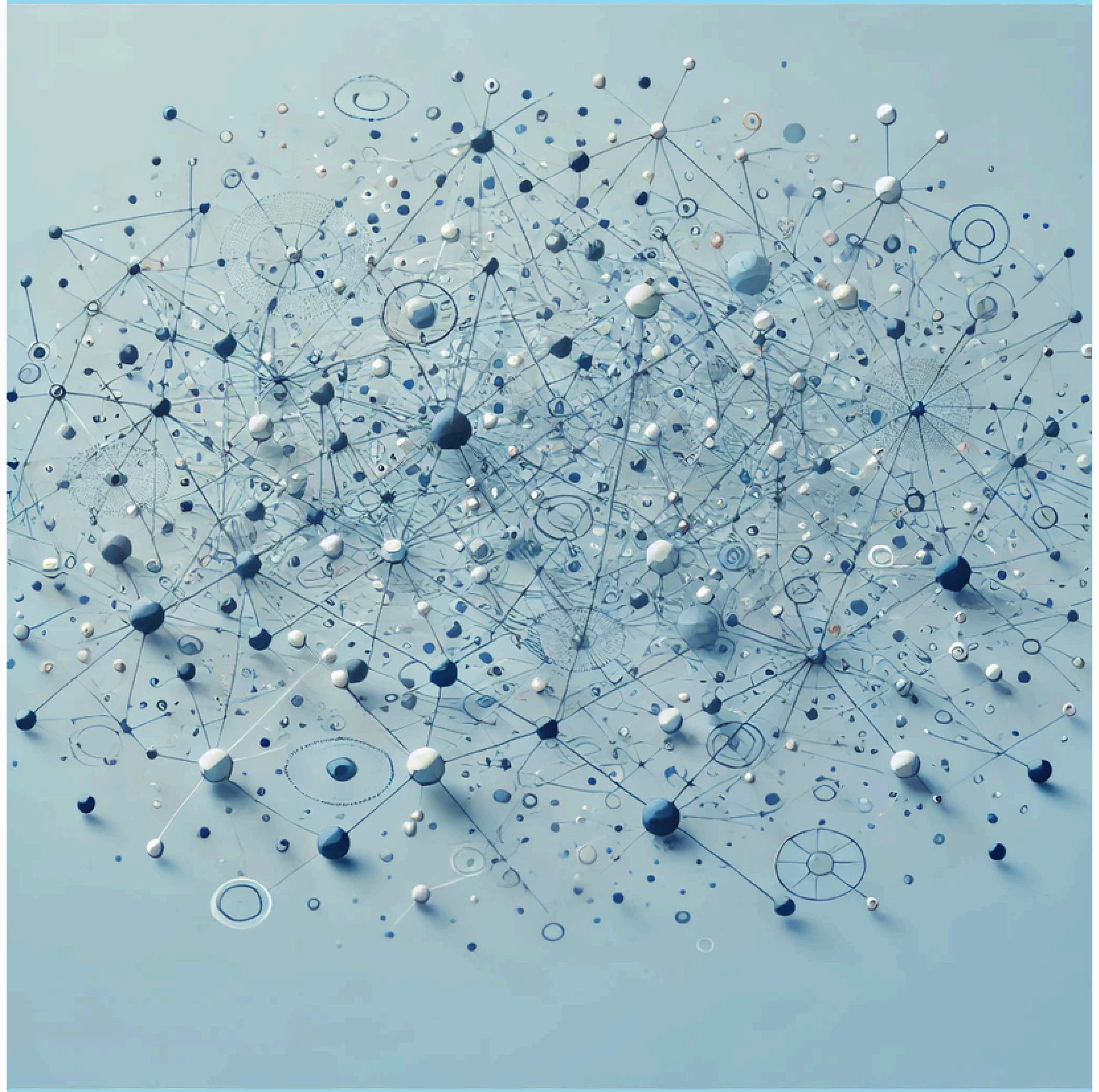
Please Evaluate the Course!

Your feedback is super important for me!

- If you have suggestions for topics to include or topics you found too boring or not very relevant, please share them
- Also if you have suggestions on how to improve the coding sessions I'll be happy to read them

Since I'm still new, it's important for me to receive the feedback of at least 6 students!



Universität Konstanz

UNIVERSITÄT KONSTANZ

Multilayer Networks

Network Science of Socio-Economic Systems

Giordano De Marzo

Calendar Change

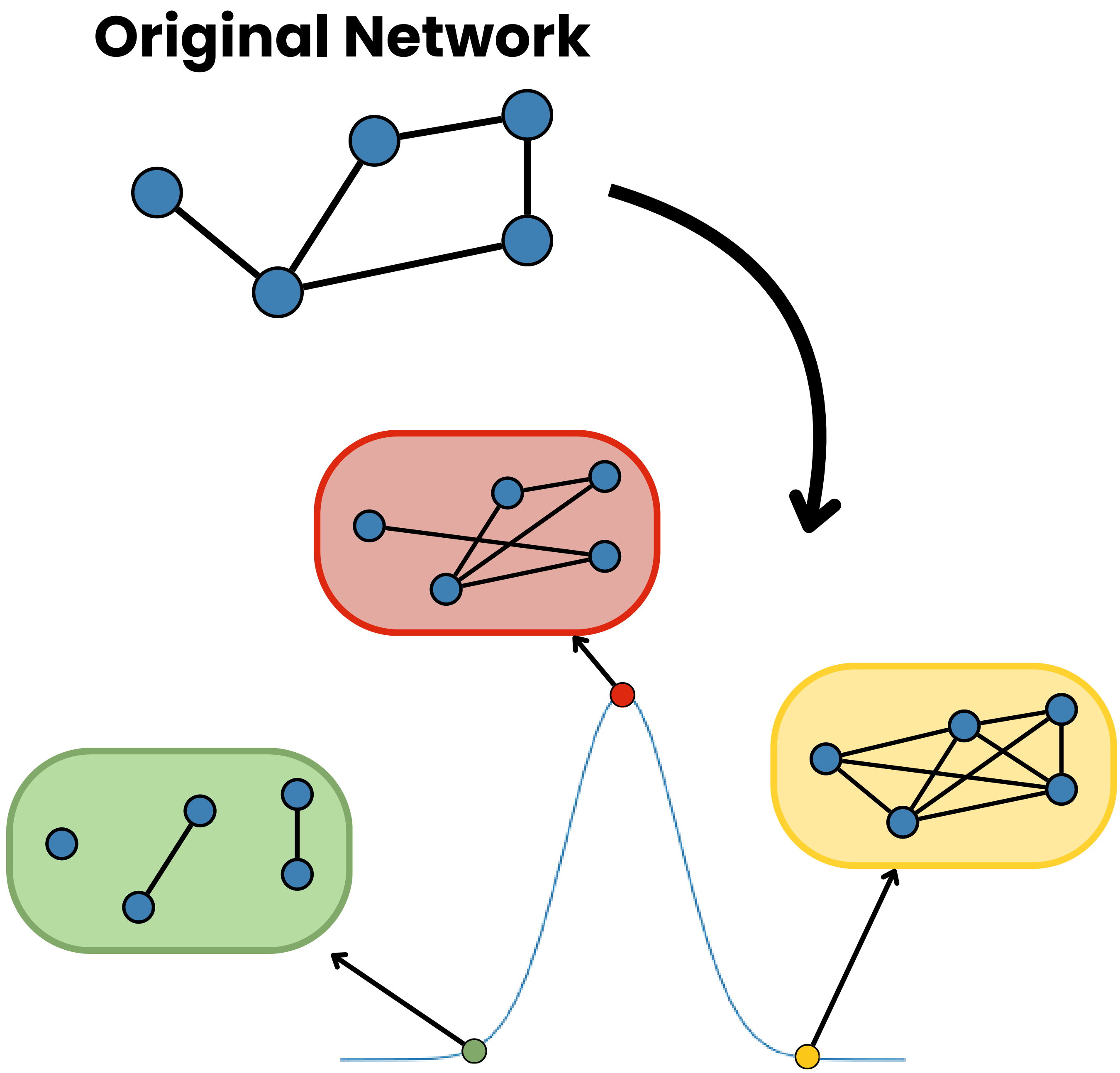
There will be no lecture tomorrow!

- I managed to move a trip so I don’t need to use tomorrow coding slot for a theory lecture
- I think this is better since some people have problems on Tuesday

This is the updated calendar (also on my website)

- December 17, 2024 - No lecture
- January 13, 2025-Economic and Financial Networks 1
- **January 20, 2025 - Economic and Financial Networks 2**
- January 27, 2025-Social Networks
- February 03, 2025-Advanced Topics in Network Science
- February 04, 2025-Students Presentations

There is no change in the coding session schedule



Recap

Null Models for Networks

Null models are crucial to validate the properties of networks

Network Ensembles

An ensemble is the set of all possible graphs with a given macroscopic property.

Applications of Null Models

Null model can be used for validating properties and to reconstruct networks

Bipartite Networks Projection

Null models allow to obtain statistically validated projection of bipartite networks

Outline

1. Basic Definitions
2. Multiplex Networks
3. Epidemic Spreading
4. Info About the Exam

Basic Definitions

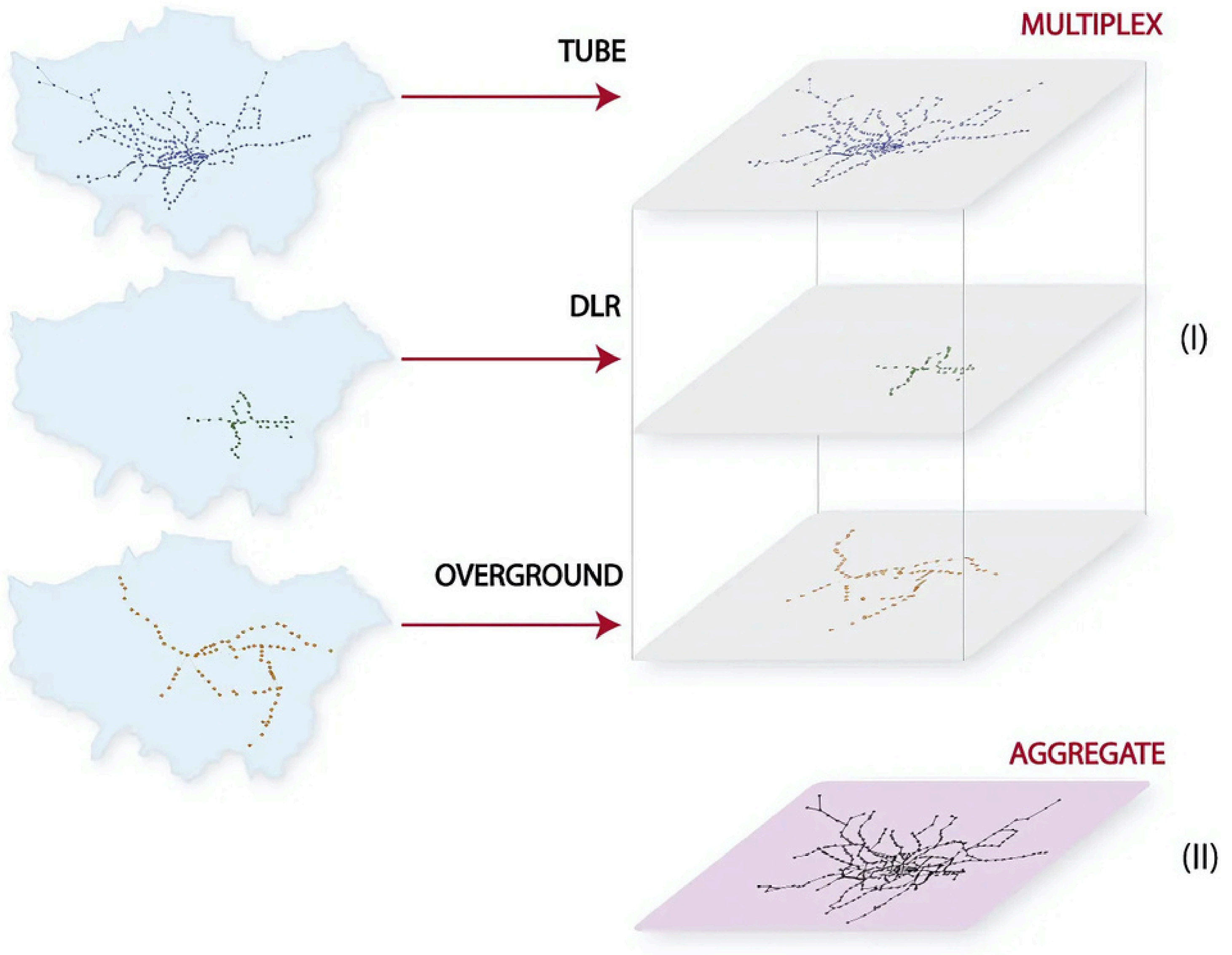
Multilayer Networks

Many phenomena take place on structures more complex than simple networks

- moving around a city, we can use different transportation systems
- social networks are characterized by links of different types

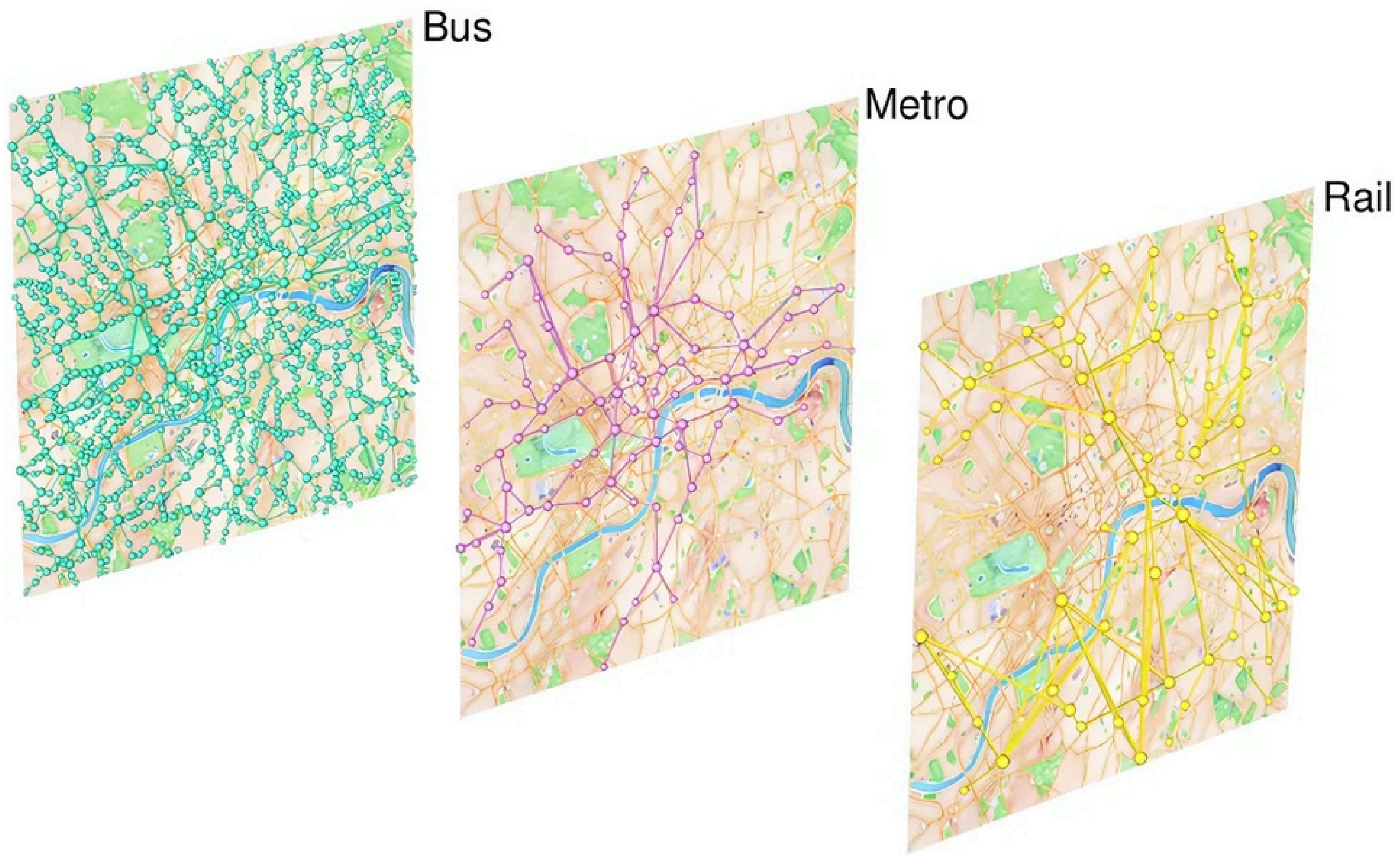
These type of phenomena can not be captured just by adding weights

