped in disordered states and consensus is never reached

Phase Diagram

The model parameters are the strength of personalized information λ , its adaptability c and the number of agents N



- 1 Two distinct phases
- 2 $\delta = c - 1$ does not play a major role
- 3 In large systems the critical λ goes to zero

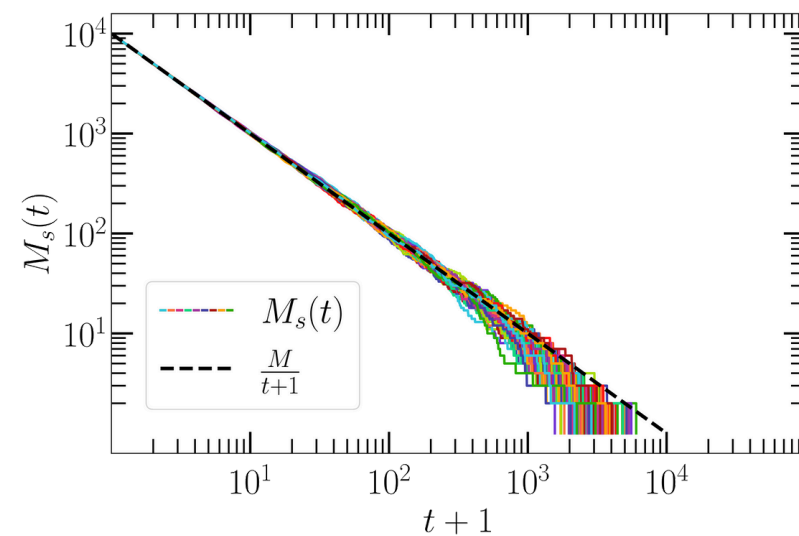
**Personalized information
has strong effects!**