- links are intrinsically different
- they are better described by multilayer networks



A multilayer network is composed of several layers, each containing a networks

Public Transport



We already introduced the examples of transportation networks

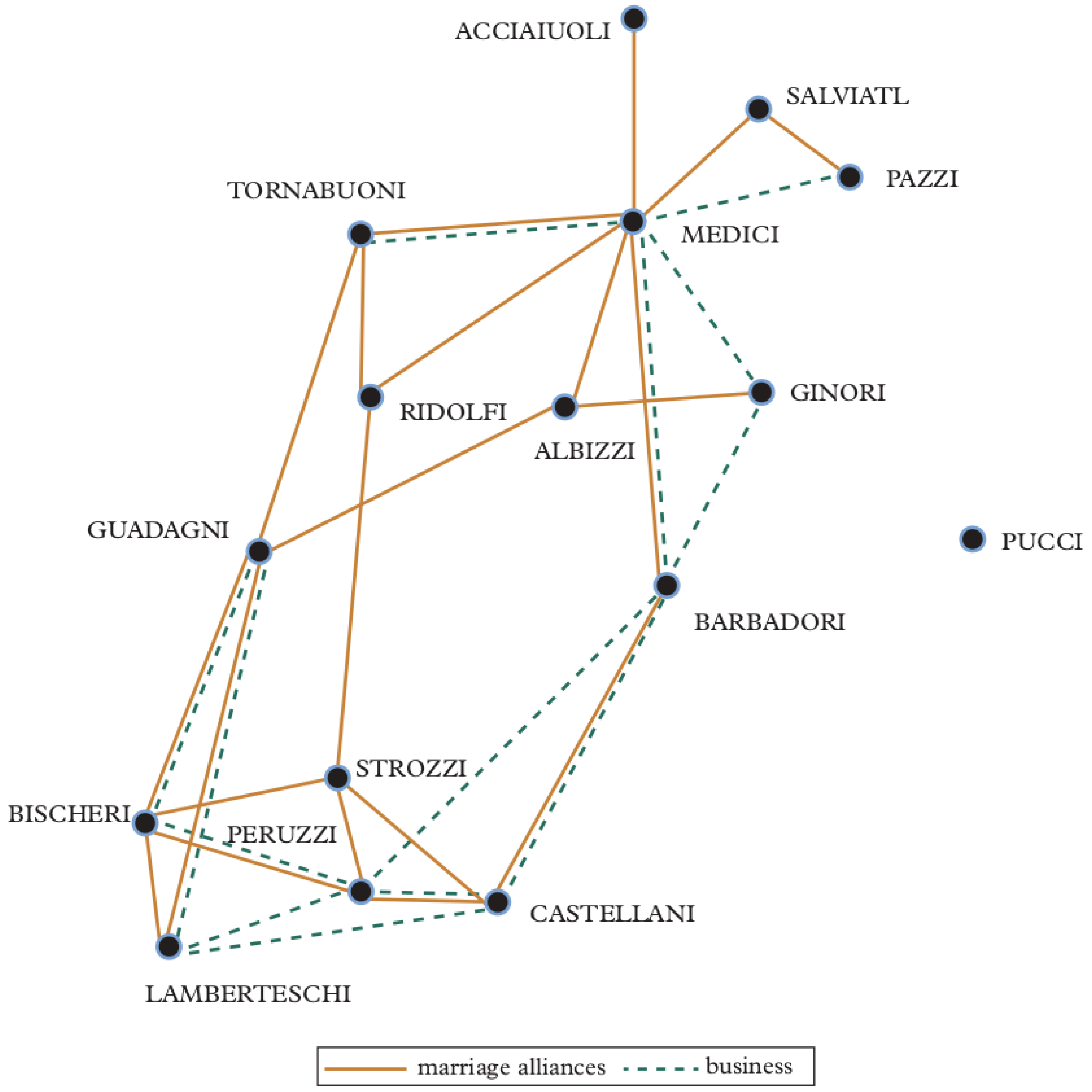
- each layer corresponds to a different mean of transport
  - bus
  - car
  - metro
  - rail
- each layer can be characterized by different nodes and links
  - not al bus stops are also metro stops
  - cars follow different paths than trains

Social Networks

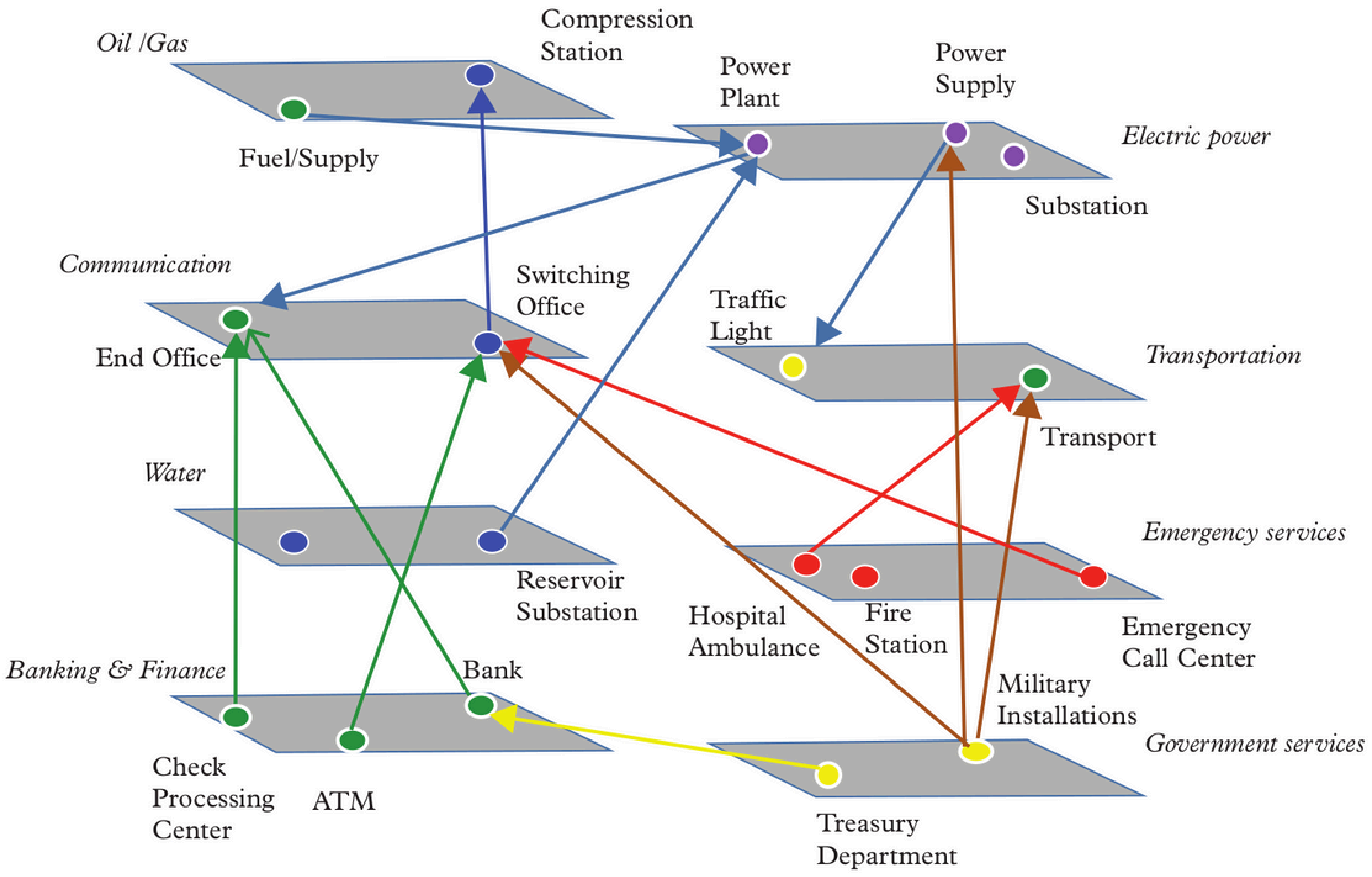
Social networks are another classical example of multilayer networks

- social relations can be of different types
  - friendship
  - sentimental relation
  - family membership
- different links carry substantially different information
  - we can’t just treat all the social relations in the same way

The figure show the social networks of Italian families during Renaissance



Infrastructures



Infrastructures are generally linked the one to the other by dependencies relations

- an hospital can’t operate without electricity
- a power plant may stop if the transport system fail

We can represent infrastructures as a series of layers

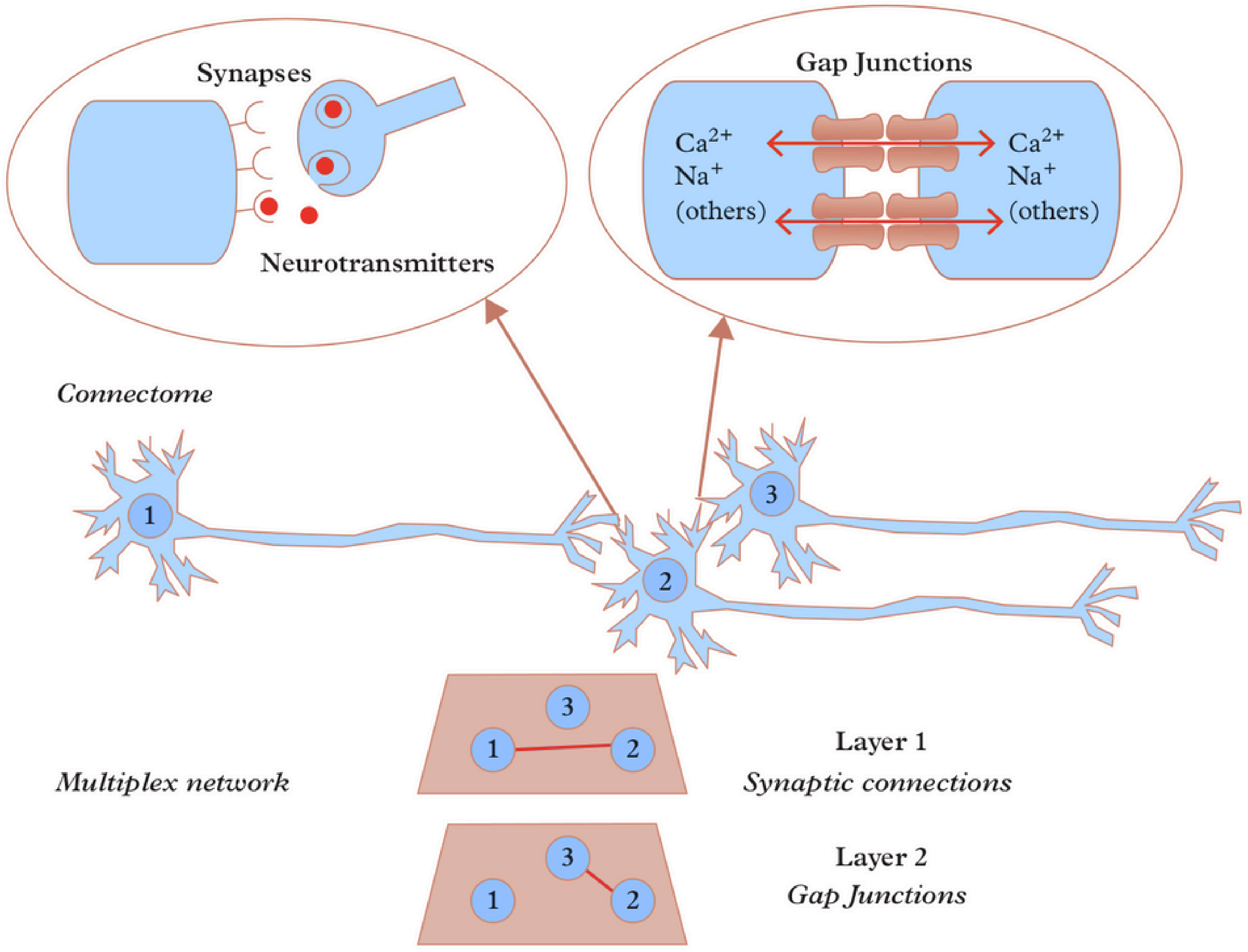
- connections may occur
  - within layers
  - between layers

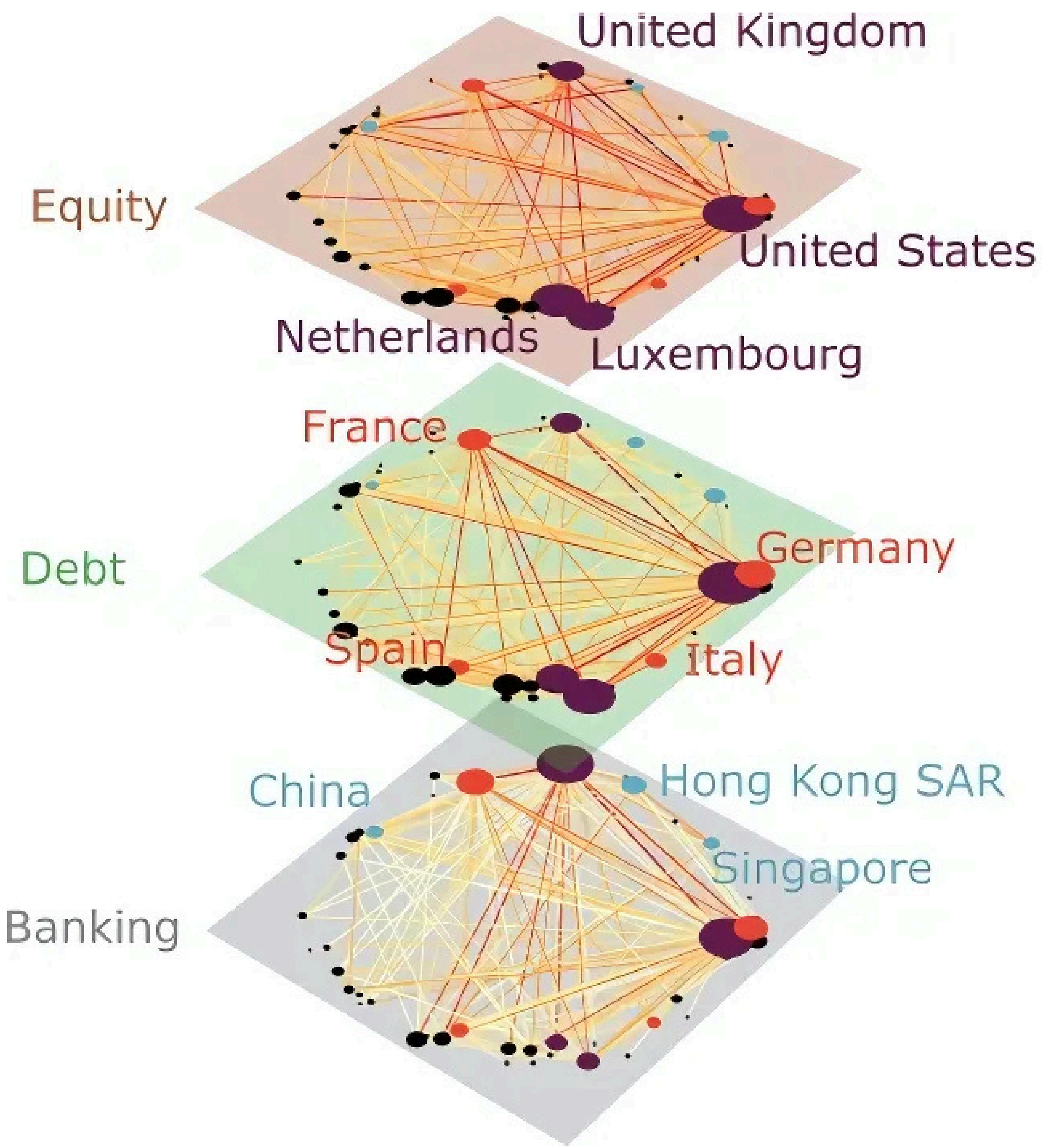
Neural Networks

Also biological neural networks are in many cases better described in terms of multilayer networks

- neurons can communicate in different ways
- synaptic connections are a possibility
- however there are also gap junctions and other links

Multilayer networks allow to treat these links separately





Financial Networks

Economic and financial actors can be linked by different types of relations

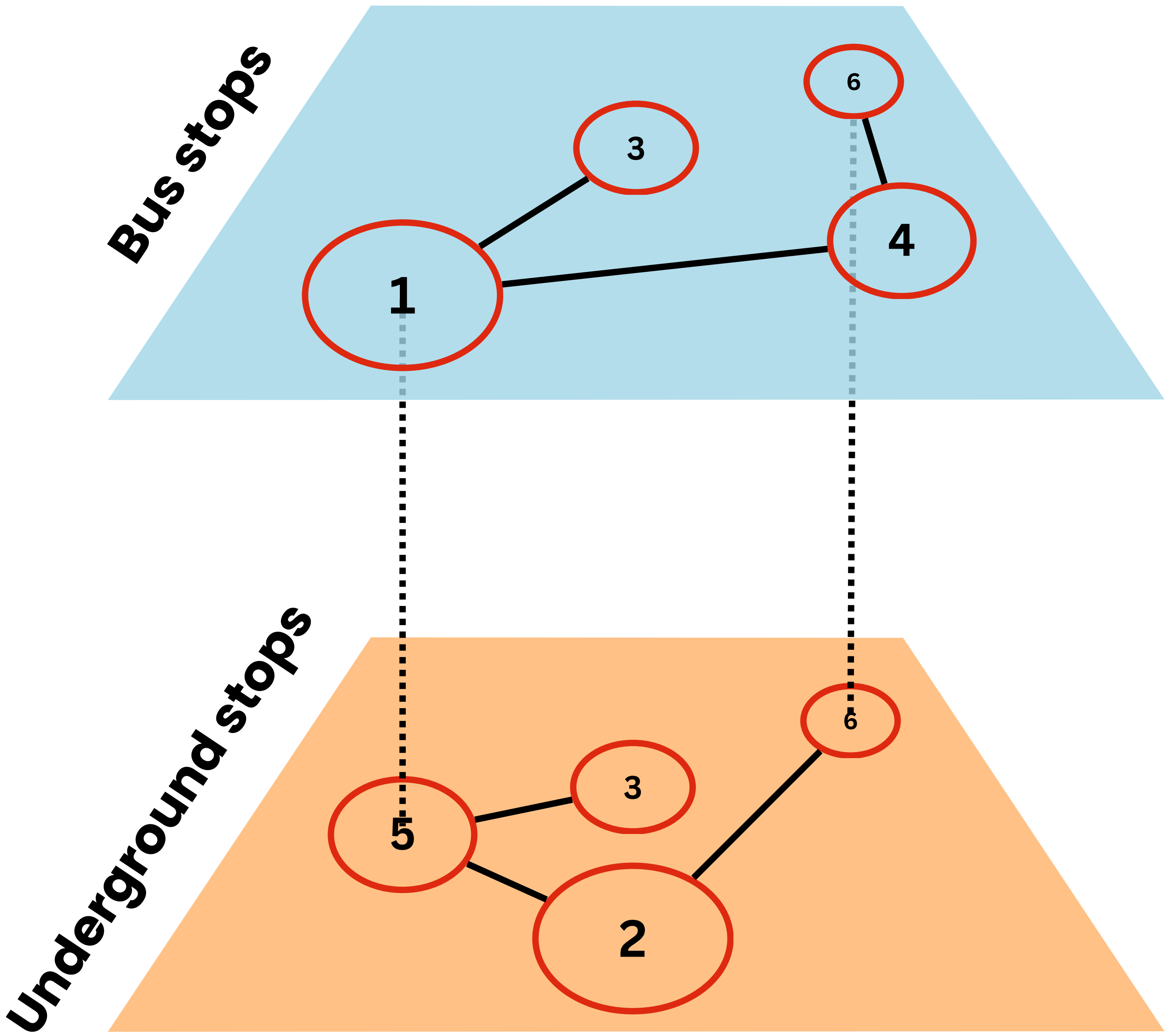
- if we consider countries, they may
  - trade goods
  - trade financial products
  - buy and sell bonds
  - invest in other countries
- this gives rise to a very complex multilayer structure

For instance the figure show the structure obtained considering Equity, Debt and Banking

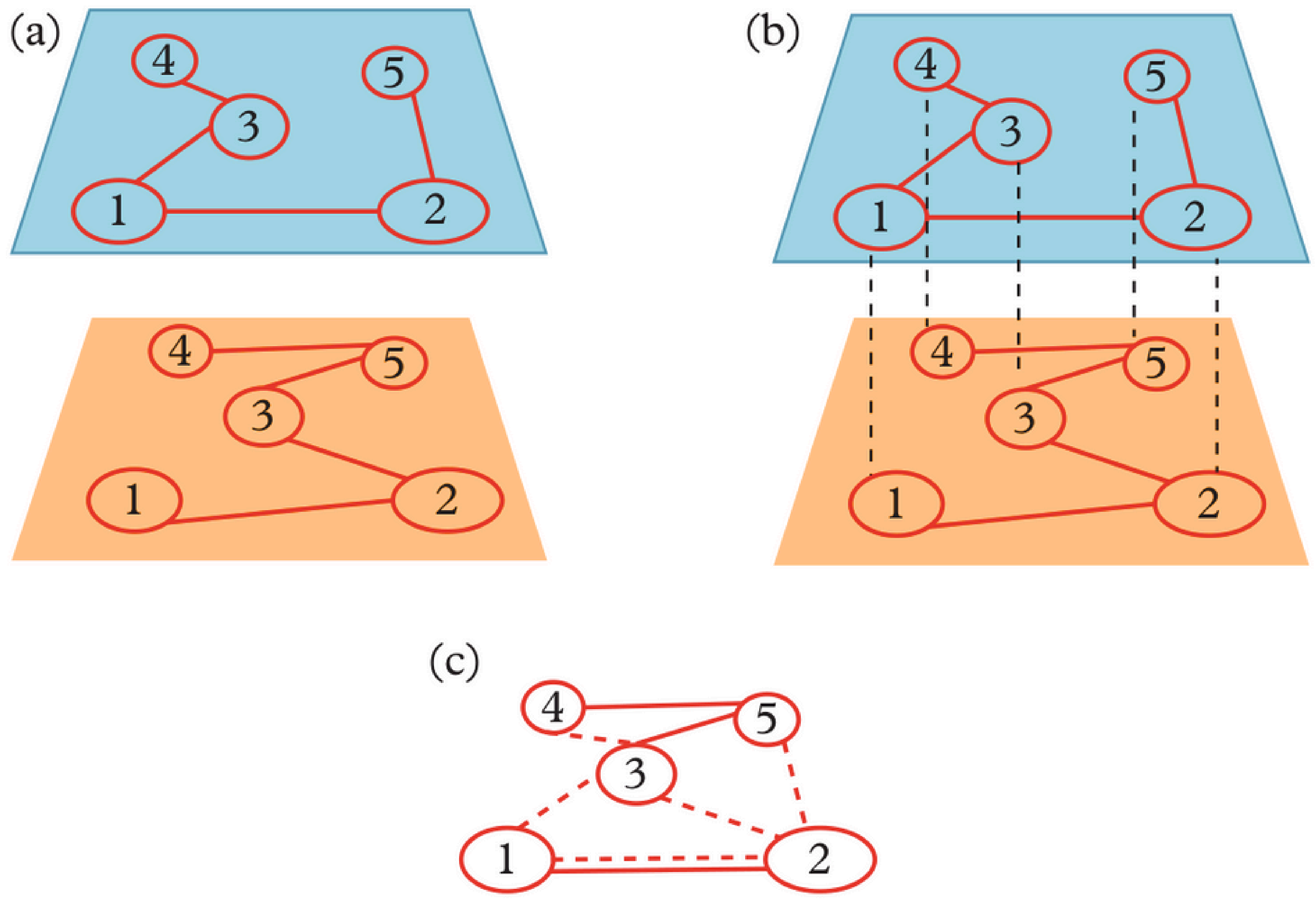
General
Definition

A multilayer networks is
- a set of M networks
- different networks can be characterized by different nodes
- connections between nodes in different layers are possible

This is like a transportation system
- two layers, bus and underground
- not all bus stops have the corresponding underground stop
- connections between layers are like elevators



Multiplex Networks



Multiplex networks are a simpler subset of multilayer networks

- each layer shares the same set of nodes
- layers only differ in the specific links
- examples are
  - the Italian families network
  - countries financial network

There are different ways to represent multiplex networks

- this also depends on the presence of links between layers

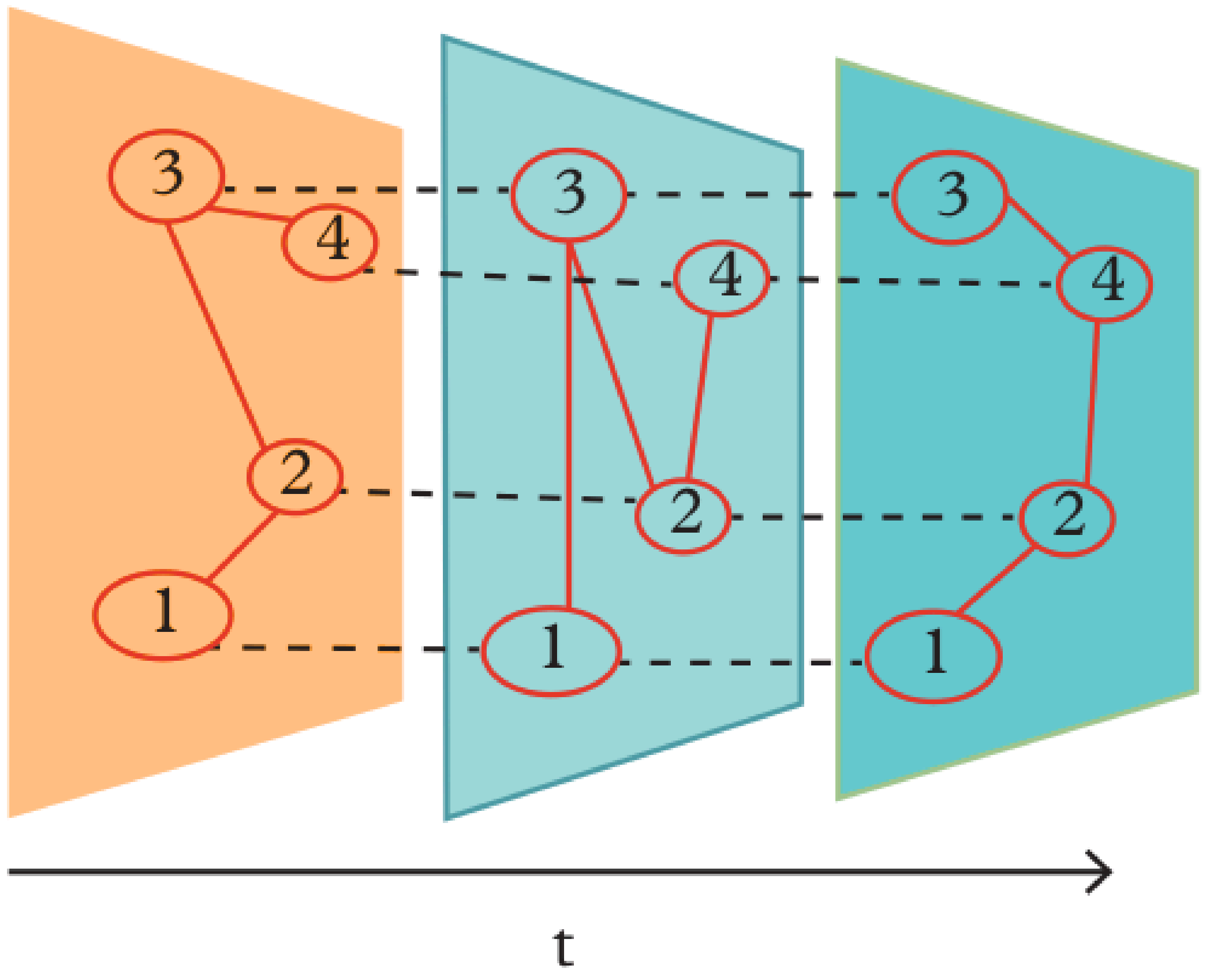
Multi-Slice Networks

Multi-slice or temporal networks are a special example of multiplex networks

- each layer corresponds to a different time steps
- they describe a network as it evolves over time