Multi-Opinion Generalization

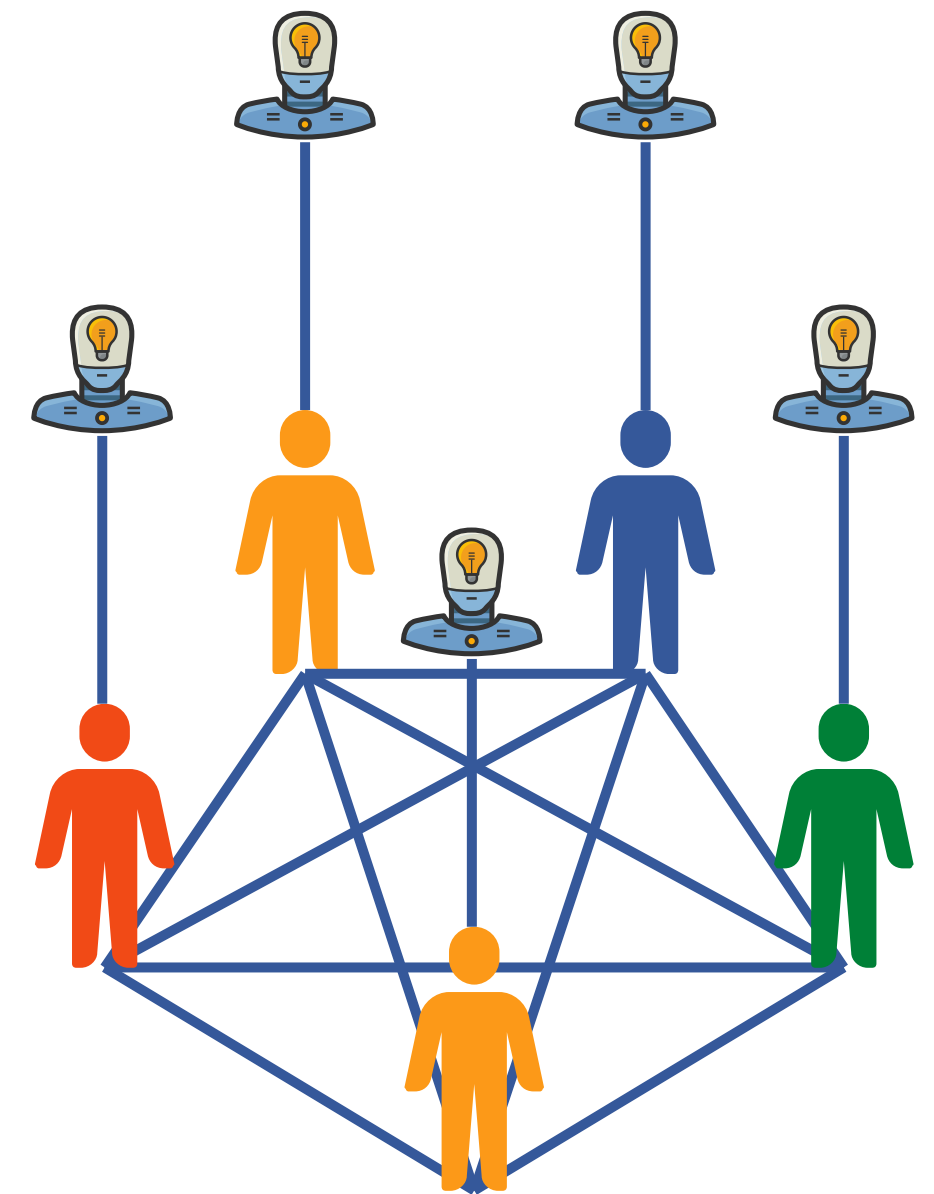
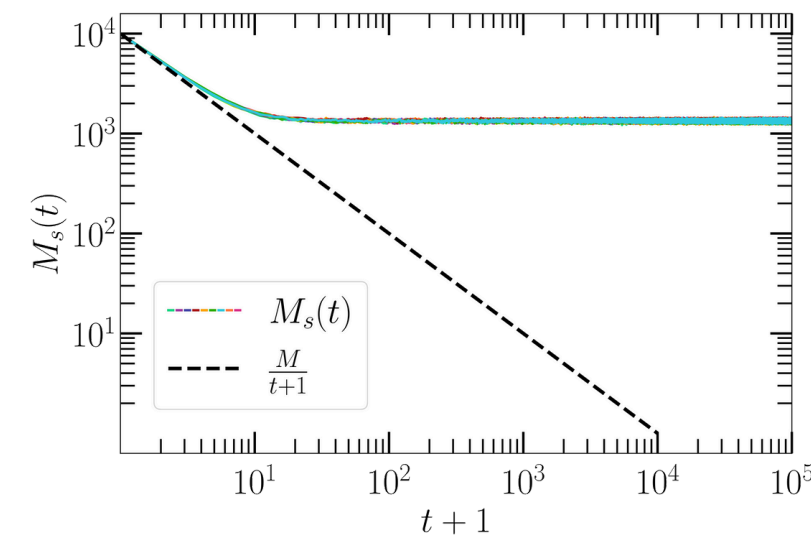
The order parameter is given by the **number of surviving opinions** $M_s(t)$:

- **low personalized information** $M_s(\infty)=1$
- **high personalized information** $M_s(\infty)>1$

A polarized state is stable if $|N_m - N_l| < Nm_c = N \frac{\lambda}{1 - \lambda}$