Temporal networks are useful for studying processes on evolving networks

- epidemic spreading
- opinion dynamics



Multiplex Networks

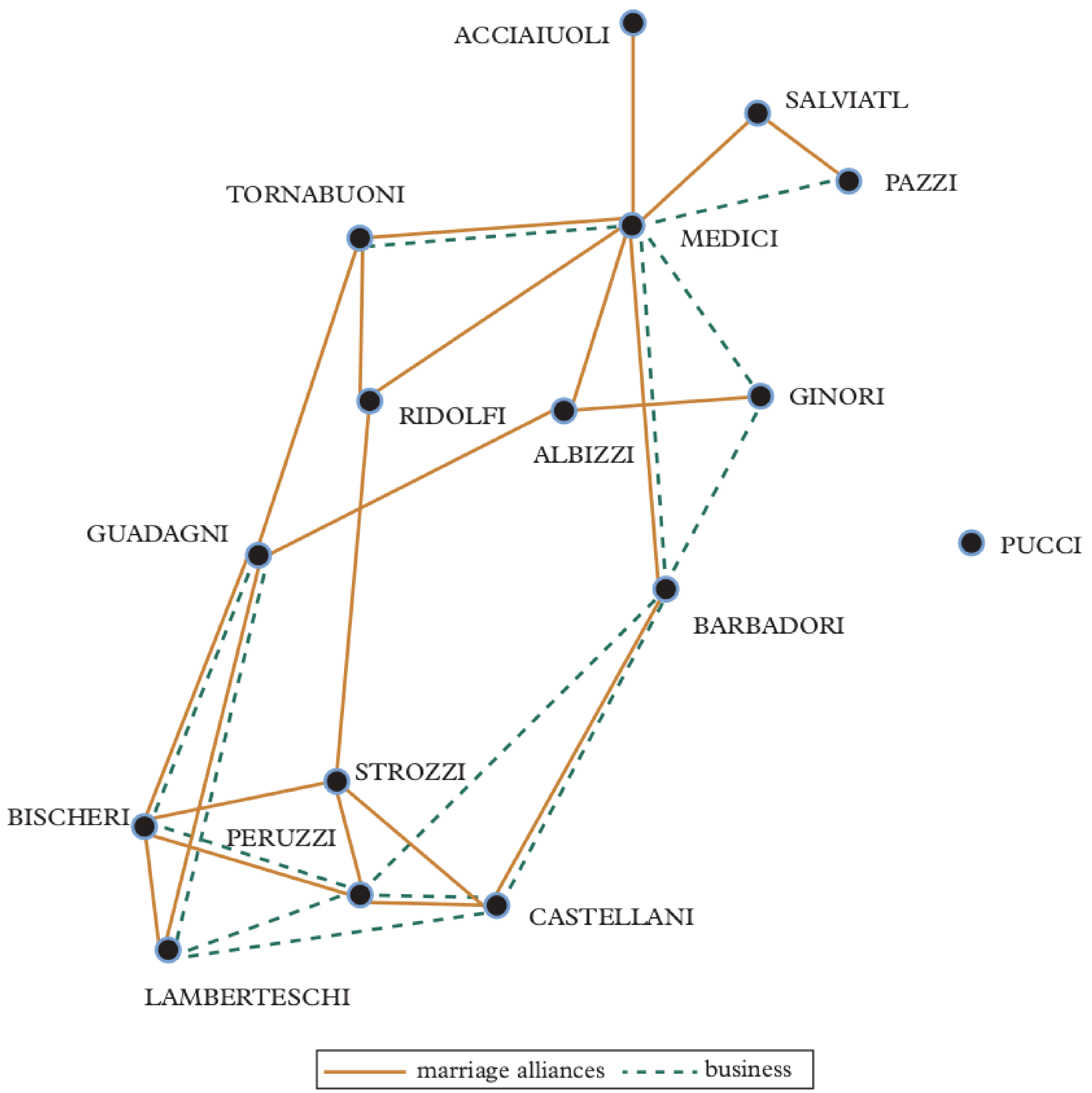
Why **Multiplex** Networks

Multiplex networks are the most simple step toward multilayer networks

- despite their simplicity they are very versatile
- in many situations nodes are the same
- differences only lay in the links

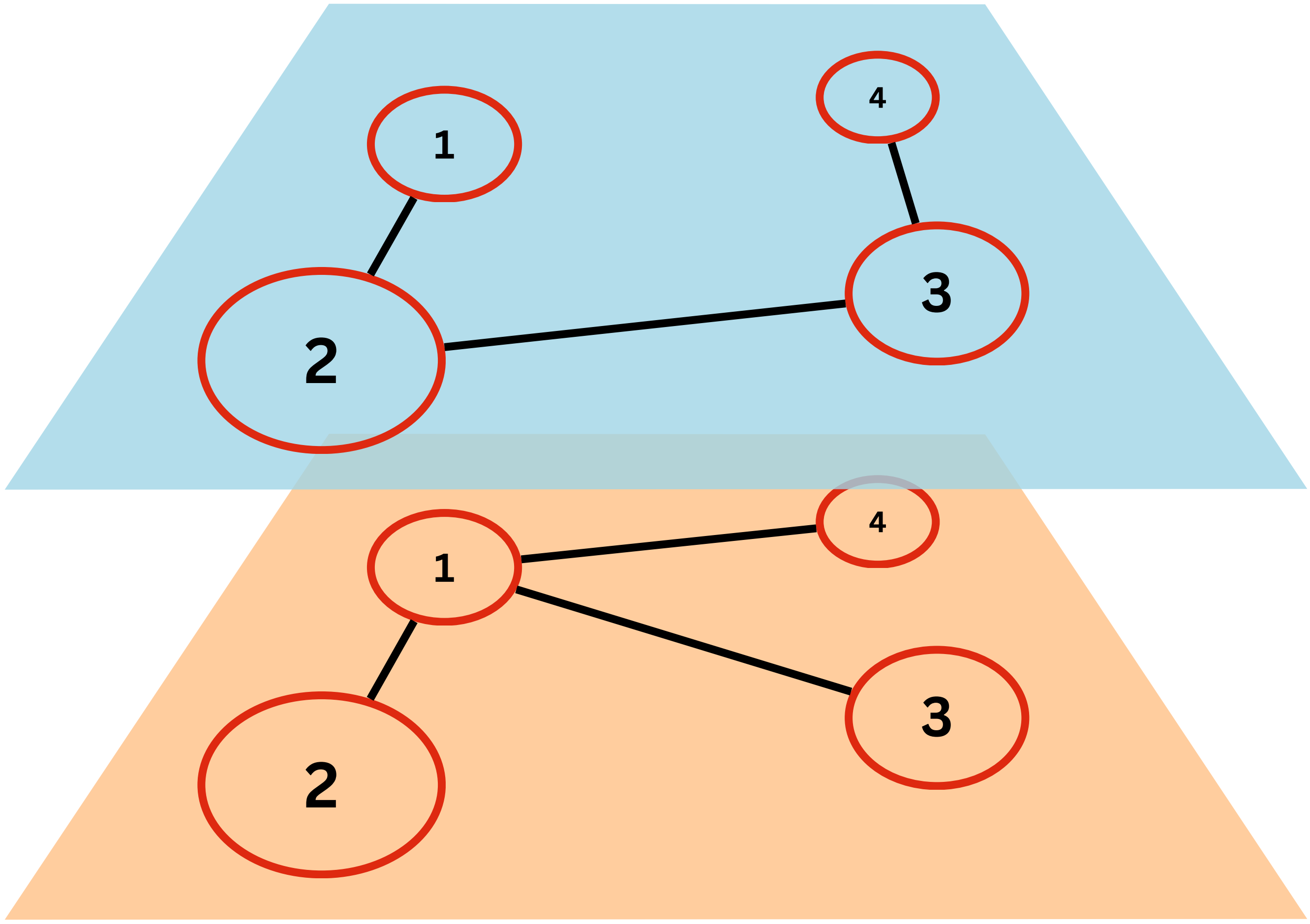
It is easy to generalize results we derived for networks to multiplexes

In the following we focus on multiplex networks



Multiplex Degree

$$k_1 = (1, 3)$$
$$k_2 = (2, 1)$$
$$k_3 = (2, 1)$$
$$k_4 = (1, 1)$$



Degree is very easy to generalize

- each node will have a (different) degree for each one of the M layers
- we can group all these degrees in a single vector for each node
- we call this vector multiplex degree

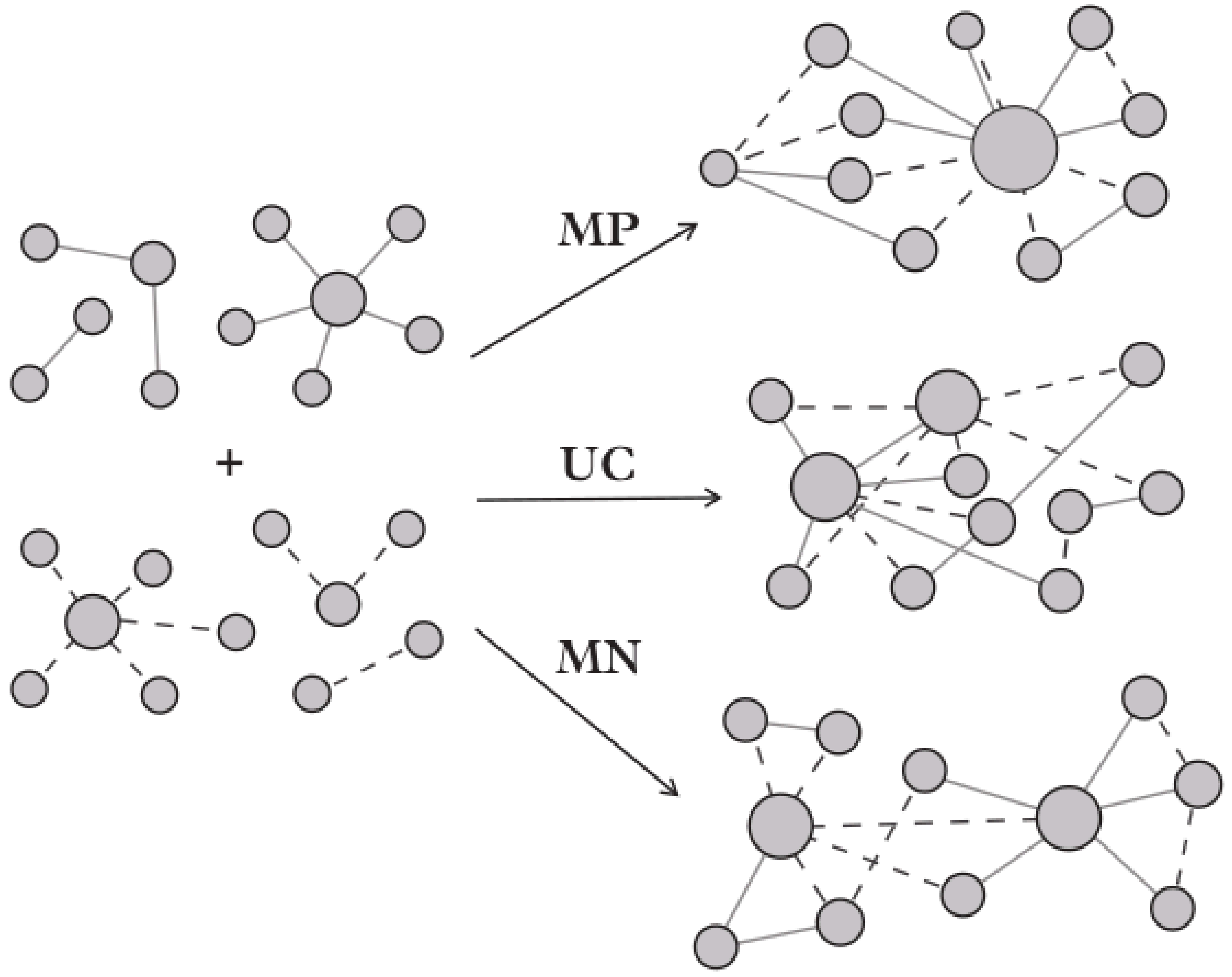
The **multiplex degree** is thus an M components vector, where the n-th component gives the degree of the node in the n-th layer

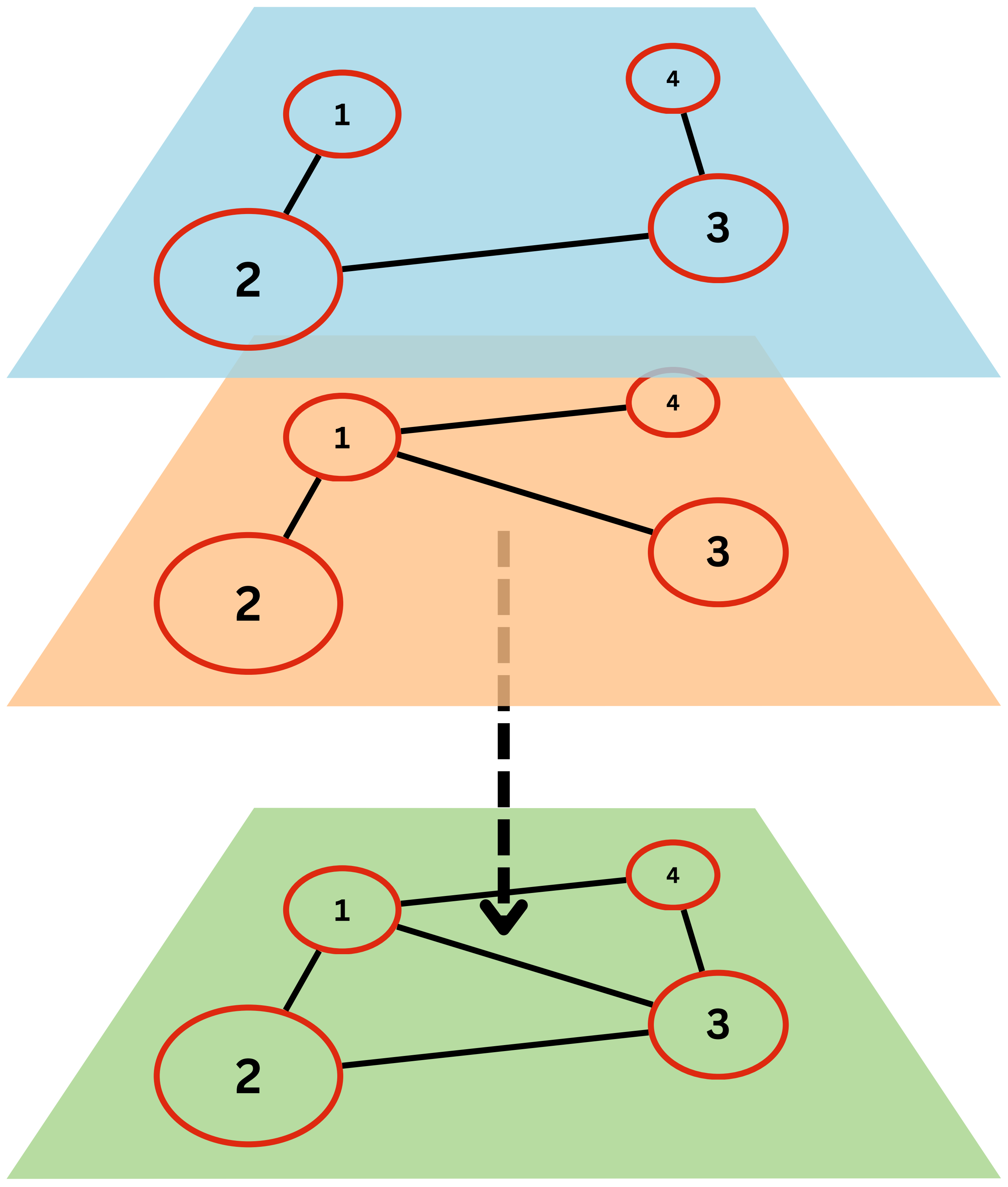
Degree Correlations

One of the most important property of multiplex networks is the type of correlations between the degrees of the same node in different layers

There are different possible scenarios, with 3 limit cases

- Maximally Positive correlated (MP)
  - similar degree in all layers
- Uncorrelated (UC)
  - no correlation between layers
- Maximally Negative correlated (MN)
  - high degree in one layer, low degree in the other layer