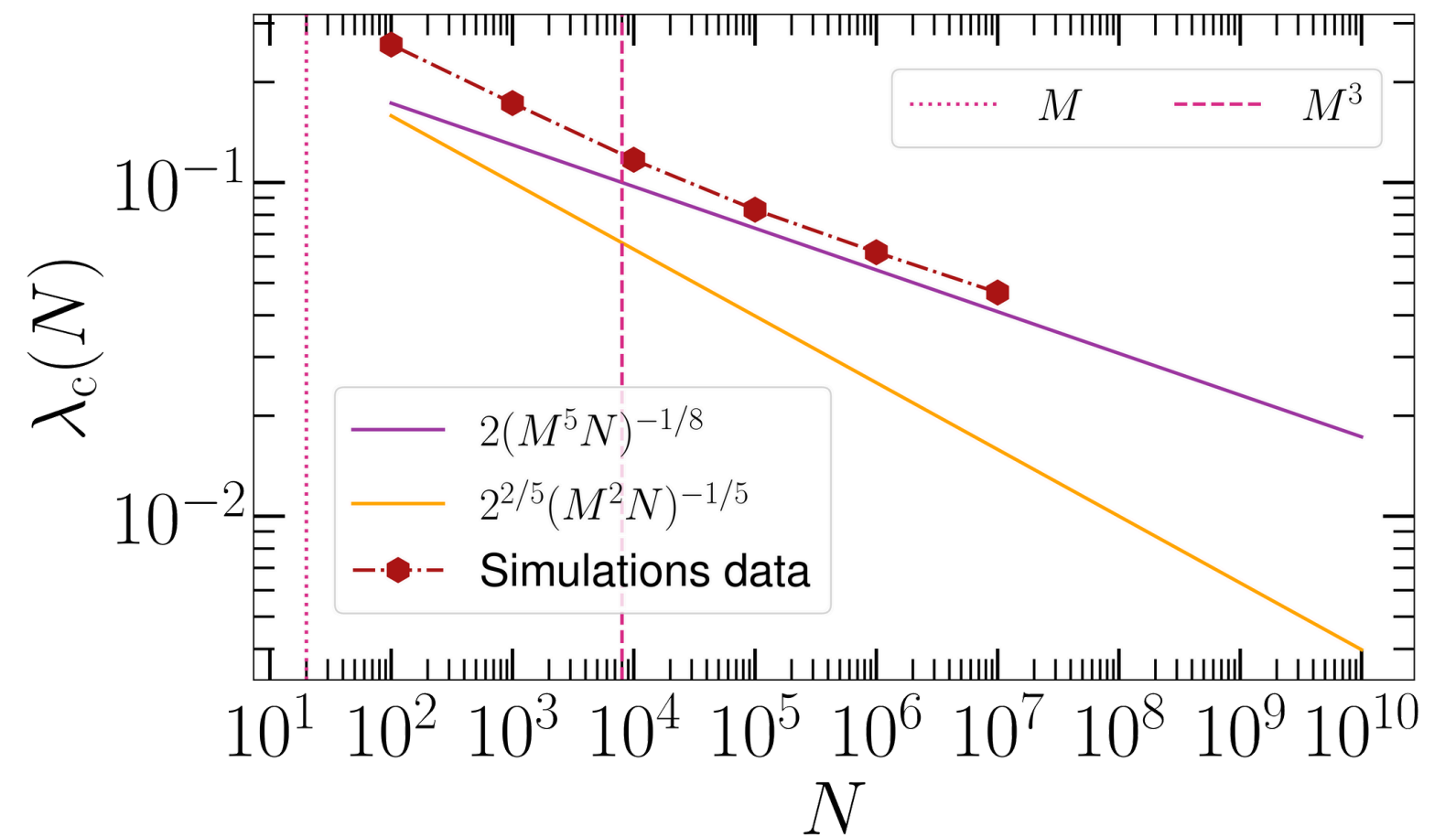
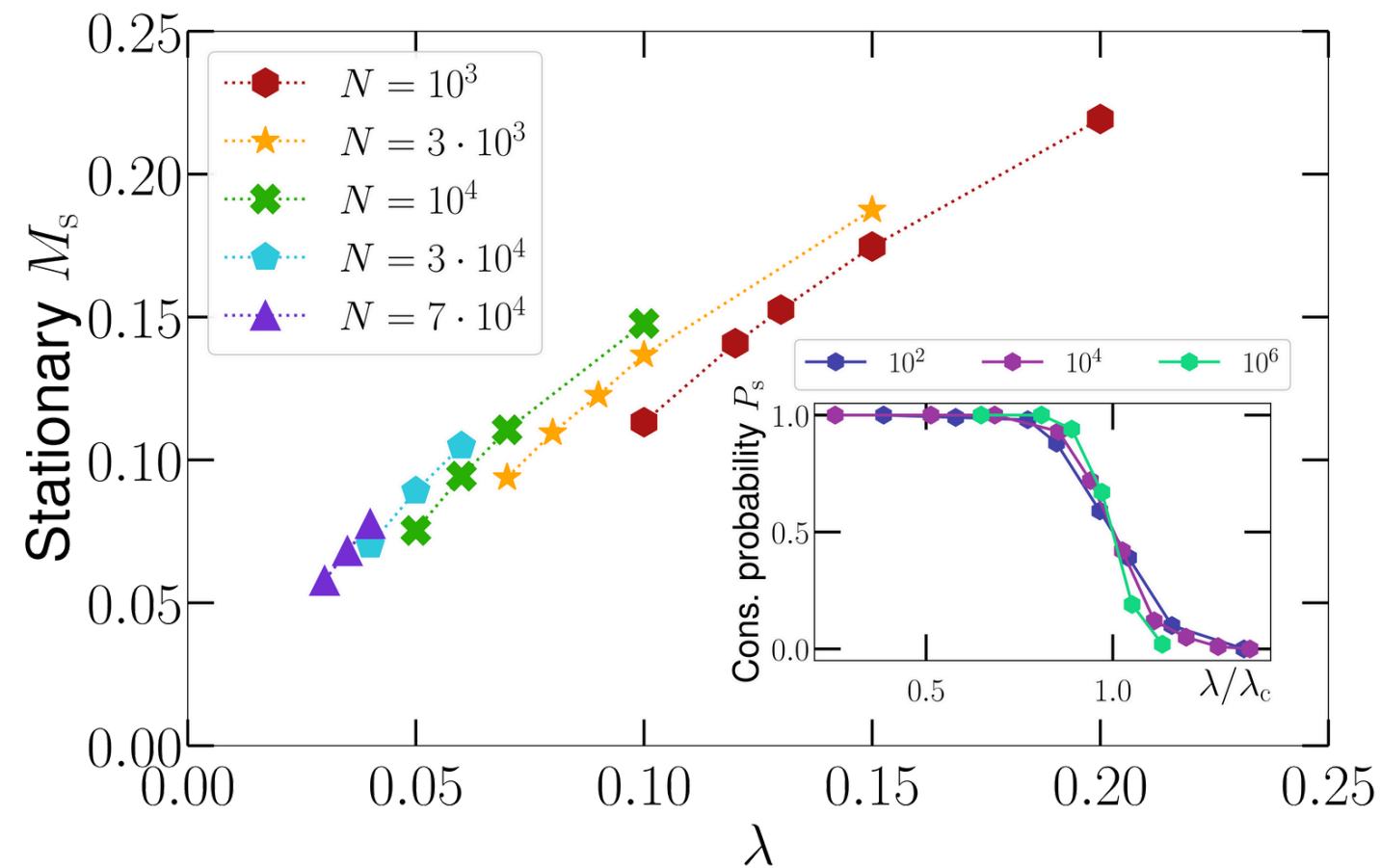
Stronger personalized information
→