Aggregated Network

A multiplex network can be aggregated projecting all links on a single layer

- two nodes are linked if they are connected at least in one layer
- this procedure returns a simple network
- we lose all information about the different nature of links

In most situations this aggregated network lack many of the properties of the multiplex network

A similar procedure can also be applied to temporal networks

Distance and Interdependence

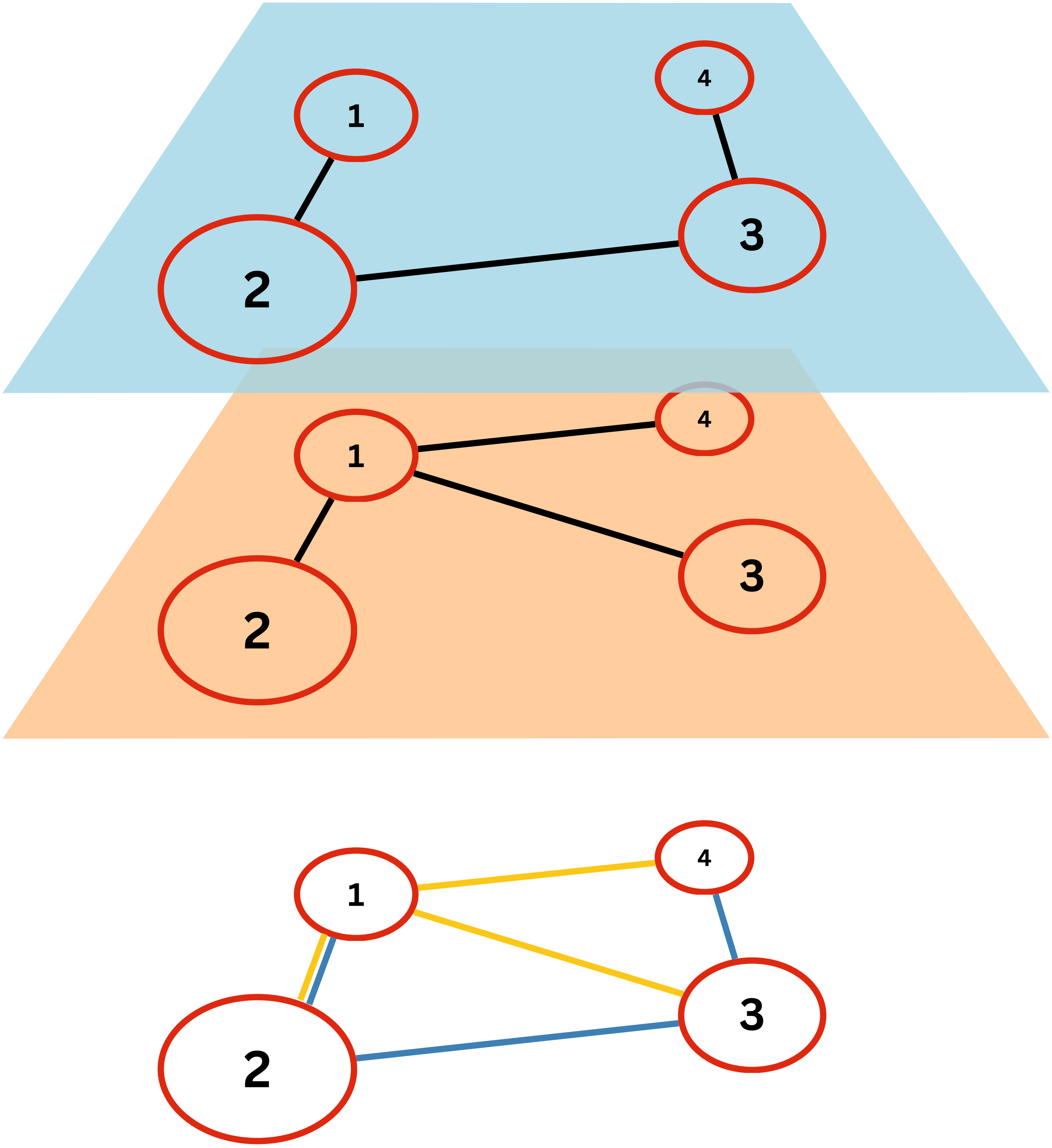
Moving in a multiplex network is easier

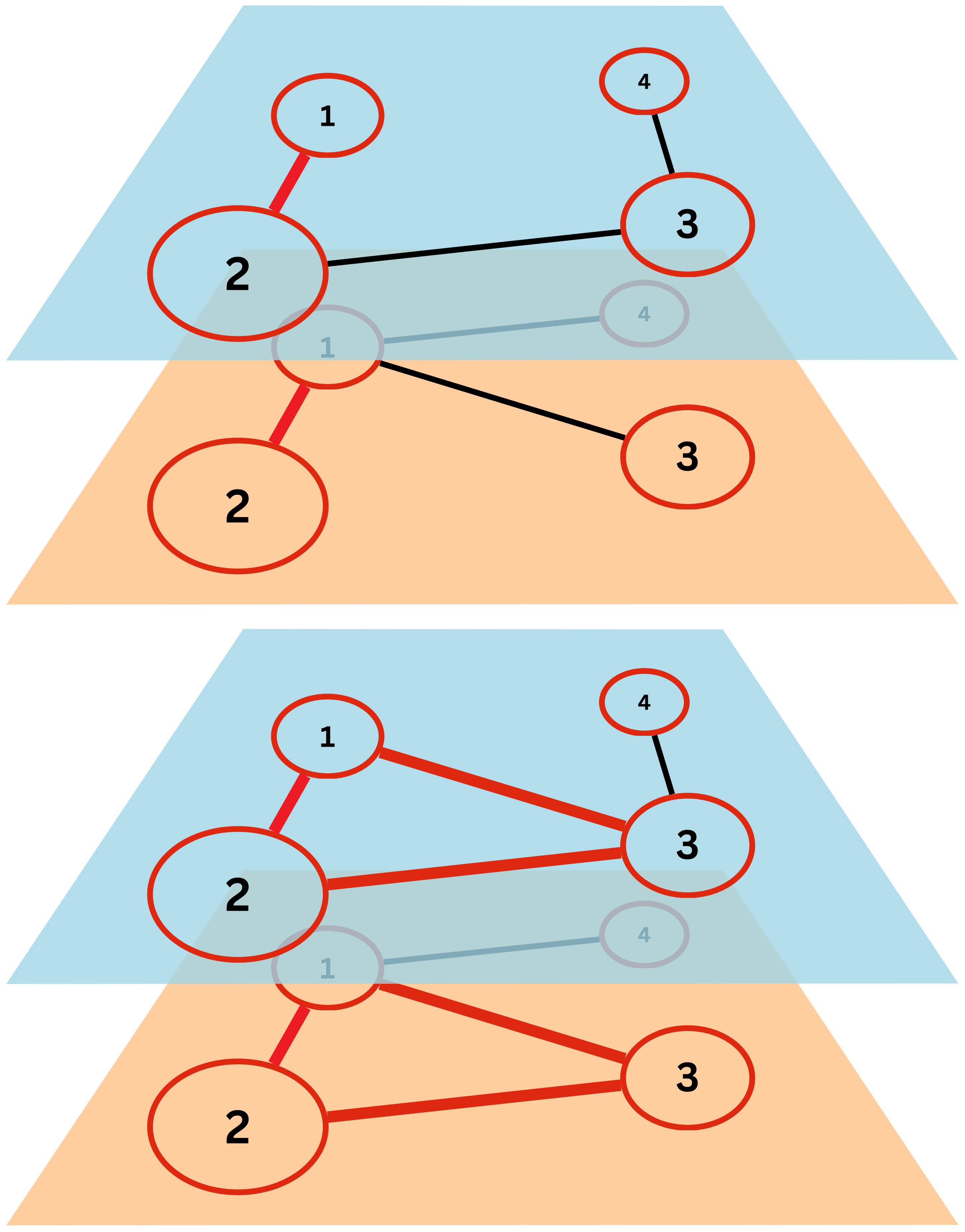
- in this case the distance is the shortest path on the aggregated layer

In order to quantify the relevance of the multiplex structure we can use the local and global **interdependence** λ

$$\lambda_i = \sum_{i \neq j} \frac{\psi_{ij}}{\sigma_{ij}} \qquad \lambda = \frac{1}{N} \sum_{i=1}^{N} \lambda_i$$

For λ = 0 all the shortest paths lie on just one layer, and for λ = 1 all the shortest paths include links of more than one layer.





Link **Overlap**

Another measure that allows to understand the importance of the multiplex structure is the link overlap O

- given two layers α and β this measure quantifies how similar in terms of links the two layers are
- it is defined as the number of common links they have

$$O^{[\alpha,\beta]} = \sum_{i<j} a_{ij}^{[\alpha]} a_{ij}^{[\beta]}$$

Two layers with very high overlap will be formed more or less by the same connections

- real networks are characterized by fairly high overlaps (they are redundant)

Clustering Coefficient

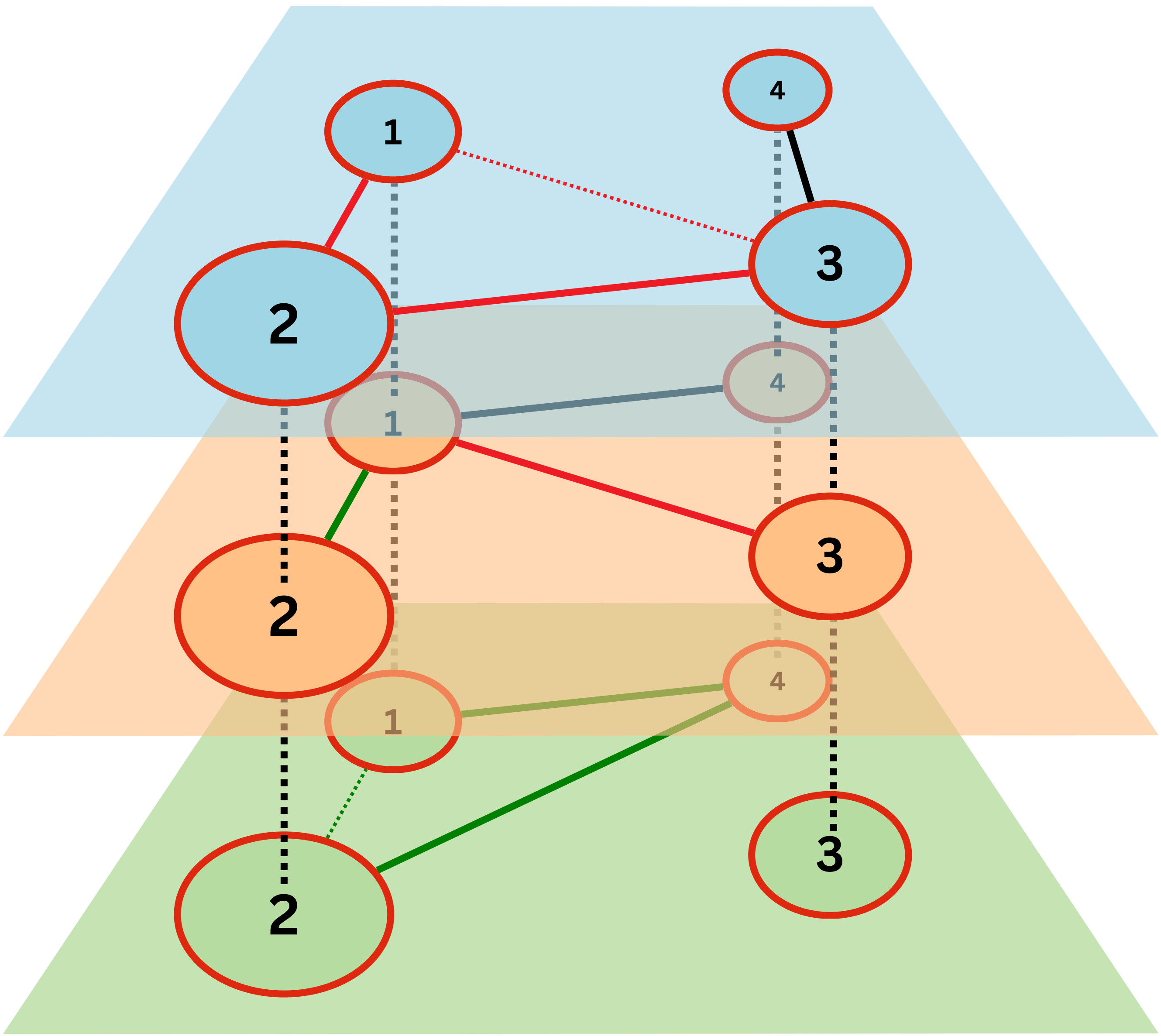
Another measure we need to generalize is the clustering coefficient

- triangles can now close in different ways
- we have to consider
  - single layer triangles (standard clustering coefficient(
  - two layers triangles
  - three layers triangles

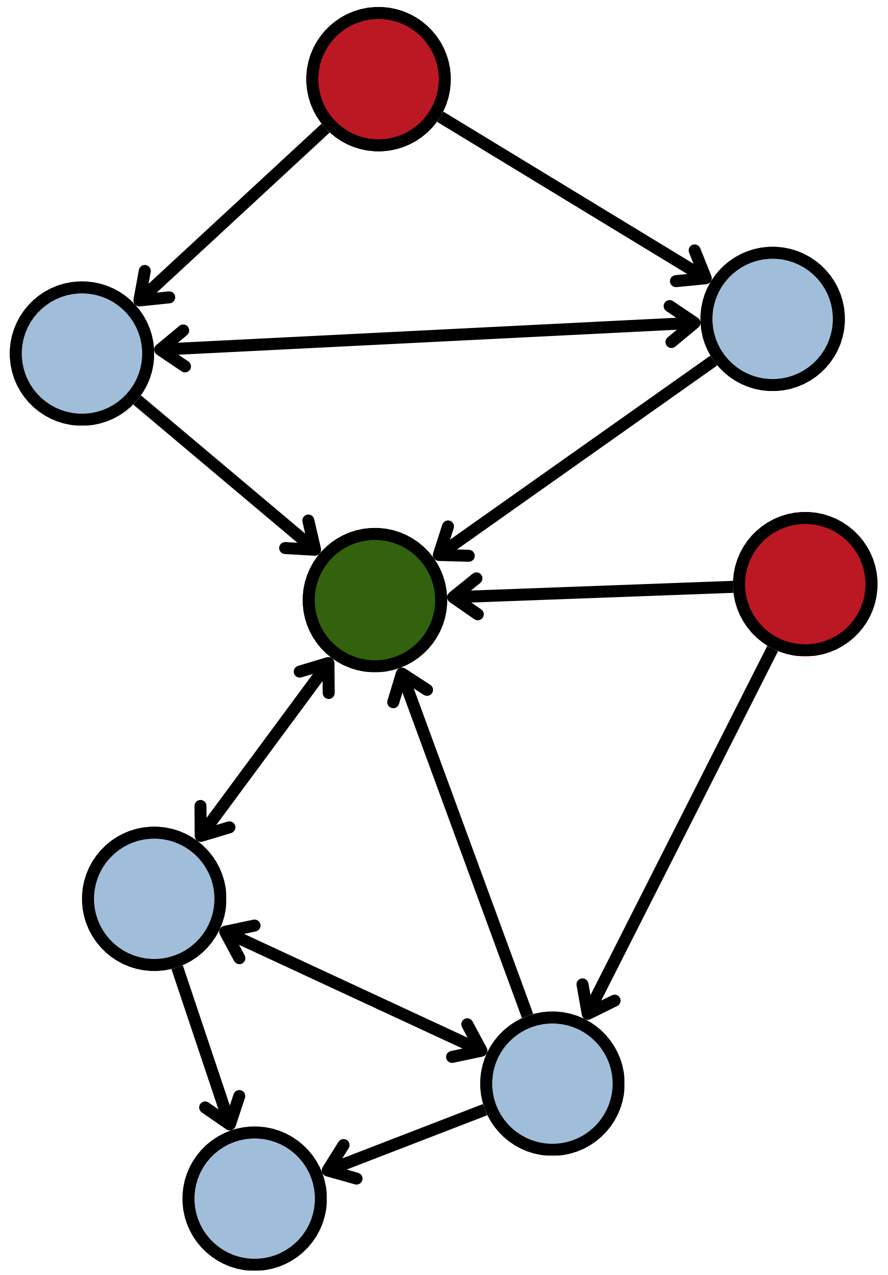
We thus need to introduce two new measures

$$C_{i,1} = \frac{\sum_{\alpha=1}^{M} \sum_{\beta|\beta\neq\alpha} \sum_{r,s} a_{ir}^{[\alpha]} a_{rs}^{[\beta]} a_{si}^{[\alpha]}}{(M-1)\sum_{\alpha=1}^{M} k_i^{[\alpha]}(k_i^{[\alpha]}-1)},$$

$$C_{i,2} = \frac{\sum_{\alpha=1}^{M} \sum_{\gamma|\gamma\neq\alpha} \sum_{\beta|\beta\neq\alpha,\gamma} \sum_{r,s} a_{ir}^{[\alpha]} a_{rs}^{[\beta]} a_{si}^{[\gamma]}}{(M-2)\sum_{\alpha=1}^{M} \sum_{\gamma\neq\alpha} k_i^{[\alpha]} k_i^{[\gamma]}}$$