Phase Transition

The system shows a continuous phase transition in λ

$$\begin{cases} t^* = \frac{2}{\lambda_c} \left\{ \sqrt{\int_0^{t^*} \left[\frac{1-\lambda_c}{M_s(t')} + \lambda_c \right] dt'} + \sqrt{\int_0^{t^*} \frac{1-\lambda_c}{M_s(t')} dt'} \right\} \\ \lambda_c = \frac{\gamma \frac{N^{1/2}}{M} (t^*)^{3/2}}{N + \gamma \frac{N^{1/2}}{M} (t^*)^{3/2}} \end{cases}$$



Take Home Messages

Recommendation Algorithms

Online platforms use recommendation algorithms to filter the content we see and maximizing our engagement.

Echo Chambers

Link-recommendation algorithms favor the formation of communities of like-minded people, called Echo Chambers.

Filter Bubbles

Content-recommendation algorithms limits the content we see, only showing us items that are close to our ideas and believes. This generates Filter Bubbles.

Opinion Dynamics Models

Opinion dynamics models can be used to understand how recommendation algorithms may affect opinion dynamics and foster polarization and fragmentation.

Conclusions

Opinion Dynamics

Opinion Dynamics studies how opinions form and get shared in groups of people or agents, leading to a global consensus or to fragmented opinions.

Voter Model

Opinion Dynamics model with binary opinion and a copy mechanism. Consensus is reached only if the dimension of the lattice is small enough or in finite systems.

Bounded Confidence

Opinion Dynamics model with continuous opinions. Agents interact only if their opinions are not too different.

Recommendation Algorithms

Opinion dynamics can let us understand the possible effects of recommendation algorithms without modifying the online platforms' functioning.