Multiplex PageRank



- Low Reputation
- High Reputation

PageRank can be generalized to multiplex networks in several ways. One possibility is the **Multiplex PageRank**

- we consider just two layers
- we first compute the PageRank in one layer
- we then compute the PageRank in the other layer
    - in this case however we bias the walker
    - it teleports more likely to nodes with high PageRank in the first layer

The idea behind this algorithm is that a person can inherit some of its popularity from another network they are part of

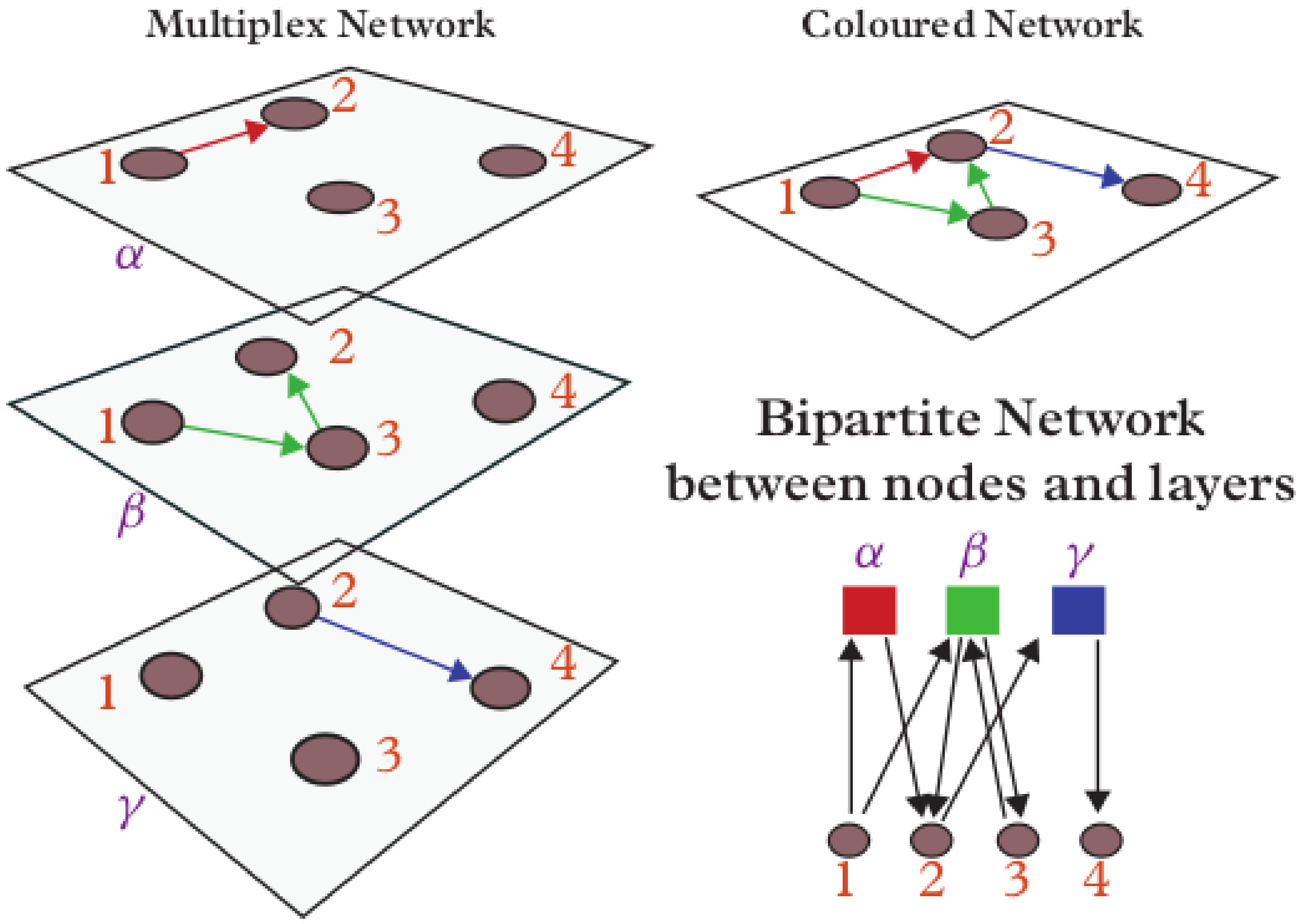
MultiRank

Multiplex PageRank works well if there is a clear hierarchy between layers

- it doesn’t generalize well to multiplexes with many layers
- also other direct approaches involving random walkers fail when there are hundreds of layers

MultiRank changes the perspective providing a ranking of layers and nodes

- this is achieved building a bipartite network of nodes and layers
- nodes are linked to the layers they are active into
- a PageRank coupled with another set of equations is used



Epidemic Spreading

Epidemics on Multiplex Networks

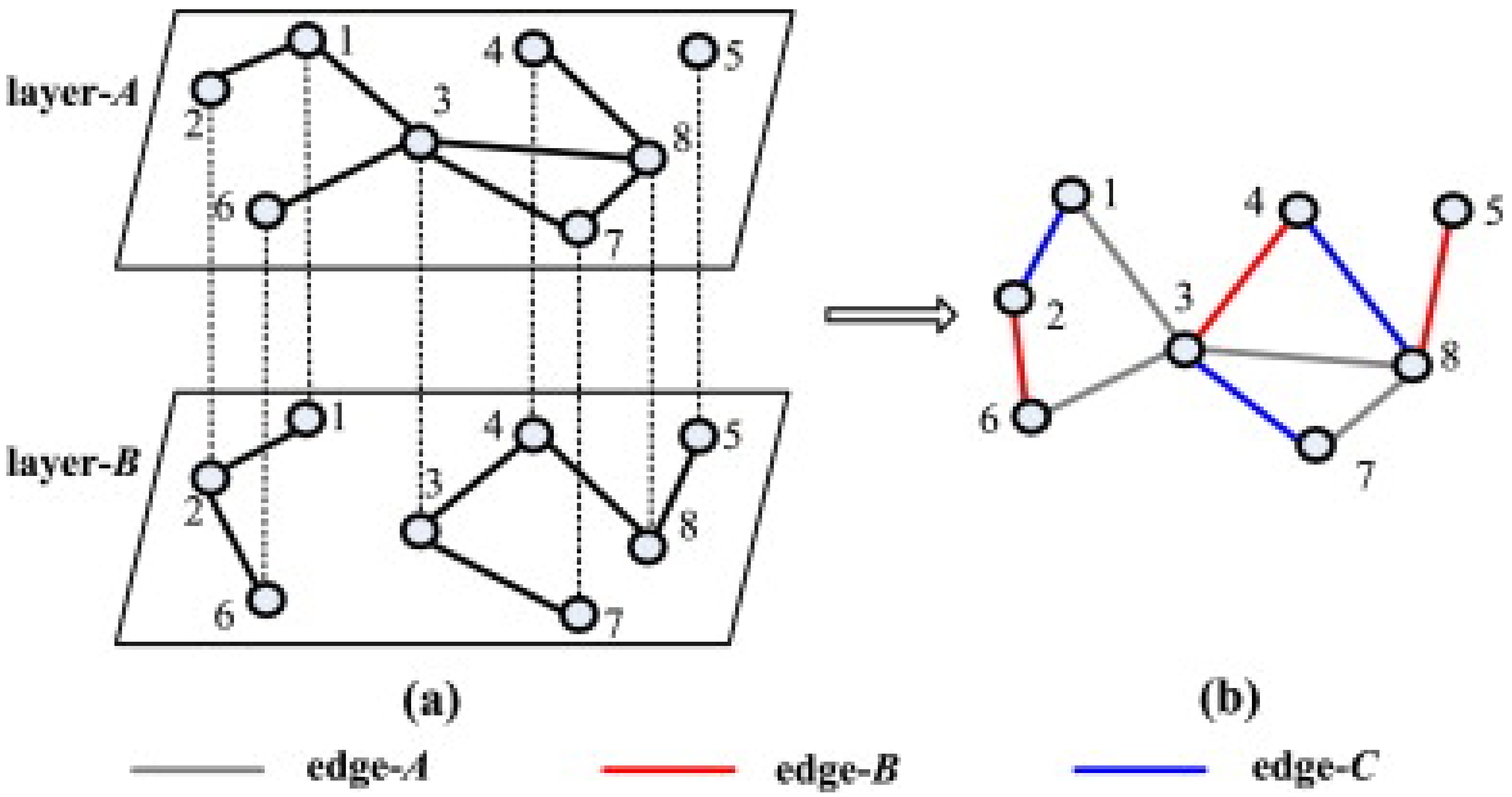
Epidemics can propagate along different path following the exiting transportation routes

- different routes can have different spreading rate
- the problem is intrinsically multilayer

Several studies focused on epidemics on multiplex networks

- different transmission routes
- different pathogens
- interaction of online and physical layers

Multiplex Networks



Zhao, Dawei, et al. "*Multiple routes transmitted epidemics on multiplex networks*." Physics Letters A 378.10 (2014): 770-776.

The most simple case consists of a single pathogen spreading on two layers forming a multiplex network

- we assume the spreading rate is different across different links
- we will have two spreading rates λ_A and λ_B
- nodes follow a SIR process

If a node gets infected on a layer, it will also spread the epidemic in the other layer

- we expect for the epidemics to be easier to propagate

Epidemic Threshold

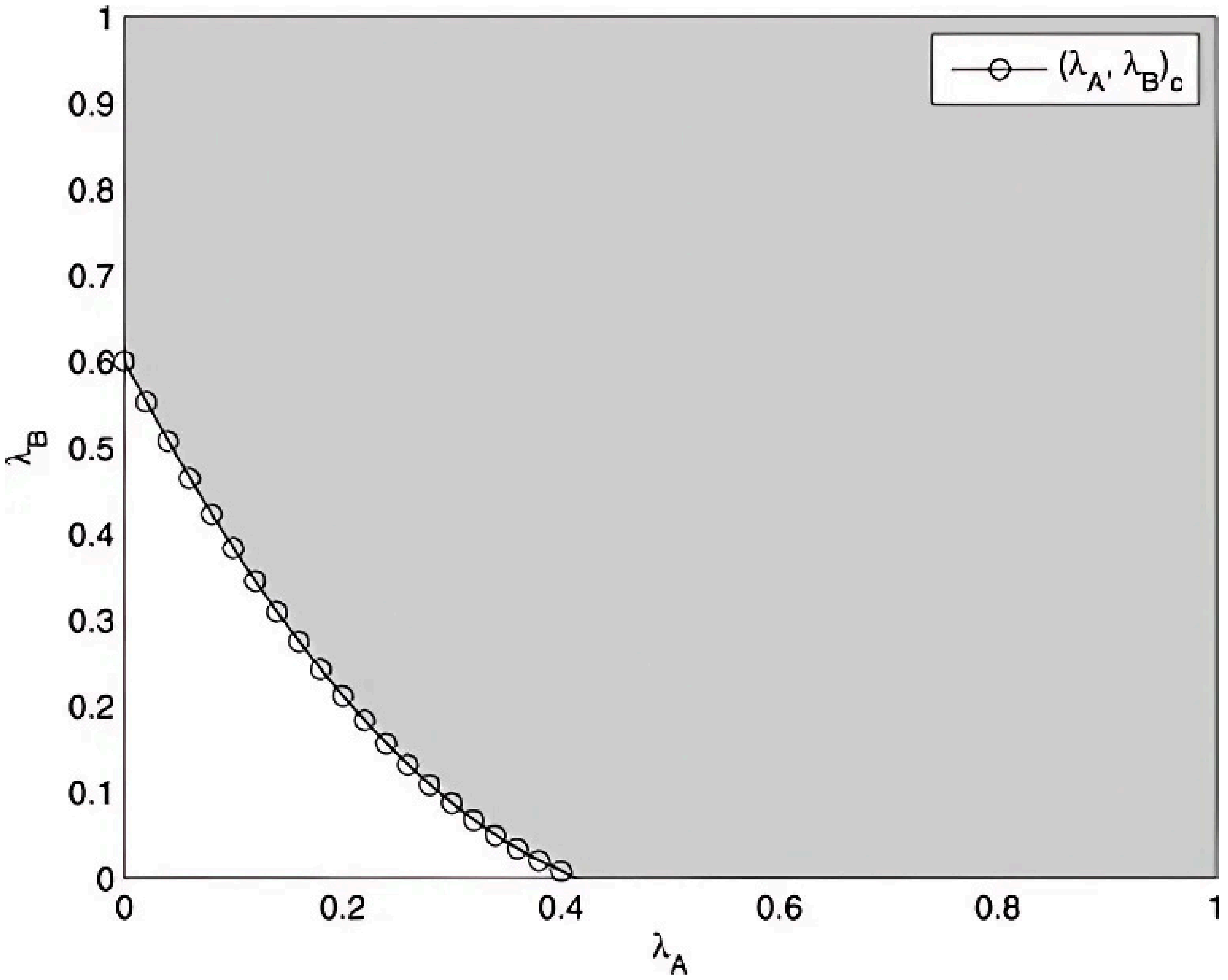
The epidemic threshold of this model will depend on

- the specific structure of the two networks
- the value of the spreading rates

The figure shows the phase diagram for the simple case of two uncorrelated random networks

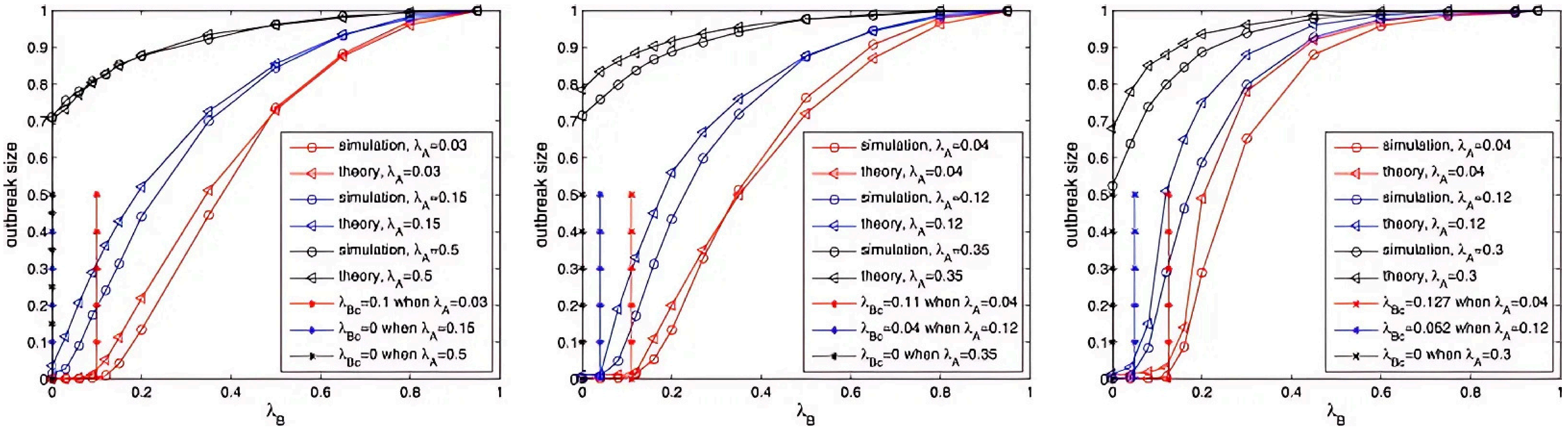
- the epidemics diffuses only for values in the grey area

Thanks to the multiplex structure, epidemics that would die on each single network, can now propagate



Outbreak Size

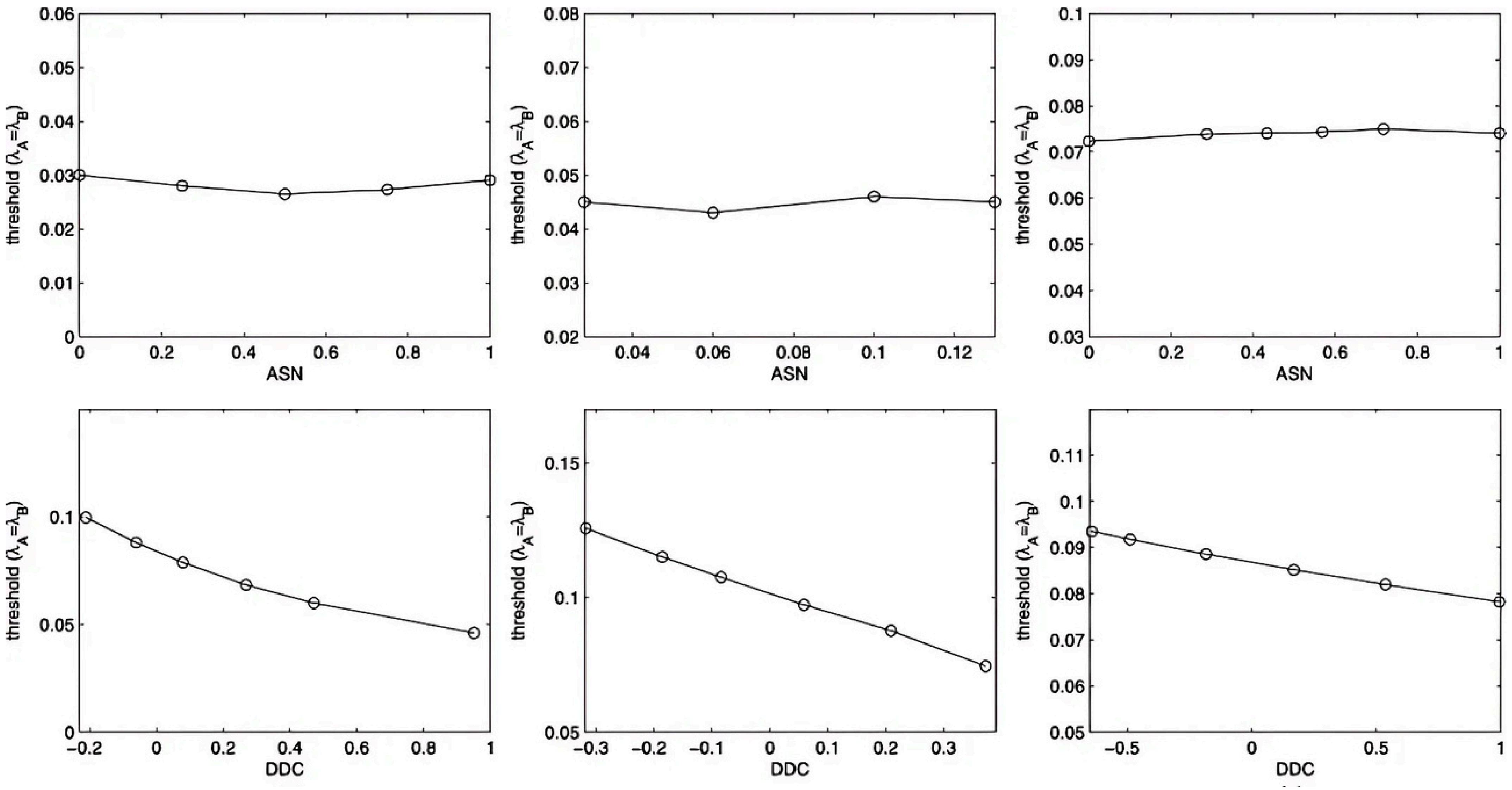
Analytical computations can be performed to derive the expected outbreak size. Also in this case we observe a non-trivial interplay between the two networks and a good agreement with simulations



The Role of Correlations

Finally we look at the role of correlations

- neighbors correlations (ASN) have basically no effect on the threshold
- degree correlations (DDC) instead sensibly reduce the epidemic threshold

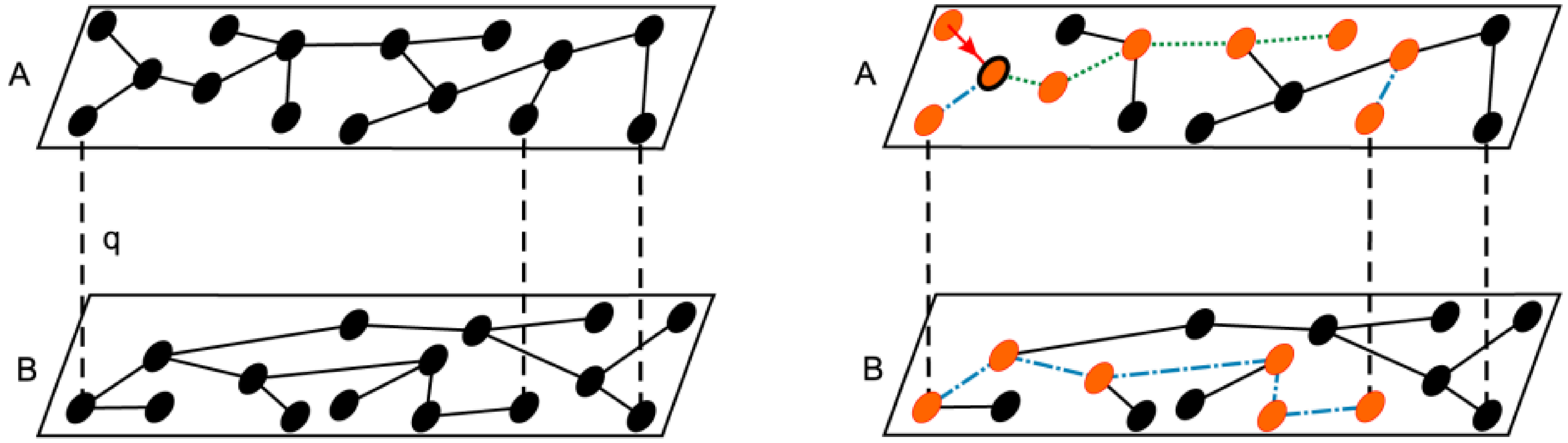


Node **Overlapping Multiplex** Networks

Multiplex networks are very versatile, but in some situations they can be limiting. In particular we could relax the requirement of nodes existing in all layers

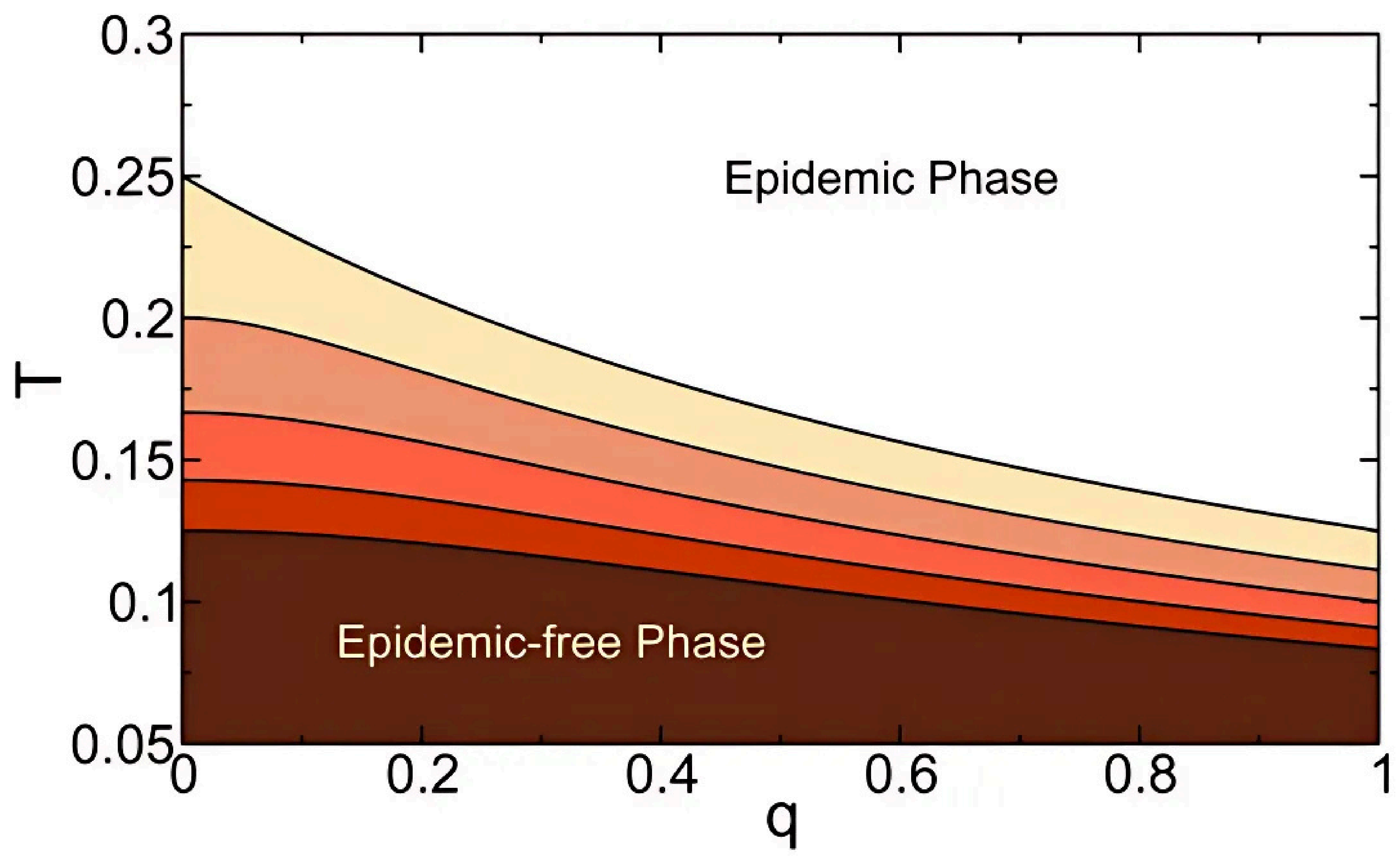
- we introduce overlapping multiplex networks
- the overlap q measures the fraction of nodes shared by both layers

In many social networks we are active only in some of the layers



Buono, Camila, et al. "*Epidemics in partially overlapped multiplex networks.*" PloS one 9.3 (2014): e92200.

Epidemic Threshold



The figure show the phase diagram of a SIR model on such a multilayer network

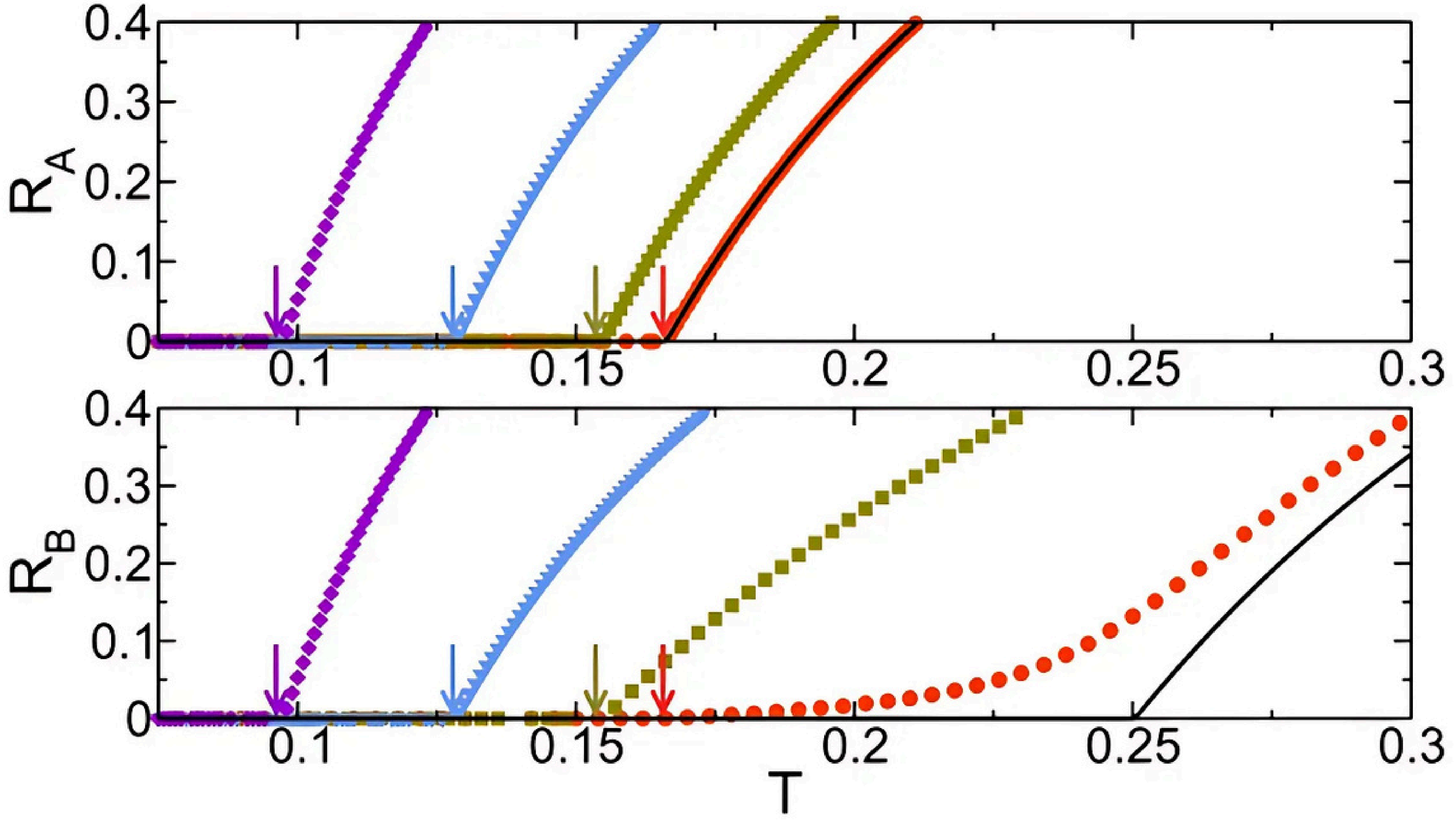
- different curves correspond to different average degree
- T is the transmittability
  - it is the probability to infect within the recovery period
- q is the overlap between layers