Quiz

- Which examples of opinions that are very easy to change?
- What about opinions we almost never change?
- What is more realistic, the Voter Model or the Bounded Confidence Model?
- What aspects are being neglected in the models we considered?
- What do you think about recommendation algorithms? Do you feel trapped in a filter bubble?
- What type of recommendation algorithms does Netflix use?
- What type of recommendation algorithms does Instagram use?

Bonus: LLM Powered Agents

In standard opinion dynamics models, the dynamics is hardly coded by humans

- in Glauber we follow the majority
- in Voter we select a random neighbor

What if we could use agents that decide by their own what to do?

We could try using LLMs as agents in simulations

- we explain them the context, but we don't tell them how they should behave
- agents decide autonomously how to update their opinions

The idea is that an LLM, trained on human data, can capture human behavior without the need of fine tuning many parameters of the model.

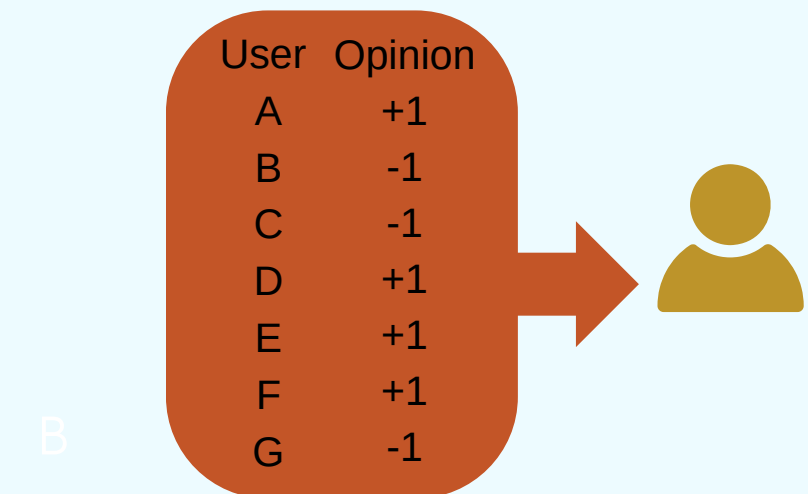
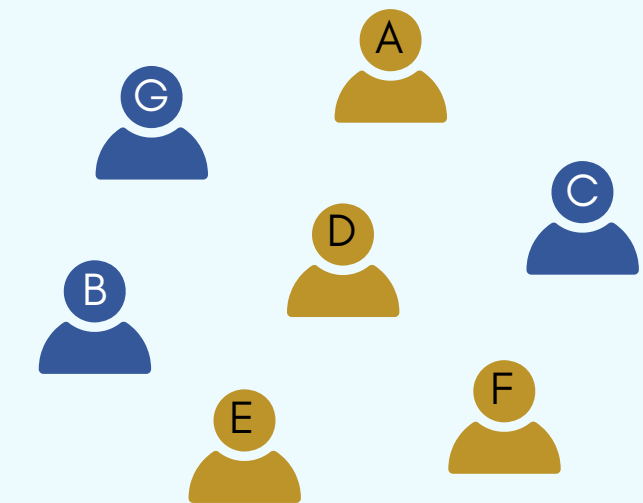
Bonus: Opinion Dynamics with LLMs

The setup is similar to the Voter Model or the Glauber Dynamics:

- at each time step we select a random agent
- we show it the full list of agents in the system with their names and the current opinions they have
- we ask the selected agent to reply with the opinion it wants to support

- You recently subscribed to a social network.
- Below you can see the list of all your friends together with the opinion they support.
- You must reply with the opinion you want to support.

```
A +1  
B -1  
C -1  
⋮
```

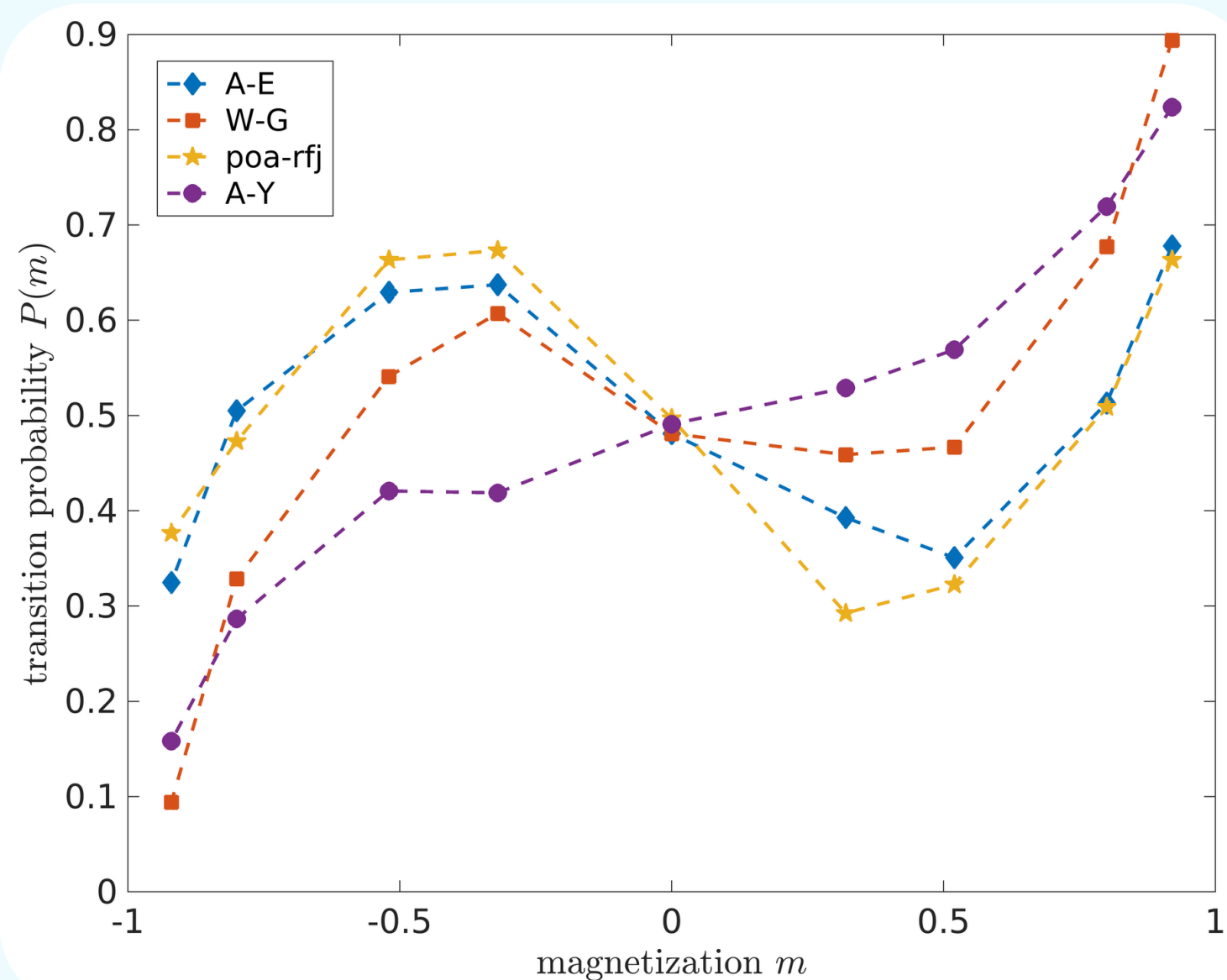


GPT3.5 Agents

First we consider agents powered by GPT3.5-turbo

- we reconstruct the transition probability of a single agent as function of the magnetization
- different opinion names give different curves
- there can be either a mild tendency to follow majority or even a tendency to go against the majority (for small m)

From the shape of the transition probability we understand that GPT3.5 agents are not smart enough to coordinate and reach consensus.