A larger value of the overlap leads to lower epidemic thresholds

The Effect of Overlap

We can see this effect in detail looking at the individual layers

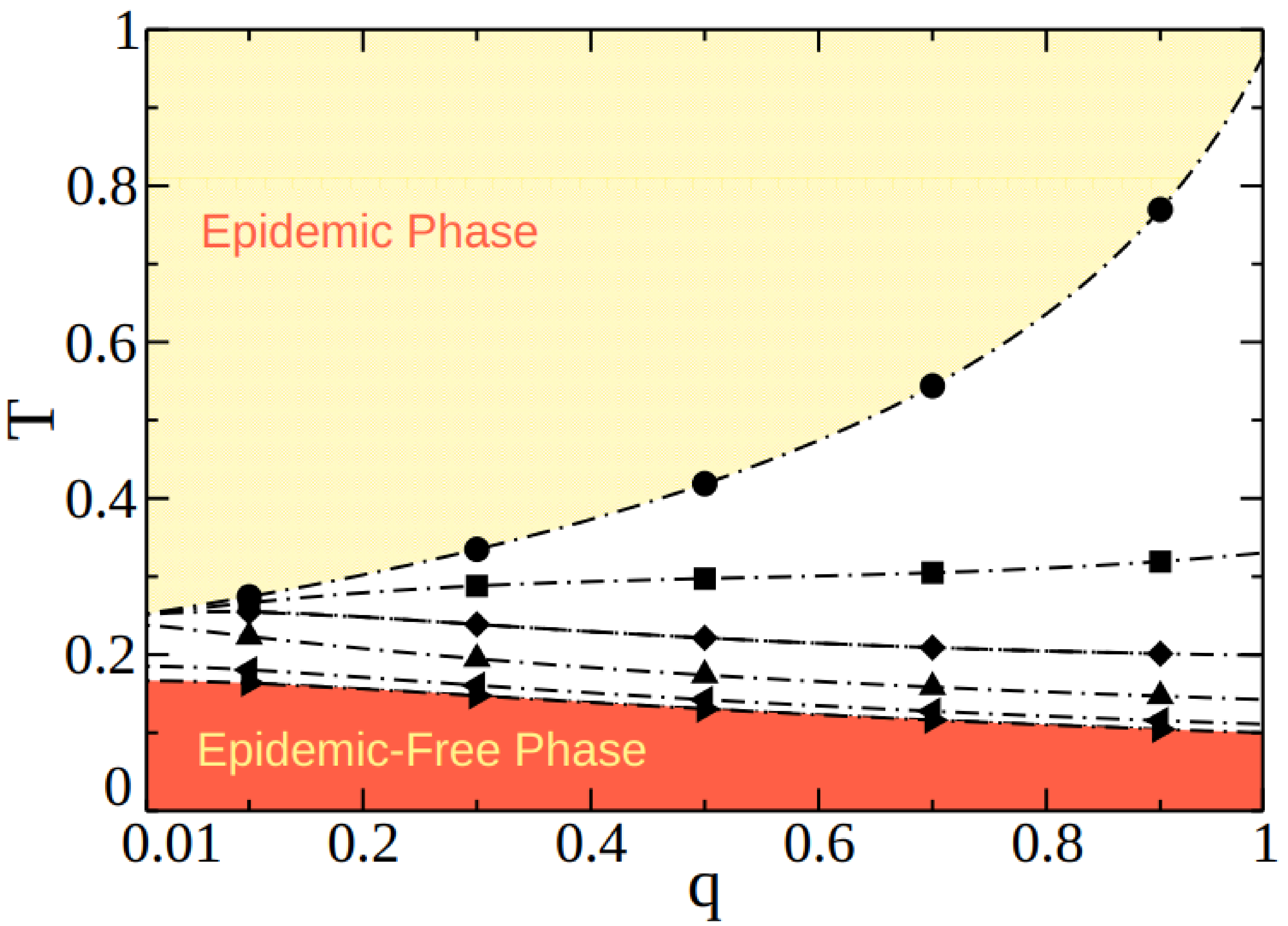
- the plot show the fraction of infected individual
- orange corresponds to very low overlap (0.01), purple to maximal overlap (1)



The second network inherits the epidemic threshold of network A already for very low values of overlap

- the epidemics is dominated by the networks with the lowest threshold

Random **Immunization**



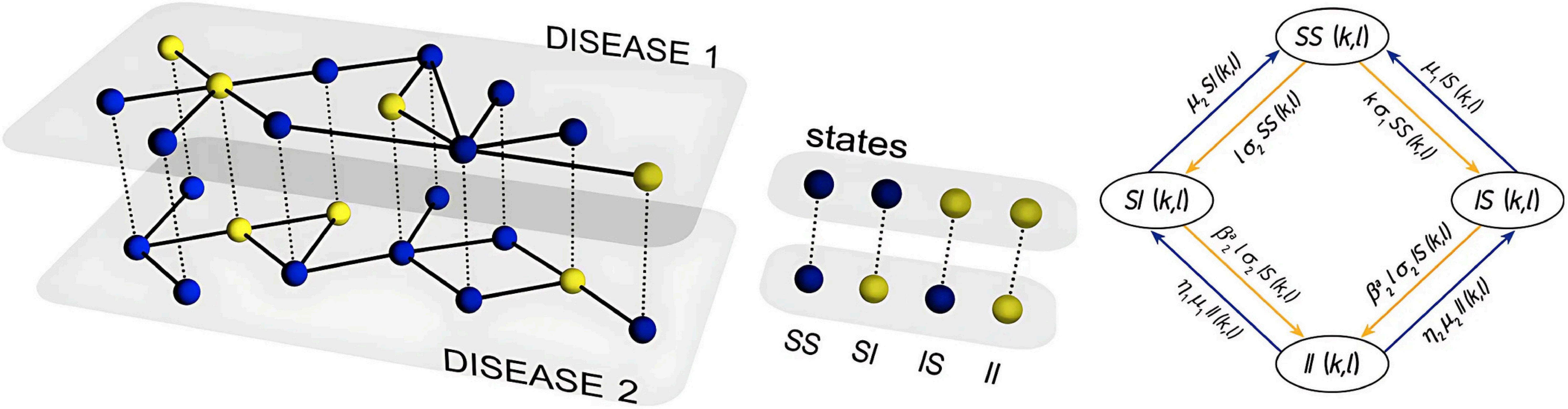
Zuzek, Lucila G. Alvarez, Camila Buono, and Lidia A. Braunstein. "*Epidemic spreading and immunization strategy in multiplex networks.*" Journal of Physics: Conference Series. Vol. 640. No. 1. IOP Publishing, 2015.

Another aspect we can study in this model is the effect of immunization

- we only immunize a fraction m of layer A nodes
- this will also immunize a fraction mq of B nodes
- the figure shows how the phase diagram for different values of m
- we need m=0.9 (top line) for obtaining substantial effects

Interacting **Diseases**

The different layers may also corresponds to different pathogens spreading in the same populations. They may be transmitted in different modalities and thus spread on different networks. Moreover they can also interact and potentially reinforce each other. Example: AIDS+Influenza

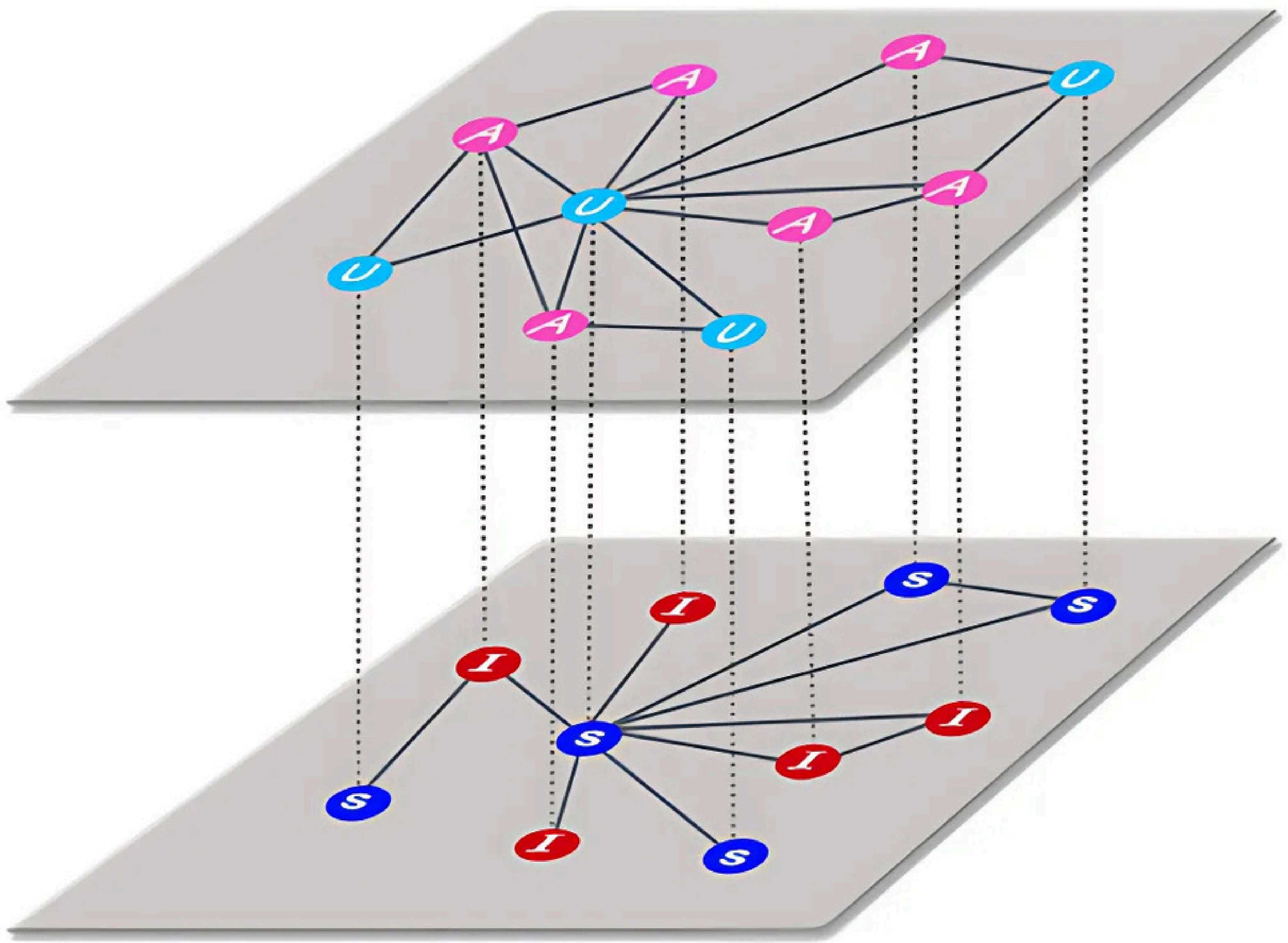


Sanz, Joaquín, et al. "*Dynamics of interacting diseases.*" Physical Review X 4.4 (2014): 041005.

Awareness Spreading

Another phenomenon we can model using multiplex networks is the interplay between ideas/knowledge and epidemic spreading

- the R0 depends on the measures adopted by people
- we can consider
  - a physical layer where people can get infected
  - a virtual layer where they can only share ideas
- in both layers the authors considered a SIS like process



Granell, Clara, Sergio Gómez, and Alex Arenas. "*Dynamical interplay between awareness and epidemic spreading in multiplex networks.*" Physical review letters 111.12 (2013): 128701.

Info About the Exam

Course Assessment

- Students select a published article or a dataset from from those listed on the course website.
- The task is to replicate the analysis performed in the paper or apply network science to the analysis of the dataset.
- Results must be summarized in a report and discussed in a presentation.

The course grade is based on:

- the student **presentation** (35%)
- the **code** (15%)
- and on the **report** (50%)

Final Report

I will upload datasets and papers on my website during the course

- the same dataset/paper can not be choose by more than one person
- you will have to communicate your choice by 26/01
- first come first served, but there is plenty of choice for everybody

The report must be structured as a scientific paper and about **4/5 pages** long (including the bibliography). It must contain

- Abstract
- Introduction
- Results/Main Analysis
- Discussion
- References
- An appendix (up to 3 pages)

The appendix should only contain additional figures and/or methods

Option 1: Reproducing a Paper

If you go for option 1 you goal will read, (hopefully) understand and then replicate the results of a paper

- you are expected to write and submit some code
- you have to write a report summarizing the main results and ideas of the paper
- you have to report some of the figures you created and comment on the procedure you followed to replicate the results

Note that not all papers will be fully reproducible

- not all papers share the dataset
- in this case you can use a synthetic dataset, another dataset or perform only part of the analysis
- if you are not sure ask me!

Option 2: Analyzing a Dataset

If you go for option 2 you will apply some of the techniques we covered during the lessons/coding sessions to analyze a real network

- you can choose from a list of datasets I will upload on the website
- also in this case you have to submit both the code and a report

The minimal analysis that you are expected to submit will include

- degree distribution
- study of clustering and path length
- at least one centrality measure
- community detection
- a nice plot of the network

Other analysis you could perform are for instance: network robustness study, using null models, comparing different centrality measures …

Some Considerations

Both options have pros and cons

- Option 1
  - what you have to do is more clear
  - reproducing a paper is a nice exercise
  - however the code may be harder to write
- Option 2
  - you have more freedom to focus on the techniques you liked the most
  - it's more close to what you experiment while doing research
  - however you don't have a clear path to follow

Option 1 will require more time to understand the paper and replicate the code, but less time to write the report
Option 2 will require some time to understand the data, not very much time to write the code, but more effort to interpret the results and write the report

Conclusions

Basic Definitions

Multilayer networks are a set of different networks, each belonging to a different layer, with nodes that can be connected also between layers. Examples include public transport, neural networks, social networks and many more.

Multiplex Networks

Multiplex networks are a subclass of multilayer networks, where the same set of nodes is found in every layer. We introduced the main properties of these networks and we generalized measures such as clustering and PageRank.

Epidemic Spreading

Epidemics spread on multiplex networks following different transport routes. Typically the network with the lowest epidemic threshold dominates the dynamics. Also other epidemic phenomena can be modeled in this way.