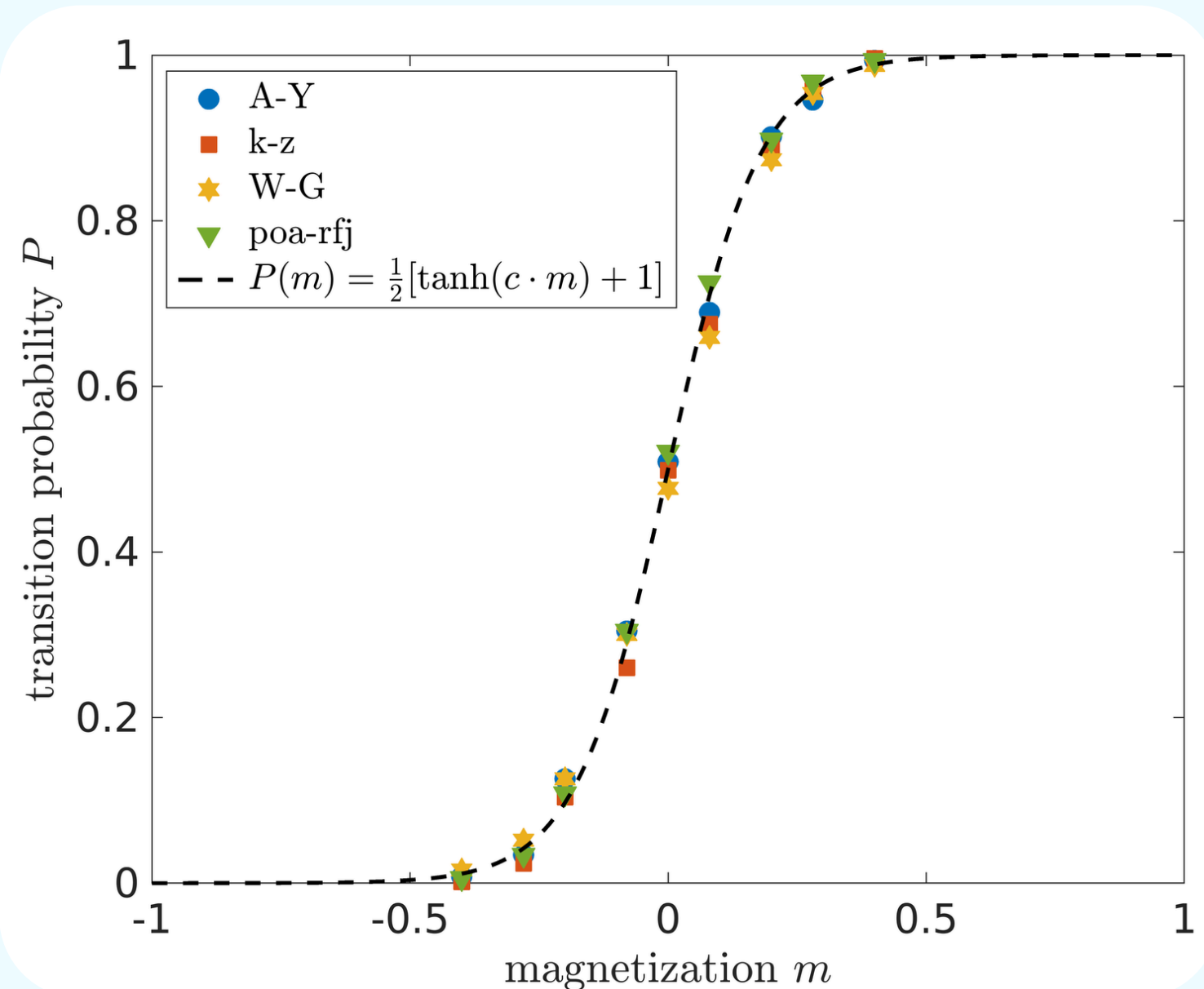


GPT4 Agents

Then we consider a more advanced LLM, namely GPT4-turbo

- now different opinion names give approximately the same curve
- the transition probability resembles the Glauber one with a small temperature
- GPT4 has a much stronger tendency to follow the majority

From the shape of the transition probability and our analysis of Glauber dynamics we expect these GPT4 agents to easily coordinate and reach consensus.



Size and Temperature

The presence of a temperature derives from cognitive limits of the AI:

- if we increase the system size (longer lists in prompts) we observe an increase of the temperature
- GPT4 struggles in identifying the majority when there are many users in the system/prompt
- this induces some randomness in GPT4 agents
- more powerful models (Claude 3) are characterized by a lower temperature

